

FEL WITH OROTRON TYPE FEEDBACK*

N.S. Ginzburg[#], A.M. Malkin, N.Yu. Peskov, R.M.Rozental, A.S. Sergeev, IAP RAS, Nizhny Novgorod, Russia

K. Kamada, R. Ando, Kanazawa University, Kanazawa, Japan.

Abstract

In this paper we discuss a novel scheme of the free-electron laser in which the electromagnetic wave excited by the electron beam is coupled to the mode of a two-mirror quasi-optical resonator by means of corrugation one of the mirrors. The described scheme allows to combine selective properties of quasi-optical resonator with relativistic frequency up-conversion of a free electron laser.

INTRODUCTION

In microwave electronics orotron [1] is widely used as a source of radiation in millimeter and submillimeter wavebands. In this device an electron beam is coupled to a mode of quasi-optical resonator by means of the periodical grating one of the resonator mirrors. In our paper we discuss a relativistic modification of such a device which can be useful for providing spatial coherence radiation from large size electron beam

In orotron (Fig.1a) a rectilinear electron beam moving along the periodical grating on the mirror of a quasi-optical cavity to be in Cherenkov synchronism to a spatial harmonic of the resonator mode. A novel FEL scheme (Fig. 1b) is suggested in which the slow wave structure of orotron is replaced by shallow Bragg corrugation which provides the coupling of the transverse (with respect to direction of beam propagation) mode of a quasi-optical resonator with longitudinally propagating wave. The latter can be excited by the relativistic electron beam wiggling in undulator field in condition of Doppler frequency up-shift. It should be noted that such a scheme is similar to FEL based on coupling between the quasi cutoff and the propagating mode in cylindrical waveguides which was described in [2,3] (above interaction mechanism will be tested on a 8 mm FEL based on high current accelerator 0.7 MeV/2 kA/200ns at Kanazawa University, Japan). In considered case we replace the waveguide by open quasi-optical resonator and suggest that FEL will be driven by large size sheet electron beam. As in [2] the transverse propagating (cut-off) mode provides the feedback in the system thus leading to self-excitation, while the longitudinal propagating wave is responsible for energy extraction in the steady-state regime of operation. But additionally as will be shown in this paper in considered geometry the cut-off mode provides spatial coherence radiation from sheet electron beam.

THE ELECTRODYNAMIC MODEL

We consider a resonator consisting of two metallic mirrors. One of the mirrors is corrugated with a period d . This corrugation provides coupling between the modes possessing longitudinal (in z direction, see Fig.1b) wavenumbers differing by factor $\bar{h} = 2\pi/d$. In the simplest case of the sinusoidal corrugation

$$b(z) = b_0 \cos(\bar{h}z),$$

where $b(z)$ is the deviation of the mirror surface, $2b_0$ is the corrugation depth. In the case of more complicated corrugation (for example, meander type corrugation) b_0 is equal to the amplitude of the first space harmonic. We assume corrugation to be shallow in the wavelength scale, so that $\bar{h}b_0 \ll 1$.

We use coupling between the propagating in z direction wave and the q -th mode of the quasi-optical resonator (in the system with the closed waveguide [2] this mode corresponds to the quasi cut-off mode). Above coupling requires that the wavenumber of the longitudinal propagating wave $h = \omega/c$ and the wavenumber of the transverse propagating mode quasi-optical resonator ($q\pi/L_y$) should be equal to the translation vector of the Bragg structure:

$$h \approx \frac{q\pi}{L_y} \approx \bar{h},$$

where ω is the operation frequency, c is the velocity of light, L_y is the distance between the mirrors (in y direction).

