

# COHERENCE OF SPACE CHARGE VIBRATION AND PARAMETERS OF ELECTRON GUNS

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## Abstract

Space charge effect always determines the motion of particles in electron guns. Coherence of space charge vibration leads to oscillation of the emittance along a gun or a charge affected beamline. This phenomenon is closely related to a technique known as emittance compensation. It has been considered in the paper. The optimal parameters of guns and the expected emittance of the beam from the optimal ones have been estimated and scaled.

## INTRODUCTION

Emittance compensation technique has been mentioned first probably in [1]. It was explained and developed further in [2] and other papers. The two basic effects, caused by the longitudinal nonuniformity of charge density and the transverse one, and their combination in uniform and nonuniform beamlines were considered in [3] - [6], also with accelerating and bunching. The main results of the latter works is that both effects separately or together can be compensated, the charge phase advance through the beamline should be  $2n\pi$  ( $n$  is integer) and the focusing should be optimal. Then the normalized emittance dilution is well estimated as

$$\epsilon_n \cong \epsilon_c x_e \sqrt{\frac{I}{I_0 \beta \gamma}}, \quad (1)$$

where  $x_e$  is the rms size of the beam at the entrance;  $I$  is the peak current;  $I_0 = 4\pi \cdot mc^2 / Z_0 |e|$ ,  $\approx 17.045$  kA for electrons;  $\beta = v/c$ ;  $\gamma = 1/\sqrt{1-\beta^2}$ ;  $v$  is the longitudinal velocity; and  $\epsilon_c$  is the dimensionless coefficient depended on the type of the beamline.

In this paper we consider electron guns in the same view. We take into account only macroscopic space charge effect and neglect thermal and grid emittance. The main difference between a gun and a beamline is the presence of metallic electrodes near the emitter. Their charge depends on the one of the beam and generates comparable fields, so exclusion of near-cathode electrodes from simulation of beam motion in a gun causes lost of accuracy.

## EMITTANCE DILUTION IN GUNS

### Phenomena and Basic Scaling

If the emitter is round and the beam is homogeneous and stationary, the gun geometry can be optimized so that the space charge effect doesn't affect the emittance, as in the well known Pierce gun. If the beam is not longitudinally uniform, the transverse phase portraits of its slices differ and their emittances are not zero. Let's consider

these phenomena and estimate the total emittance.

Particle motion in the same gun is similar if its voltage and current meet Child-Langmuir law  $I \propto U^{3/2}$ . In this case the emittance (not normalized!) doesn't depend on the current [5] 4.1. At the same time, the brightness is  $I/\epsilon_n^2 \propto \sqrt{U}$ . If all the dimensions of a gun are changed proportionally, its quality factor  $\epsilon_c$  preserves while its brightness is  $\propto \sqrt{U}/r^2$ . Thus, one should find  $\epsilon_c$  and the optimal compensation beamline for any gun.

### Charge Amplitude and Phase

General equation of small charge vibrations has been derived in [3] (3). It generates a transformation matrix between two arbitrary points of a beamline [5] (2.16). The charge vibration phase is defined in [5] (2.19). Now we can define the differential characteristics of a bunch [5] 4.1, [6]. The local charge phase is

$$\varphi = \arctan\left(\frac{-C'x}{C\sqrt{j}}\right) = \arctan\left(\frac{\frac{dx'}{dI} - \frac{x'}{x} \frac{dx}{dI}}{\left(\frac{1}{2I} - \frac{1}{x} \frac{dx}{dI}\right)\sqrt{j}}\right), \quad (2)$$

where  $x$  is the rms-size of a slice,  $j = II_0$ ,  $C$  and  $C'$  are the transformation matrix elements associated with cos-like trajectories. The quadrant is chosen so that the signs of  $\sin\varphi$  and  $\cos\varphi$  coincide the ones of the numerator and the denominator respectively.  $x$  and  $x'$  are considered as functions of the slice current  $I$ .

It is also useful to define the relative amplitude of charge vibrations to estimate emittance dilution in a compensation beamline [5] 4.1, [6] (5):

$$a = \sqrt{(C'x/\sqrt{j})^2 + C^2}. \quad (3)$$

Then the relative amplitude of a slice is [5] (4.10), [6] (13):

$$A = \sqrt{\left(\frac{2I}{x} \frac{dx}{dI} - 1\right)^2 + \frac{1}{j} \left(2I \left(\frac{dx'}{dI} - \frac{x'}{x} \frac{dx}{dI}\right)\right)^2}. \quad (4)$$

### Basic Gun

A simple diode gun has been simulated first. Its geometry is shown in fig. 1. The emitter radius was 5 mm, the distance between the electrodes was 123 mm, while the beam was observed at 200 mm from the cathode. The perveance was very close to the "natural" one, so the optimal current was 2 A at 300 kV. SAM simulation code [7] was used to calculate beam motion in the gun. As usually for emittance compensation, a bunch has been divided by slices, and each slice was considered independently as a steady-state beam. The current density at the cathode was always homogeneous.

