THEORY OF NONLINEAR HARMONIC GENERATION IN FELS WITH HELICAL WIGGLERS

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Abstract

Nonlinear Harmonic Generation (NHG) is of importance for both short wavelength FELs, in relation with the achievement of shorter wavelengths with a fixed electron-beam energy, and high-average power FEL resonators, in relation with destructive effects of higher harmonics radiation on mirrors. We present a treatment of NHG from helical wigglers with particular emphasis on the second harmonic. Our study is based on an analytical solution of Maxwell's equations, derived with the help of a Green's function method. We demonstrate that NHG from helical wigglers vanishes on axis. Our conclusion is in contrast with literature, that includes a kinematical mistake in the description of the electron motion. **A detailed report with relevant references is given in [1]**.

INTRODUCTION

NHG is of undisputed relevance in the field of FELs. It is generated by bunching of the electron beam at higher harmonics, driven by interaction with the fundamental. In general, NHG can be treated in terms of an electrodynamical problem where Maxwell's equations are solved with given macroscopic sources in the space-frequency domain. These sources are obtained through the solution of self-consistent equations for electrons and fields. Further on, solution of Maxwell's equations characterizes harmonic radiation in the space-frequency domain. The dependence of sources in the space-frequency domain on transverse and longitudinal coordinates is complicated because is the result of the above-mentioned self-consistent process. However, here we deal with an FEL setup where an ultrarelativistic electron beam is sent, in free space, through an undulator with many periods. Then, paraxial and resonance approximation can be applied. In particular, for a fixed transverse position, the longitudinal dependence is always slow on the scale of an undulator period. NHG has been dealt with in the case of a planar wiggler, both theoretically and experimentally. Odd harmonics have maximal power on axis[1] and are linearly polarized. Even harmonics have been shown to have vanishing on-axis power and to exhibit both horizontal and vertical polarization components. Here we present the first theory of NHG from helical wigglers, based on a solution of Maxwell's equations in the space-frequency domain, obtained with a Green's function technique.

[1]Here we assume that the bunching wavefront is perpendicular to the (longitudinal) FEL axis.