Note that due to the symmetry of the considered system not only the forward $+z$ direction propagating wave is coupled to the transverse mode, but also a

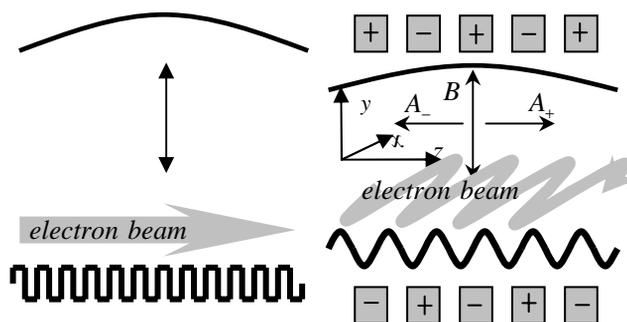


Fig.1. Schematic of the orotron (a) and FEL with orotron-type feedback (b).

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[#]ginzburg@appl.sci-nnov.ru

backward propagating one. This wave in the FEL does not interact with electrons but provide an additional channel of extraction of electromagnetic energy out of the system.

We present the electromagnetic fields of the coupled modes in the following form, separating its transverse (over y -axis) structures and the slowly varying amplitudes over z and x coordinates:

$$\vec{E}_\pm = \text{Re}\left(A_\pm(t, z, x)\vec{E}_A(y)e^{\mp ih_x z}e^{i\omega t}\right) \quad (1)$$

$$\vec{E} = \text{Re}\left(B(t)f(x, z)\vec{E}_B(y)e^{i\omega t}\right) \quad (2)$$

Here z is the coordinate along the corrugation vector, y is directed between the mirrors and x is the transverse coordinate parallel to the mirrors (fig.1b).

The coupling equations describing the scattering of the three modes in assumption of shallow corrugation can be derived from the Maxwell equations using the surface magnetic current technique developed in [4]. These equations include the equations for the propagating modes (1) and the equation of resonator mode excitation (2).

$$\pm \frac{\partial A_\pm}{\partial z} + \frac{1}{c} \frac{\partial A_\pm}{\partial \tau} + i\delta A_\pm = i\alpha B f(x, z) \quad (3)$$

$$\frac{\partial B}{\partial t} + \frac{\omega}{2Q} B = ic\alpha \iint f(x, z)(A_+ + A_-)dXdZ, \quad (4)$$

Here $\delta = \omega - \frac{q\pi c}{L_z}$ is the geometrical detuning from the wave synchronism,

$$\alpha = \frac{hb_0}{4L_y} \frac{1}{\sqrt{hL_z}}$$

is the coupling coefficient proportional to the depth of the corrugation, the factor Q describes the quality factor of the mode of regular two-mirror resonator (this factor takes into account both diffraction losses of the mode and Ohmic losses in the resonator walls).

We use the open boundary conditions for the amplitudes of the propagating modes:

$$A_+(z = L_z) = 0, \quad A_-(z = 0) = 0. \quad (5)$$

where L_z is the length of the mirrors in z direction. We also assume that the corrugation is shallow so that the perturbation of the quasioptical resonator modes is small which structure specifies by the function $f(x, z)$.

It should be noted that the spatial structure of the propagating waves $A_\pm(z, x)$ is not fixed and can be found from simulations.

FEL INTERACTION IN THE QUASIOPTICAL RESONATOR WITH CORRUGATED MIRROR

A FEL oscillator can be realised within the described electrodynamic system. We consider a sheet beam of electrons and wiggling in the periodical undulator field and interacting with synchronous A_+ wave propagating in $+z$ direction. Corresponding synchronism condition can be presented as

$$\omega - hv_\parallel \approx \Omega,$$

where Ω is the oscillation (bounce) frequency of the particles in the undulator field, v_\parallel is the electrons longitudinal velocity. Electrons wiggling parallel to the y -axis generate RF current, which density can be written in the form

$$j(x, z, t) = I_0(x)J(x, z, t), \quad (6)$$

$$J(x, z, t) = \frac{1}{\pi} \int_0^{2\pi} e^{-i\theta(x, z, t)} d\theta_0.$$

Here $I_0(x)$ is the unperturbed beam current density,

$$\theta = \omega t - hz - \int \Omega dt$$

is the slowly varying electrons phase in the field of synchronous wave, θ_0 is its value in the point of entry in the interaction space. Current (6) excites the synchronous wave so the current density enters the right hand part of Eq.(3) in a usual way.