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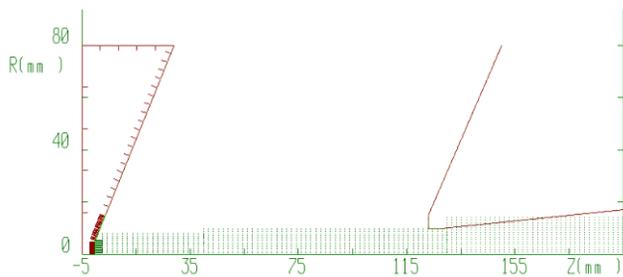


Figure 1: The geometry of the basic gun, red solid lines are electrodes.

The calculated beam parameters depending on the beam current are depicted in fig. 2. They were calculated by the following formulae:

$$x = \sqrt{\langle x^2 \rangle},$$

$$x' = \langle xx' \rangle / \sqrt{\langle x^2 \rangle}, \quad (5)$$

$$\varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}.$$

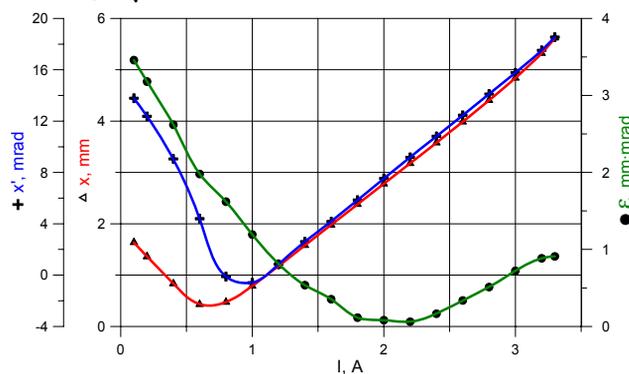


Figure 2: Beam parameters vs. beam current: rms-size, its derivative and emittance.

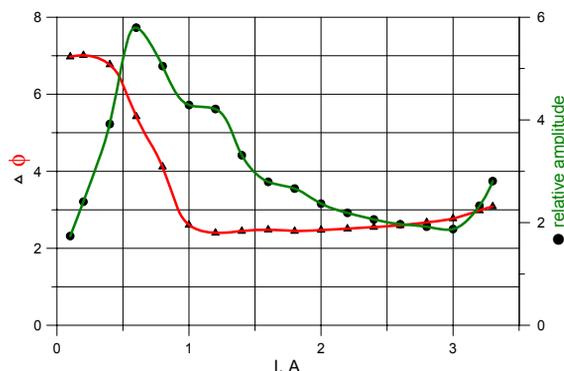


Figure 3: Charge phase and relative amplitude vs. beam current.

The dependencies of the phase and the amplitude on the current for the mentioned gun are shown in fig. 3. It is seen that the phase is almost constant within current limits from 1 to 3 A and its value is  $\approx 2.5 \approx 0.8\pi$ . Thus, if an ideal uniform beamline (where the phase advance doesn't depend on the amplitude) with the phase advance  $\approx 1.2\pi$  is placed after the gun, one should expect the minimal emittance. The following questions are still left: (i) what peak current of a bunch gives the minimum emittance in

this system, (ii) which slice should be matched to the compensation beamline, and (iii) what is the optimal phase advance of the latter.

Each slice was considered as a number of particles, representing the motion of the appropriate ring. The longitudinal charge distribution was presumed as Gaussian. An ideal beamline with phase advance  $\varphi$  matched with a slice with the current  $j_0$  and the state  $x_0$  and  $x_0'$  has the following produces the following linear transformation:

$$x' \rightarrow x' - x \cdot (x_0' / x_0),$$

$$\begin{cases} x \rightarrow x_0 + (x - x_0) \cos \varphi + x_0' x_0 / \sqrt{j} \sin \varphi, \\ x' \rightarrow x' \cos \varphi - (x / x_0 - 1) \sqrt{j} \sin \varphi. \end{cases} \quad (6)$$

Then  $\varphi$  and  $j_0$  were optimized for each peak current value. For example, the minimum for 2.2 A is 1.03 mm-mrad and is situated at  $I = 1.043$  A and  $\varphi = 3.91 \approx 1.246\pi$ . The phase is quite near the predicted value.

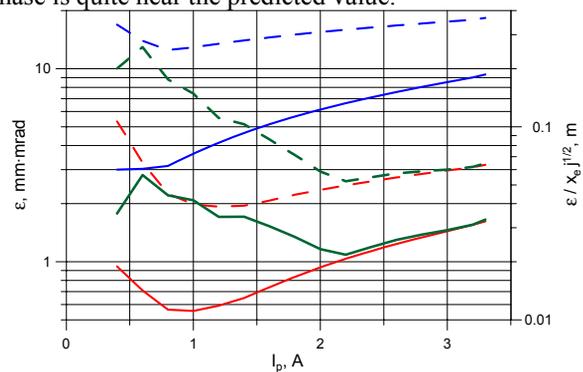


Figure 4: Emittance (solid) and  $\varepsilon_c$  (dashed) vs. peak current. Blue – no compensation, red – ideal beamline, green – non-uniform beamline.