NHG IN HELICAL WIGGLERS

Analysis of the harmonic generation mechanism

We use paraxial Maxwell's equations in the space-frequency domain to describe radiation from ultra-relativistic electrons. Let us call the transverse electric field in the space-frequency domain $\vec{\bar{E}}_\perp(z,\vec{r}_\perp,\omega)$, where $\vec{r}_\perp = x\vec{e}_x + y\vec{e}_y$ and z identify a point in space. From the paraxial approximation follows that the electric field envelope $\vec{\widetilde{E}}_\perp = \vec{\bar{E}}_\perp \exp\left[-i\omega z/c\right]$ does not vary much along z on the scale of the reduced wavelength $\lambdabar = \lambda/(2\pi)$. Maxwell's equation in paraxial approximation reads: $\mathcal{D}\left[\vec{\widetilde{E}}_\perp(z,\vec{r}_\perp,\omega)\right] = \vec{f}(z,\vec{r}_\perp,\omega)$, where $\mathcal{D} \equiv {\nabla_\perp}^2 + (2i\omega/c)\partial/\partial z$, ${\nabla_\perp}^2$ being the Laplacian operator over transverse cartesian coordinates. The source-term vector $\vec{f}(z,\vec{r}_\perp)$ is specified by the trajectory of the source electrons, and can be written in terms of the Fourier transform of the transverse current density, $\vec{\bar{j}}_\perp(z,\vec{r}_\perp,\omega)$, and of the charge density, $\bar{\rho}(z,\vec{r}_\perp,\omega)$, as $\vec{f} = -4\pi[(i\omega/c^2)\vec{\bar{j}}_\perp - \vec{\nabla}_\perp\bar{\rho}]\exp[-i\omega z/c]$. In this paper we will treat $\vec{\bar{j}}_\perp$ and $\bar{\rho}$ as macroscopic quantities, without investigating individual electron contributions. $\vec{\bar{j}}_\perp$ and $\bar{\rho}$ are regarded as given data, that can be obtained from any FEL code. Codes actually provide the charge density of the modulated electron beam in the time domain $\rho(z,\vec{r}_\perp,t)$. A post-processor can be used in order to perform the Fourier transform of ρ that can always be presented as $\bar{\rho} = -\widetilde{\rho}(z,\vec{r}_\perp - \vec{r'}_{o\perp}(z),\omega)\exp\left[i\omega s_o(z)/v_o\right]$, where the minus sign on the right hand side is introduced for notational convenience only. Quantities $\vec{r'}_{o\perp}(z)$, $s_o(z)$ and v_o pertain a reference electron with nominal Lorentz factor γ_o that is injected on axis with no deflection and is guided by the helical undulator field. Such electron follows a helical trajectory $\vec{r'}_{o\perp}(z) = r'_{ox}\vec{e}_x + r'_{oy}\vec{e}_y$. We assume that $r'_{ox}(z) = K/(\gamma_o k_w)[\cos(k_w z) - 1]$ and $r'_{oy}(z) = K/(\gamma_o k_w)\sin(k_w z)$, where $K = \lambda_w e H_w/(2\pi m_e c^2)$ is the undulator parameter, $\lambda_w = 2\pi/k_w$ being the undulator period, $(-e)$ the negative electron charge, H_w the maximal modulus of the undulator magnetic field on-axis, and m_e the rest mass of the electron. The correspondent velocity is described by $\vec{v}_{o\perp}(z) = v_{ox}\vec{e}_x + v_{oy}\vec{e}_y$. Finally, $s_o(z)$ is the curvilinear abscissa measured along the trajectory of the reference particle. Introduction of $\widetilde{\rho}$ is useful when $\widetilde{\rho}$ is a slowly varying function of z on the wavelength scale. This property is granted by the fact that the charge density distribution under study originates from an FEL process. From this fact it also follows that $\widetilde{\rho}$ is slowly varying on the scale of the undulator period λ_w and is peaked around each harmonic of the fundamental $\omega_r = 2k_w c\bar{\gamma}_z^2$, that is fixed imposing resonance condition between electric field and reference particle. The word "peaked" means that the bandwidth of each harmonic component obeys $\Delta\omega/(h\omega_r) \ll 1$ for each positive integer value h. Here $\bar{\gamma}_z = 1/\sqrt{1 - v_{oz}^2/c^2}$ is the longitudinal Lorentz factor. Finally, the relative deviation of the particles energy from $\gamma_o m_e c^2$ is small, i.e. $\delta\gamma/\gamma_o \ll 1$. It follows that for a generic motion we have $\vec{f} = 4\pi \exp\left[i\int_0^z d\bar{z}\omega/(2\bar{\gamma}_z^2 c)\right]\left[i\omega/(c^2)\vec{v}_{o\perp}(z) - \vec{\nabla}_\perp\right]\widetilde{\rho}[z;\vec{r}_\perp - \vec{r'}_{o\perp}(z)]$. We account for a possible deflection angle $\vec{\eta}^{(c)}$ in the the trajectory of the reference electron. Therefore $\vec{r'}_{o\perp}(z) \longrightarrow \vec{r'}^{(c)}_\perp(z,\vec{\eta}^{(c)}) = \vec{r'}_{o\perp}(z) + \vec{\eta}^{(c)}z$ and $\vec{v}_{o\perp}(z) \longrightarrow \vec{v}_\perp(z,\vec{\eta}^{(c)}) = \vec{v}_{o\perp}(z) + c\vec{\eta}^{(c)}$. Using $\vec{v}_\perp(z,\vec{\eta}^{(c)})$ in place of $\vec{v}_{o\perp}(z)$ implies that $\gamma_z(z,\vec{\eta}^{(c)})$ is now a function of both z and $\vec{\eta}^{(c)}$. In particular, $1/\gamma_z^2(z,\vec{\eta}^{(c)}) = 1 - v_z^2(z,\vec{\eta}^{(c)})/c^2$, where $v_z^2 = v^2 - v_\perp^2$ is the square of the electron longitudinal velocity. It follows that $1/\bar{\gamma}_z^2$ in $\vec{f}$ should also be substituted by $1/\gamma_z^2(z,\vec{\eta}^{(c)})$. With these prescriptions, we find a solution to paraxial Maxwell's equation with the help of a Green's function technique, without any other assumption about the parameters of the problem. We are interested in the total power emitted and in the directivity diagram of the radiation in the far zone. Thus, we introduce the observation angle $\vec{\theta} = \vec{r}_{\perp o}/z_o$, setting $\theta \equiv |\vec{\theta}|$, taking the limit for $z_o \gg L_w$, where $L_w = N_w\lambda_w$ is the undulator length. Moreover, we are interested in studying frequency near the fundamental harmonic $\omega_r = 2k_w c\bar{\gamma}_z^2$ or its h-th integer multiple. We specify "how near" the frequency ω is to the h-th harmonic by defining a detuning parameter $C_h = \omega/(2\bar{\gamma}_z^2 c) - hk_w = \Delta\omega/\omega_r k_w$. Here $\omega = h\omega_r + \Delta\omega$. Altogether, using Anger-Jacobi expansion, we find

