HIGH ORDER MODE ANALYSES FOR THE ROSSENDORF SRF GUN

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Abstract

High Order Modes (HOM) excited by the beam in a superconducting RF gun (SRF gun) could destroy the quality of the electron beam. This problem is studied on the base of frequency domain description by considering of the equivalent RLC circuit contour for each HOM, periodical excited by a pulsed current source [1].

Expression for the voltage, the field amplitude and the phase of the excited HOM has been obtained. The equations for the coupling impedances of monopole TM-HOM and TE-HOM in the RF gun cavity has been derived. In this calculation the change of the particle velocity due to acceleration is taken into account.

Resonance frequencies, coupling impedances, unloaded and external quality factors, excitation voltages and field distributions for each HOM including trapped HOM are calculated for Rossendorf SRF gun up to the frequency of 7.5 GHz, using the complex field solver CLANS. The dependence of the calculated parameters from a cavity deformation has been studied.

The influence of the seven most dangerous HOM on the beam quality has been estimated by particle tracking using the ASTRA code.

DERIVATION OF THE THEORY

In the following considerations the electron beam is represented by a pulsed current with the repetition round frequency Ω. The time dependence of each pulse is given by the Delta-function [2].

Excitation Voltage

The voltage ΔU=ω·(R/Q)·q of each HOM is excited when a bunch pass through the cavity. It can be presented as a vector in the complex space, which rotates with the round resonance frequency ω of the HOM:

$$U_0(t) = \Delta U \cdot e^{j\omega t - \omega t/2Q_L}\,, \qquad (1)$$

whereas R/Q is coupling impedance of the HOM; $\omega/2Q_L$ is attenuation factor; Q_L is loaded quality factor of the HOM and q is bunch charge.

The voltage U_N is the result of N bunch-HOM interaction, where the voltage is changed each time by the amount ΔU and N phase rotation ϕ between the interaction moments. This phase angle is determined by the HOM and the bunch repetition frequency and given by *ϕ=2π·Fraction(ω/Ω)*. Finally for U_N one obtains:

$$U_N(t) = \Delta U e^{i(\omega t+\varphi)} \sum_{n=0}^{N} \left(e^{i\cdot 2\pi\omega/\Omega - 2\pi\omega/\Omega\cdot 2Q_L}\right)^n . \qquad (2)$$

where φ is phase angle at that time, when the bunch starts at cathode of the SRF gun. In the stationary limit N→∞ the amplitude of excitation voltage U(t) becomes constant. From (2) follows:

$$U(t) = U_N(t)\big|_{N\to\infty} = \frac{\Delta U}{1 - e^{i\cdot 2\pi(\omega/\Omega)} \cdot e^{-2\pi(\omega/\Omega)/2Q_L}} \cdot e^{i\cdot(\omega t+\varphi)}\,, \qquad (3)$$

The time evolution of U in the complex plane is graphical represented in Fig.1.

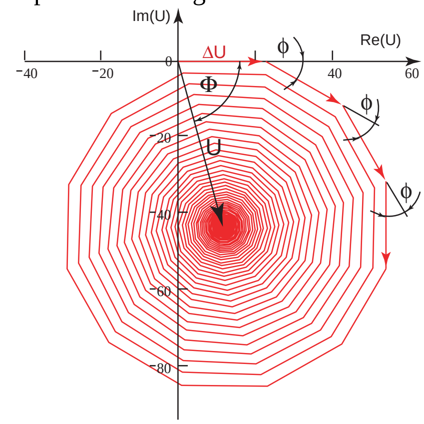

Figure 1: Evolution of excitation voltage of all bunches for low Q values of the HOM.

In the real cavity this excited voltage is equal to the change of the particle energy due to the interaction with the HOM field

$$U \equiv \int dW/e, \qquad (4)$$

whereas W is the change of the kinetic particle energy which can be measured in an experiment.