Plots of the emittance and the quality factor vs. the peak current of a Gaussian bunch with and without the ideal compensation beamline are depicted in fig. 4. One can see that (i) the beamline reduces the emittance six times and (ii)  $\varepsilon_c$  weakly depends on the peak current. A non-ideal compensation beamline increases the quality factor by ([3]: (5), (15), Table 1)

$$0.037 \frac{x_0}{x_e} A^3 \frac{\varphi}{2\pi} \approx 0.13, \quad (7)$$

where all the parameters belong to the matched slice. Thus, imperfection of the compensation beamline can weaken the compensation significantly.

Consider then a simplest non-uniform beamline that consists of two gaps of different lengths and a thin lens between them. Then the motion of charged rings through it is described by the following equation [3] (12):

$$x'' = \frac{2\tilde{j}}{x}, \quad (8)$$

where  $\tilde{j}$  is the current within the ring. The motion is presumed as laminar. Both lengths and the lens strength were optimized. The optimal emittance and  $\varepsilon_c$  are also placed in fig. 4. It is clear that both beamlines give almost equal results if the peak current is bigger than 2 A.

### Other Guns

Seven other guns have been simulated in the same way to investigate the influence of the gun geometry. In five first ones the emitter radius was the same while the length was varied. The electrodes were shaped to make perfect electric field. Additional electrodes were added to the guns "Short 2", "Long 2" and "Long 3" to equalize their perveance to the primary one. The cathode electrode in "Long 3" is planar. The optimal current in all the cases above was  $\approx 2$  A. The two last guns are similar to first half-cells of 1.3 GHz photo-electron RF guns. The emitter radii in these cases are 2 mm. The results are placed in table 1. The values in parentheses in the second column mean the observation points. The last three columns contain quality factors of non-compensated guns, with ideal lines and with non-uniform lines respectively.

Table 1: Simulated guns parameters

Gun	Length, mm	U, kV	$\varepsilon_c$ n/c, m	$\varepsilon_c$ ideal, m	$\varepsilon_c$ n/u, m
Basic	123 (200)	300	0.3	0.05	0.065
Short	61.5 (100)	150	0.3	0.04	0.05
Short 2	61.5 (100)	300	0.35	0.055	0.15
Long	246 (400)	850	0.43	0.085	0.14
Long 2	246 (400)	300	0.26	0.04	0.065
Long 3	246 (400)	300	0.4	0.07	0.023
RF 1	50 (70)	1000	0.28	0.05	0.02
RF 2	50 (70)	2000	0.28	0.05	0.02

### PARAMETERS OF EXISTING GUNS

It is interesting to compare the theoretical parameters above with ones achieved in real guns. One can find them in table 2. Only guns demonstrated record brightness were selected there. The lengths and voltages at the first half-cell of guns were used to calculate  $\varepsilon_c$ .

Table 2: Existing guns parameters

Gun	Ref.	$E_{kin}$ , MeV	$I_p$ , A	$x_e$ , mm	$\varepsilon_n$ , mm-mrad	$\varepsilon_c$ , m
LCLS*	[8]	2.0	50	1	0.83	0.034
DESY	[9]	1.05	100	1	2.7	0.06
" *	"	"	43	0.57	1.7	0.10
BNL IV	[10]	1.33	67	0.3	1.4	0.14
"*	"	"	133	"	1.35	0.095
MIT	[11]	0.27	50	0.5	3.5	0.14

\* Guns with bunch shaping.

The best  $\varepsilon_c$  of existing guns are close to ones estimated above, although exceed them. On the one hand, bunch shaping permits to reduce the emittance, as the Gaussian longitudinal distribution was considered in estimations. On the other hand, the rectangular transverse distribution in estimations yields better emittance than the Gaussian one typical for photo-electron guns. Also the temperature of photo-electrons was not taken into account. For example, the emittance of LCLS gun is lower than the temperature limit [9]:

$$\varepsilon_n = x_e \sqrt{\frac{2E_k}{3m_e c^2}} \cong 1 \text{ mm} \cdot \text{mrad} . \quad (9)$$

There could be two explanations: (i) the light spot size was reduced and/or (ii) the temperature of electrons leaving copper cathode is less than of ones from CsTe cathode (0.8 eV by [9]).

### CONCLUSIONS

- Appropriate emittance compensation applied to an electron gun always improves emittance by factor 3...15.
- Both effects of the longitudinal nonuniformity of charge density and the transverse one can be compensated well.
- The expected normalized emittance of a well-designed gun with an optimal compensation beam-line is

$$\varepsilon_n \approx 0.02 \dots 0.07 x_e \sqrt{\frac{I}{I_0 \beta \gamma}} = 0.01 \dots 0.035 r_e \sqrt{\frac{I}{I_0 \beta \gamma}} , \quad (10)$$

where  $x_e$  is the rms beam size at the emitter and  $r_e$  is the emitter radius.

- Quality factors  $\varepsilon_c$  of best existing guns approach theoretical limitation. Further improvement is possible with beam shaping.

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