$$\begin{aligned}\vec{\widetilde{E}}_\perp &= \frac{i\omega}{cz_o}\int_{-\infty}^{\infty} dl'_x \int_{-\infty}^{\infty} dl'_y \int_{-L_w/2}^{L_w/2} dz'\widetilde{\rho}\left(z', l'_x, l'_y\right) \\ &\times \exp\left\{i\frac{\omega}{c}\left[\frac{z_o\left(\theta_x^2+\theta_y^2\right)}{2} + \frac{K(\theta_x - \eta_x^{(c)})}{k_w\gamma} - (\theta_x l'_x + \theta_y l'_y)\right]\right\} \\ &\times \sum_{m,n=-\infty}^{\infty} J_m(u)J_n(v)\exp\left[\frac{i\pi n}{2}\right]\exp\left[i(n+m+h)k_w z'\right] \\ &\times \exp\left\{i\left[C_h + \frac{\omega}{2c}\left(\theta_x - \eta_x^{(c)}\right)^2 + \frac{\omega}{2c}\left(\theta_y - \eta_y^{(c)}\right)^2\right]z'\right\} \\ &\times \left\{\left[\frac{K}{2i\gamma}\left(\exp[ik_w z'] - \exp[-ik_w z']\right) + \left(\theta_x - \eta_x^{(c)}\right)\right]\vec{e}_x\right. \\ &\left. - \left[\frac{K}{2\gamma}\left(\exp[ik_w z'] + \exp[-ik_w z']\right) - \left(\theta_y - \eta_y^{(c)}\right)\right]\vec{e}_y\right\} \quad (1)\end{aligned}$$

with $u = -K\omega(\theta_y - \eta_y^{(c)})/(c\gamma k_w)$ and $v = -K\omega(\theta_x - \eta_x^{(c)})/(c\gamma k_w)$. Whenever $C_h + \omega/(2c)[(\theta_x - \eta_x^{(c)})^2 + (\theta_y - \eta_y^{(c)})^2] \ll k_w$, the second phase factor in z' in Eq. (1) (the one containing C_h) is varying slowly on the scale of the undulator period λ_w. As a result, simplifications arise when $N_w \gg 1$, because fast oscillating terms in powers of $\exp[ik_w z']$ effectively average to zero. We further select frequencies such that $|\Delta\omega|/\omega_r \ll 1$, i.e. $|C_h| \ll k_w$. Note that this condition on frequencies automatically selects observation angles of interest $h(\vec{\theta} - \vec{\eta}^{(c)})^2 \ll 1/\bar{\gamma}_z^2$. Independently of the value of K and for observation angles of interest we have $|v| \ll 1$ and $|u| \ll 1$, and we can expand $J_n(v)$ and $J_m(u)$ in Eq. (1).

From now on we consider the case $h = 2$. Performing expansion, and accounting for terms giving a non-zero contribution after integration in dz' we obtain

$$\begin{aligned}\vec{\widetilde{E}}_\perp &= \frac{i\omega^2\left(\vec{e}_x + i\vec{e}_y\right)K^2}{2cz_o\omega_r(1+K^2)}\left[\left(\theta_x - \eta_x^{(c)}\right) + i\left(\theta_y - \eta_y^{(c)}\right)\right] \\ &\times \int_{-\infty}^{\infty} dl'_x \int_{-\infty}^{\infty} dl'_y \int_{-L_w/2}^{L_w/2} dz'\widetilde{\rho}\left(z', l'_x, l'_y\right)\exp\left[iC_2 z'\right] \\ &\times \exp\left\{i\frac{\omega}{c}\left[\frac{z_o\left(\theta_x^2+\theta_y^2\right)}{2} - \left(\theta_x l'_x + \theta_y l'_y\right)\right]\right\} \\ &\times \exp\left\{i\frac{\omega}{2c}\left[\left(\theta_x - \eta_x^{(c)}\right)^2 + \left(\theta_y - \eta_y^{(c)}\right)^2\right]z'\right\}. \quad (2)\end{aligned}$$