TM-HOM Excitation by Accelerated Beam

The energy exchange between the HOM and the particles moving through the cavity is defined as:

$$dW = \frac{1}{m\gamma}\cdot\left(dP_\theta \cdot P_\theta + dP_r \cdot P_r + dP_z \cdot P_z\right) = \frac{1}{m\gamma}\cdot\left(d\vec{P}\cdot\vec{P}\right) \qquad (5)$$

Here P_z, P_θ, P_r are particle momentum components which commonly depend from the accelerating and focusing fields in the cavity; The components of the $d\vec{P}$ are the momentum transfer due to the HOM-particle interaction. They are given by:

$$d\vec{P} = e\cdot\vec{E}\cdot dt + \frac{e}{m\gamma}\cdot\left[\vec{P}\times\vec{B}\right]\cdot dt\,, \qquad (6)$$

whereas e and m are the electron charge and the electron mass respectively, γ is relativistic factor, E and B is the electric and magnetic RF field of the HOM.

For the monopole TM HOM field at cavity axis the equations B=0; $\bar{E} = E_z(z)\cdot\sin(\omega t+\varphi)$ are hold. From (4) and (5) follow:

$$U = \int_0^T E_z(z)\cdot\sin(\omega t+\varphi)\cdot\beta(t)c\cdot dt \qquad (7)$$

The integration interval T is the time of HOM-particle interaction. $E_z(z)$ is the electric field distribution of the HOM on the cavity axis; β(t) and z=z(t) are the particle velocity and its coordinate determined by the acceleration and can be obtained from (6) if $\bar{E}$ and $\vec{B}$ are the accelerated and focusing fields. The phase φ must be equal to phase of excited HOM when a multiple harmonics n·Ω of the repetition is equal to the HOM frequency ω. In that case U has the maximal possible value. By this consideration and using (7) we obtain for U and φ

$$U=\sqrt{\left[\int_0^T E_z(z)\cdot\sin(\omega t)\cdot\beta c\cdot dt\right]^2+\left[\int_0^T E_z(z)\cdot\cos(\omega t)\cdot\beta c\cdot dt\right]^2} \quad (8)$$

$$\varphi=\arctan\left[\int_0^T E_z(z)\cdot\sin(\omega t)\cdot\beta(t)dt \Big/ \int_0^T E_z(z)\cdot\cos(\omega t)\cdot\beta(t)dt\right] \quad (9)$$

dJ = dq·U – exchanging of HOM field saved energy with the particle energy according to energy balance;

dU = ω(R/Q)·dq – HOM voltage excited by the particles in the equivalent circuit;

From previous follow dJ=U·dU/ω(R/Q), integration gets:

$$R/Q=\frac{U^2}{2\omega J} \quad (10)$$

The R/Q values of the Rossendorf SRF gun calculated for different accelerating gradients are presented in table 1. Its values are decreased for a low accelerating gradient.

Excitation of the TE Mode by Accelerated Beam

The TE mode has only one field component $\bar{E}\equiv E_\theta$, which is responsible for the energy transfer between beam and HOM field. Therefore from (4) and (5) follows:

$$dW=\frac{e}{m\gamma}\cdot\left(\vec{P}\cdot\vec{E}\right)\cdot dt+\frac{e}{(m\gamma)^2}\cdot\left(\vec{P}\cdot\left[\vec{P}\times\vec{B}\right]\right)\cdot dt=\frac{e}{m\gamma}\cdot P_\theta\cdot E_\theta\cdot dt \quad (11)$$

$P_\theta/m\gamma$ is the azimuthally velocity component. It is only different from zero in the present of an external magnetic field. Only in this case the TE HOM can be excited. Therefore we propose the feed of a second TE mode with the axis field $B_{zo}(z,t)=B_{zo}(z)\cdot\sin(\omega_o t+\varphi_o)$. This mode has been used in [3] for focusing and emittance compensation of the electron beam. In the paraxial limit assuming r=const the pulse is given by:

$$P_\theta(t)\cong -e\cdot\frac{r}{2}\cdot B_{zo}(z)\cdot\sin(\omega_o t+\varphi_o)\cdot \quad (12)$$