Equations of electron motion can be written in the forms

$$\left(\frac{\partial}{\partial Z} + \frac{1}{\beta_\parallel} \frac{\partial}{\partial \tau}\right) \theta = \text{Re}(\hat{A}_+ e^{i\theta}) \quad (7)$$

Here we use the following dimensionless variables and parameters: $Z = zCh$, $X = 2x/L_x$, $\tau = \omega Ct$, β_\parallel is normalized electron velocity, $\hat{A}_\pm = e\kappa\mu A_\pm / mc\bar{\omega}\gamma_0 C^2$ is the propagating wave norm, $\theta = \bar{\omega}t - hz - \int \Omega dt$ is the electron phase in the field of the synchronous wave,

$$C = \left(\frac{eI_0}{mc^3} \frac{c\lambda^2 K^2 \mu}{4\pi^2 \gamma_0 N_A}\right)^{1/3}$$

is the Pierce parameter, μ is an inertial bunching parameter, K is the electron-wave coupling parameter proportional to the amplitude of transverse oscillation of electrons, N_A is the norm of the propagating mode.

Electron efficiency of the oscillator is defined by relations

$$\eta = \frac{C}{\mu(1-\gamma_0^{-1})} \hat{\eta}, \quad \hat{\eta} = \frac{1}{2\pi} \int_0^{L_x} \int_0^{2\pi} \left(\frac{\partial \theta}{\partial Z} - \Delta\right) \Big|_{Z=L} d\theta_0 dx.$$

We suppose the characteristic time of the changing of the resonator mode amplitude B to be substantially greater than the electrons transit time l/v_\parallel and forward and backward waves propagation time l/v_{gr} . In this case it is possible to assume that amplitude of the above mode is constant during intervals l/v_\parallel and l/v_{gr} and neglect time derivatives in the equations of electron motion (7) as well as the propagating wave excitation equations (3).

It is convenient to overwrite the coupled-waves equations in the form:

$$\begin{aligned} \frac{\partial \hat{A}_+}{\partial Z} + i\delta \hat{A}_+ &= i\alpha' B f(x, z) + J \\ \frac{\partial B}{\partial t} + \frac{\omega}{Q_{\text{mod}}} B &= i\alpha' \iint f_n(X) A_+ dXdZ \\ \left(\frac{\partial \theta^2}{\partial Z^2} \right) &= \text{Re}(\hat{A}_+ e^{i\theta}) \end{aligned} \quad (8)$$

These equations form the self-consistent set describing the dynamics of electron-wave interaction in the system under consideration.

The function $f(x, z)$ for the lowest mode of two-mirror resonator can be approximated as follows:

$$f(z) = \sin \frac{\pi z}{L_x} \sin \frac{\pi z}{L_z}.$$

Deriving the equations (8) we took into account that the backward wave A_- is not coupled to the electrons, consequently the equation for the backward wave with boundary condition (5) can be integrated. After substitution the result of integration the cutoff mode equation and the second integration one can get the value of losses coefficient which takes into account the scattering of the mode into the backward wave and write the equation (9) entering the modified Q-factor

$$Q_{\text{mod}} \approx \left(\frac{1}{Q} + \frac{2\alpha^2 \pi^2 L_z^2 \cos^2(\delta L_z / 2)}{(\delta^2 L_z^2 - \pi^2)^2} \right)^{-1}.$$

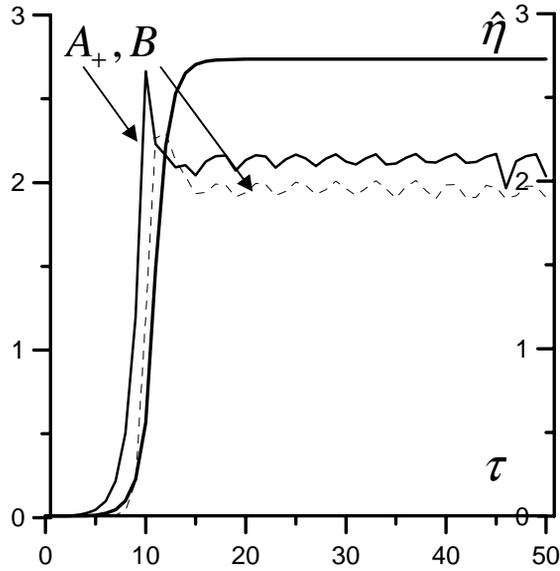


Fig.2. The process of establishment of steady-state oscillations.