The electric field is left circularly polarized and vanishes at $\vec{\theta} = \vec{\eta}^{(c)}$. Polarization characteristics are the same as for the fundamental harmonic, although the fundamental does not vanish at $\vec{\theta} = \vec{\eta}^{(c)}$.

We conclude our analysis of NHG in helical wigglers studying on-axis harmonic generation. We can do so in all generality, i.e. for any harmonic number, with the help of Eq. (1). We set $\vec{\theta} - \vec{\eta}^{(c)} = 0$. It follows that Eq. (1) can be rewritten as

$$\begin{aligned}\vec{\widetilde{E}}_\perp &= \frac{i\omega}{cz_o}\int d\vec{l'}\int_{-L_w/2}^{L_w/2} dz'\widetilde{\rho}\left(z', \vec{l'}\right)\exp\left[iC_h z'\right] \\ &\times \left\{\left[\frac{K}{2i\gamma}\left(\exp[i(h+1)k_w z'] - \exp[i(h-1)k_w z']\right)\right]\vec{e}_x\right. \\ &\left. + \left[-\frac{K}{2\gamma}\left(\exp[i(h+1)k_w z'] + \exp[i(h-1)k_w z']\right)\right]\vec{e}_y\right\}. \quad (3)\end{aligned}$$

Since $\widetilde{\rho}$ is a slowly function of z' on the scale of the undulator period and $C_h \ll k_w$, we see by inspection that, after integration in dz', one obtains non-zero on-axis field only for $h = 1$. This result is in open contrast with what reported in reference [2]: we will address this fact in the discussion section.

Analysis of a simple model

We treat a particular case to exemplify our results. We consider the case when $C_2 = 0$ (i.e. $\omega = 2\omega_r$) and $\widetilde{\rho} = I_o a_2/(2\pi c\sigma_\perp^2)\exp[-l'^2_x + l'^2_y/(2\sigma_\perp^2)]\exp[i2\omega_r/c\cdot(\eta_x^{(c)}l'_x + \eta_y^{(c)}l'_y)]H_{L_w}(z)$, with H_{L_w} a window function equal unity inside the undulator and zero everywhere else. Here I_o is the bunch current, a_2 is a constant determining the strength of the bunching and $\sigma_\perp$ the rms transverse size of the electron beam. This corresponds to a modulation wavefront perpendicular to the direction of motion of the beam. Direct substitution in Eq. (2) and calculations yield

$$\begin{aligned}\vec{\widetilde{E}}_\perp = &\frac{2iI_o a_2 L_w\omega_r\left(\vec{e}_x + i\vec{e}_y\right)}{c^2 z_o}\left(\frac{K^2}{1+K^2}\right)\\ &\times\left[\left(\theta_x - \eta_x^{(c)}\right) + i\left(\theta_y - \eta_y^{(c)}\right)\right]\exp\left[i\frac{\omega_r}{c}z_o(\theta_x^2+\theta_y^2)\right]\\ &\times\exp\left\{-\frac{2\sigma_\perp^2\omega_r^2}{c^2}\left[\left(\theta_x - \eta_x^{(c)}\right)^2 + \left(\theta_y - \eta_y^{(c)}\right)^2\right]\right\}\\ &\times\mathrm{sinc}\left\{\frac{L_w\omega_r}{2c}\left[\left(\theta_x - \eta_x^{(c)}\right)^2 + \left(\theta_y - \eta_y^{(c)}\right)^2\right]\right\}. \qquad (4)\end{aligned}$$