We assume here $B_{zo}(z) >> B_z(z)\cdot\sin(\omega t+\varphi)$ – the field of excited TE mode. In the same approximation we obtain:

$$E_\theta=-\frac{r}{2}\cdot\frac{\partial B_z(z)\cdot\sin(\omega t+\varphi)}{\partial t}=-\frac{r}{2}B_z(z)\cdot\omega\cdot\cos(\omega t+\varphi) \quad (13)$$

From (11), (12) and (13) follows:

$$U=\frac{e}{m}\cdot\frac{r^2}{4}\cdot\omega\cdot\int_0^T\frac{1}{\gamma}B_{zo}(z)\cdot B_z(z)\cdot\sin(\omega_o t+\varphi_o)\cdot\cos(\omega t+\varphi)\cdot dt \quad (14)$$

After averaging the radius r by $<r^2>=\rho^2/2$ we are able to calculate the coupling impedances for the TE modes using the same arguments as in the proceeding sections:

$$R/Q_{TE}=B_{\max}^2\cdot B_{o\_\max}^2\cdot\rho^4\cdot\omega\cdot f(\omega,\omega_o,\varphi_o)/2J \quad (15)$$

The expression $f(\omega,\omega_o,\varphi_o)$ is proportional to $\exp(-(\omega+\omega_o))/\gamma^2$. B_{max} and B_{o_max} are the maximum axis magnetic fields for the excited HOM and for the external focusing TE HOM respectively. Usually the values of $(R/Q)_{TE}$ are in the order of several nanoohms.

The excitation of TE HOM is possible only in the present of external magnetic field. The phase of excitation TE HOM field φ depends from phases φ_o of the external TE mode as $\sim\varphi_o$. If the resonance frequency of the external TE mode is not equal to a multiple of the repetition frequency ($\omega_o\neq k\Omega$) then these phase φ changes with the bunch number n as $\varphi(n)\sim 2\pi\cdot(\omega_o/\Omega)\cdot n$. Assuming this n dependence in the vector sum of the excited voltages (see fig.1) we find, that the voltage excitation of TE HOM can be neglected. These modes could be of interest in the presence of a static magnetic field only.

HOM OF THE ROSSENDORF SRF GUN

In the next step we have calculated the field distribution of the HOM for the 3 ½ cell gun cavity using the CLANS field solver. The external loading caused by the circuits disposed immediately after the choke (Q_{CC}); by the Cu stem surface loses (Q_{Cu}) and by the beam pipe (Q_{BP}), which is responsible for the trapped mode dumping, is taken into account. The resonance frequency, the coupling impedance, the unloaded and the three external quality factors and the field distribution for the TM monopole HOM up to 7.5 GHz has been calculated. The corresponding values are given in Tab.1. Fig. 2 shows the field pattern of a trapped TM HOM with the mentioned three loads.

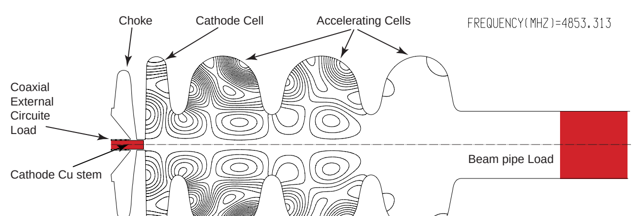

Figure 2: The 3 ½ cell cavity of the SRF gun with the field pattern of a trapped TM HOM.

These loads are simulated by low conducting materials with ε=1, μ=1, $tg\delta_\varepsilon=1.5$, $tg\delta_\mu=1.5$. All incident RF power is dissipated without of any reflecting.

The unloaded quality factor Q_o is equal G/R(f), where G is geometry factor of the HOM; $R(f)=R_{1300}\cdot(f/1300MHz)^2$ is the niobium surface resistance with R_{1300}=23.2 nOhm at a temperature of 2K.