The equations (8) were solved numerically. The results of simulations are presented in Fig.2,3 at $L_x = 2$, $L_z = 4.5$, $\Delta = -1$, $\delta = 1.7$, $\alpha' = 2$. In Fig 2 one can see the process of establishment of steady-state oscillation regime. Fig.3 demonstrates the transverse and

longitudinal structures of the propagating wave A_+ . It is important to note that output phase distribution of propagating wave $\arg A_+(x)$ does not depend on initial conation that actually means spatial synchronization of output radiation associated with this wavethrough the coupling with the cut-off mode B . Thus considered FEL scheme provides mode control in oversized microwave system. According to estimations the dimension of the system in the transverse (x and y) directions is sufficient for using above scheme for producing powerful spatially coherent radiation in short millimeter and submillimeter bands from moderately relativistic sheet electron beams.

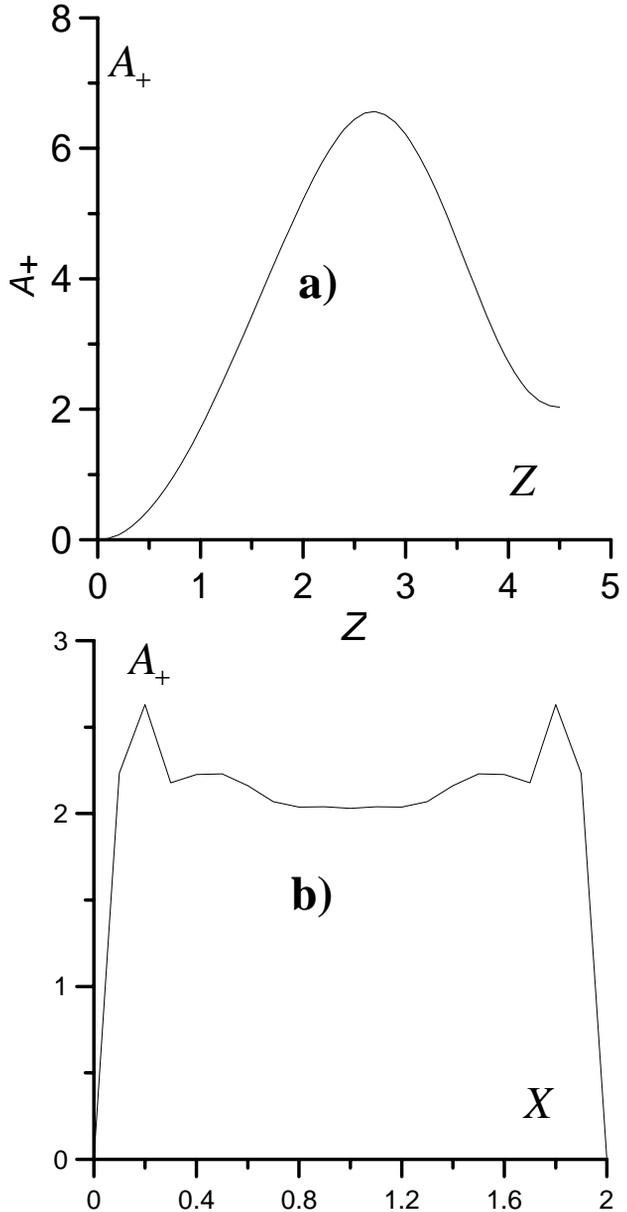


Fig.3. Longitudinal (a) and transverse (b) structure of the propagating wave in the steady state regime.

Note in conclusion that it is interesting to consider situation when the transverse (over y axis) structure of propagating mode A_+ is not fixed and is defined by its interaction with electron beam.

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