Going back to our particular case in Eq. (4), a subject of particular interest is the angular distribution of the radiation intensity, which will be denoted with I_2. Upon introduction of normalized quantities $\hat{\theta}_{x,y} = \sqrt{2\omega_r L_w/c}\ \theta_{x,y} = \sqrt{8\pi N_w}\ \bar{\gamma}_z\theta_{x,y}$, $\hat{\eta}_{x,y}^{(c)} = \sqrt{2\omega_r L_w/c}\,\eta_{x,y}^{(c)} = \sqrt{8\pi N_w}\,\bar{\gamma}_z\,\eta_{x,y}^{(c)}$ and of the Fresnel number $N = 2\omega_r\sigma_\perp^2/(cL_w)$, one obtains

$$I_2 \propto \left|\vec{\hat{\theta}} - \vec{\hat{\eta}}^{(c)}\right|^2\exp\left\{-N\left|\vec{\hat{\theta}} - \vec{\hat{\eta}}^{(c)}\right|^2\right\}\mathrm{sinc}^2\left\{\frac{1}{4}\left|\vec{\hat{\theta}} - \vec{\hat{\eta}}^{(c)}\right|^2\right\} \qquad (5)$$

In the limit for $N \ll 1$, Eq. (5) gives back the directivity diagram for the second harmonic radiation from a single particle. The directivity diagram in Eq. (5) is plotted in Fig. 1 for different values of N as a function of $|\vec{\hat{\theta}} - \vec{\hat{\eta}}^{(c)}|$, normalized to the maximal intensity $I_2^{\max}$ at each value of N. The next step is the calculation of the second harmonic power that is given by $W_2 = c/(2\pi)\int_{-\infty}^{\infty}dx_o\int_{-\infty}^{\infty}dy_o|\vec{\widetilde{E}}_\perp(z_o,x_o,y_o)|^2$. It is convenient to present the expressions for W_2 in a dimensionless form. After appropriate normalization it is a function of one dimensionless parameter only, that is $\hat{W}_2 = F_2(N) = \ln[1 + 1/(4N^2)]$. Here $\hat{W}_2 = W_2/W_o^{(2)}$ is the normalized power, while the normalization constant $W_o^{(2)}$ is given by $W_o^{(2)} = (2K^2/1+K^2)^2(I_o^2/c)a_2^2$. The function $F_2(N)$ is plotted in Fig. 2. The logarithmic divergence in $F_2(N)$ in the limit for $N \ll 1$ imposes a limit on the meaningful values of N. However, in the case $N < N_w^{-1}$ we deal with a situation where the dimensionless problem parameter N is smaller than the accuracy of the resonance approximation $\sim N_w^{-1}$, and for estimations we should replace $\ln(N)$ with $\ln(N_w^{-1})$.

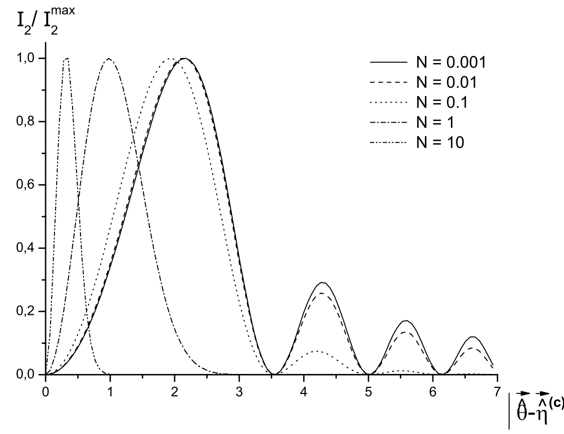

Figure 1: Directivity diagram for the intensity.

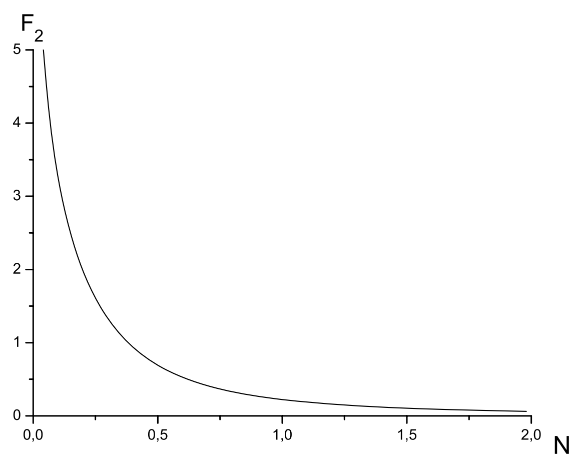

Figure 2: Illustration of the behavior of $F_2(N)$.