The cathode Cu stem surface resistance is defined as $R_{Cu}(f)=2\pi\cdot(10^{-7}\cdot f/\sigma_{77K})^{1/2}$, where $\sigma_{77K}=5.4\cdot10^8$ 1/Ohm·m is specific conductivity of copper at 77K.

Now we are able by means of (3) to calculate the excitation voltage U in dependence on the repetition frequency Ω. The result is given in Fig. 3. The upper part contains the pass band modes. The repetition frequency Ω = 26 MHz is far from the resonance condition and the excitation voltages for the HOM are lower than 25V. Also in the lower part of the picture no harmonics of 26 MHz is close to a HOM frequency.

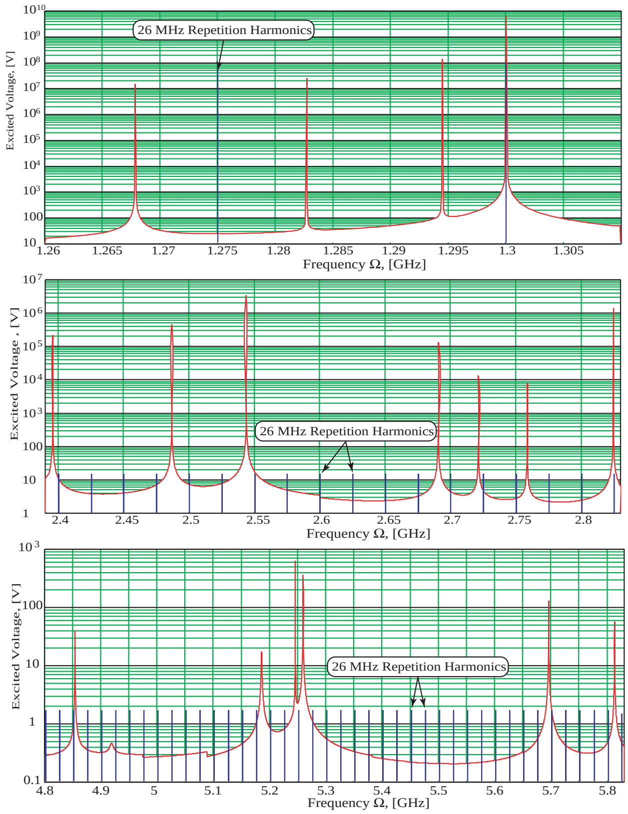

Figure 3: The voltages of the TM HOM.

Table1: Field parameters of the HOM. Trapped HOM marked by blue color. $1/Q_L = 1/Q_o + 1/Q_{CC} + 1/Q_{Cu} + 1/Q_{BP}$.

ω/2π MHz	Q_L	Q_o	R/Q, Ohm		
			β=1	E=25	E=12.5
1268	2.61e8	6.16e9	29.5	21.8	14.55
1283	3.65e9	9.98e9	4.35	2.98	1.694
1294	6.78e9	9.01e9	13.3	9.15	5.196
1300	9.35e9	1.0e10	168.6	170.2	171.2
2395	2.16e8	4.93e9	8.07	8.05	7.948
2487	1.13e7	4.27e9	16.8	12.6	9.957
2544	4.44e7	4.39e9	21.0	20.0	18.94
2690	7.36e6	4.67e9	5.86	5.82	4.536
2721	1.95e6	5.09e9	3.11	2.49	1.706
2758	1.63e6	5.43e9	1.55	1.28	1.243
2824	6.52e7	3.26e9	18.5	11.0	5.395
4043	4273	2.78e9	1.48	1.16	1.086
4853	4.48e4	2.36e9	3.49	1.27	0.22
5184	6631	2.35e9	0.87	0.89	0.641
5244	8.06e5	1.9e9	7.66	2.59	0.195
5257	9.07e6	2.22e9	1.66	1.14	1.383
5694	7.34e4	1.11e9	9.17	3.22	0.713
5811	3.63e7	1.57e9	4.13	1.57	0.291
5944	2.92e4	1.58e9	0.39	0.38	0.358
5992	745.4	2.57e9	1.21	0.89	0.657
6077	6.77e4	1.67e9	0.29	0.18	8.7e-2
6128	5060	1.83e9	0.44	0.37	0.253
6668	1.62e4	1.69e9	2.47	0.25	6.7e-3
7131	3.47e4	1.00e9	0.46	0.22	3.4e-2
7213	1.38e5	1.23e9	0.73	0.34	0.105
7467	4464	1.69e9	29.5	21.8	14.55

DEPENDENCE OF THE HOM SPECTRUM FROM THE CAVITY SHAPE DETUNING

In this section we examine the stability of the obtained result by a simple model. SRF gun is equipped by the mechanism allowing detuning of main resonance frequency of 1300 MHz. The detuning is being due to changing of total length of 3.5 cell cavity. We change the cavity shape by changing the inclination of the straight parts of the cavity cells marked by red colour in Fig.4.

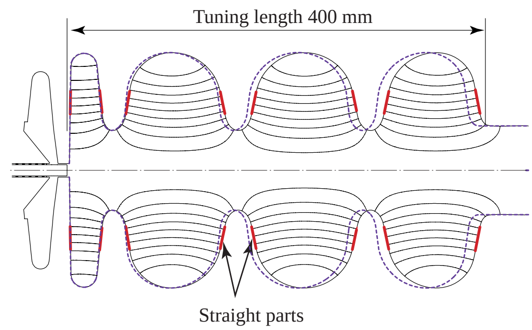

Figure 4: Change of the cavity shape.

Together with the cavity shape changes the HOM frequencies ω and loaded quality factors in dependence of the length variation δ. For a given repetition rate the voltages of (3) are also function of δ. In Fig.5 we plot the sum of all HOM voltages for different repetition rates and constant current

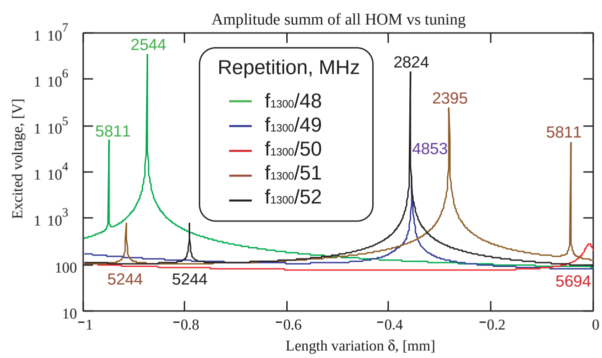

Figure 5: Amplitude sum of all TM HOM excited by 2 mA beams with different repetition frequencies. f_{1300} is accelerating mode frequency changing versus δ.

Again we obtain typical resonance behavior. But in this case the HOM voltages can achieve several MV if a HOM frequency ω approaches to a harmonics of the repetition frequency. This result shows that a certain HOM can destroy the beam quality at a special cavity shape or cavity length and a special repetition rate. We can avoid these approaches by switching between different repetitions frequencies.

In the table 2 the change of HOM parameters in 3.5 cell SRF gun cavity versus of –1 mm detuning is presented. The loaded quality factors changes significantly. It shows the high sensitivity of the trapped HOM with respect to the cavity deformation.

Table 2: The change of HOM parameters in the 3.5 cell SRF gun cavity at δ=–1 mm.

ω/2π, MHz	Δ ω/2π, MHz	ΔQ/Q, %	ΔR/Q, Ohm
1268	-0.72	-4.01	0.05
1283	-0.72	-5.388	0.046
1294	-0.68	-3.958	0.87
1300	-0.66	-0.465	-0.7
2395	0.86	1.431	0.054
2487	1.05	-0.0088	-0.01
2544	1.29	-1.075	-0.22
2690	0.63	-0.6233	-0.017
2721	0.73	-1.786	0.016
2758	0.91	-2.017	0.007
2824	0.76	-0.848	-0.07
4043	1.34	-32.98	-0.039
4853	2.21	-5.549	-0.011
4918	2.15	-2.612	-0.0158
5184	6.79	25.23	0.093
5244	5.23	5.901	-0.202
5257	4.17	-94.95	0.052
5694	2.56	1.148	-0.162
5811	9.36	15.88	0.274
5944	1.56	9.594	-0.0224
5992	3.89	-17.51	0.016
6077	1.31	-30.89	-0.0133
6128	1.96	2.292	0.00468
6668	7.02	405	-0.0075
7131	0.69	-0.9913	-0.001
7213	1.71	38.85	-0.0113
7467	2.9	-3.586	0.0028

THE INFLUENCE OF THE HOM ON THE BEAM PROPERTIES

The influence of HOM on bunch properties has been estimated by particle tracking using the ASTRA code. For this purpose the field of the seven most dangerous HOM was added separately in ASTRA input file.

Bunch charges of 77 and 308 pC, an accelerating field of 12.5 MV/m and a laser spot size of 4 mm with σ=3.5 ps Gaussian temporal distribution has been used in the calculations.

In the first step the optimal values of the launch phase and the field strength of a focusing solenoid disposed after the RF gun are determined by the emittance minimization without HOM fields. In the next step the influence of the HOM with an axis field E_{max}=1 MV/m is calculated. The results are presented in table 3. The values Δε and $\Delta\sigma_w$ are the changes of the emittance and the energy spread with respect to the undisturbed values.

It is interesting, that for certain HOM and for certain E_{max} values the transverse emittance decreases. The energy spread is always diminishing. In figure 6 the emittance and energy spread dependencies from excited voltage are shown for 2824 MHz HOM.

Table 3: Influence of TM HOM for E_{max}=1 MV/m on the beam properties.

ω/2π	77 pC			308 pC		
	Δε, %	$\Delta\sigma_w$ %	U, KeV	Δε, %	ΔW, %	U, KeV
2395	1.65	-0.37	32.7	1.11	-4.12	32.7
2543	2.27	-0.85	48.5	2.29	-5.46	48.3
2824	-11.7	-14.5	132*	0.93	-14.4	19.
4853	-0.8	-4.14	5.3	0.02	-4.8	5.11
5693	-1.11	-5.88	6.63	2.31	-7.11	6.13
5811	1.33	-6.82	6.01	0.39	-5.05	5.75
5992	1.33	0.29	6.88	3.58	0.33	6.69
Without HOM	ε, μm	σ_w, KeV	-	ε, μm	σ_w, KeV	-
	0.46	17.9	-	1.47	24.5	-

* The excitation corresponds to minimum of emittance at E_{max}=7.72 MV/m

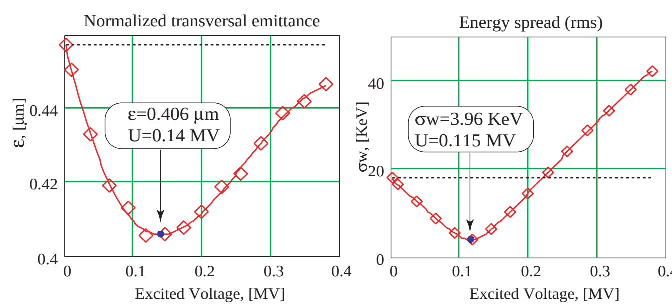

Figure 6: Emittance and energy spread versus excited voltage of the 2824 MHz HOM.

CONCLUSION

At a repetition (cw) frequency of 26 MHz the monopole TM-HOM of the 3 ½ cell SRF gun cavity have no influence on the beam quality. But a relatively small deformation of the cavity can change the situation drastically. In order to avoid this situation it would be useful to have a possibility to change the repetition frequency in a dedicated area.

It is very interesting that for a specific parameter field the excitation of special monopole TM-HOM improve the beam quality.

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