DISCUSSION

NHG in a helical wiggler has been addressed in [2], where a numerical study-case guarantees that paraxial and resonance approximation can be applied. The main result of [2] is that characteristics of helical undulator radiation from an extended source, i.e. a bunched electron beam, are drastically different compared to those from a single electron. In particular, NHG does not vanish on-axis, in open contrast with our conclusions. In our understanding, results in [2] are incorrect. Any linear superposition of a given field harmonic from single electrons conserves single-particle characteristics like parametric dependence on undulator parameters and polarization. In particular, since field harmonics from a single electron vanishes on-axis, they must vanish on-axis for the linear superposition as well. This also applies in the case of NHG, because the dependence of charge and current density distributions of the bunched beam on the longitudinal coordinate is slow on the scale of the undulator period. This argument suggests that results in reference [2] are incorrect. A flaw can be pinpointed in the azimuthal resonance condition proposed in [2]: "The azimuthal electron motion in helical wigglers is $\theta = k_w z$ (k_w is the wave number for the wiggler period λ_w), which couples to circularly polarized waves that vary as $\exp(i\phi_h)$, where $\phi_h = kz + h\theta - \omega t$ is the wave phase. Hence, the phase along the particle trajectories varies as $\phi_h = (k + hk_w)z - \omega t$, and the hth order azimuthal mode corresponds to the hth harmonic resonance [i.e., $\omega \approx (k + hk_w)v_z$]". There θ indicates an azimuthal position as in Fig. 3. Note that the phase $\phi_h = kz + h\theta - \omega t$ pertains a circularly polarized wave whose electric field is written in terms of unit vectors $\vec{e}_\rho$ and $\vec{e}_\theta$ and not of $\vec{e}_x$ and $\vec{e}_y$ ($\vec{e}_\rho$, $\vec{e}_\theta$, $\vec{e}_x$ and $\vec{e}_y$ are shown in Fig. 3). In fact one may write the electric field of a (e.g. left) circularly polarized plane wave at position $\vec{r}$ and time t as $E_o(\vec{e}_r + i\,\vec{e}_\theta)\exp[i\phi_h]$, E_o being a constant field strength. We argue that it is incorrect to use relation $\theta \approx k_w z$ in the expression for ϕ_h as done in [2]. On the one hand, the radiation diffraction size for a single particle is of order $\sqrt{\lambda\!\!\!^{-} L_w}$. On the other hand, the electron rotation radius is given by $r_w = (K/\gamma)\lambda\!\!\!^{-}_w$. It follows that $r_w^2/\lambda\!\!\!^{-} L_w \ll 1$, that holds independently of the value of K, because $N_w \gg 1$. Thus, the electron rotation radius is always much smaller than the radiation diffraction size. Moreover, straightforward geometrical considerations show that it makes sense to talk about a transverse beam size $\sigma_\perp$ only when $\sigma_\perp^2 \gg r_w^2$. This is the case in practical situations of interest. There is still room to compare the beam size $\sigma_\perp$ with the radiation diffraction size. In the case $\sigma_\perp \ll \sqrt{\lambda\!\!\!^{-} L_w}$ we deal with a filament electron beam. In the opposite case the filament beam approximation breaks down. We are interested in this last case, where single-particle results cannot be used. We have therefore established a hierarchy in the characteristic scales of interest: $\sigma_\perp^2 \gtrsim \lambda\!\!\!^{-} L_w \gg r_w^2$. It follows that the azimuthal coordinate of each electron, θ, is fixed during the motion inside the undulator with the accuracy of the resonance approximation, scaling as $1/N_w$. In contrast to this, the identification $\theta \approx k_w z$ is made in [2]. This is a kinematical mistake. If $\theta \approx k_w z$ each electron would be rotating around the origin of the coordinate system, that is not the case. Thus, the azimuthal resonance condition is a misconception following from this mistake. This misconception is subsequently passed on to simulations in [2], resulting in incorrect outcomes. Harmonic emission exists for a single electron, and it also exists for an electron bunch. However, qualitative properties are different with respect to what has been predicted in [2]. In particular, as we have seen before, on-axis power vanishes.

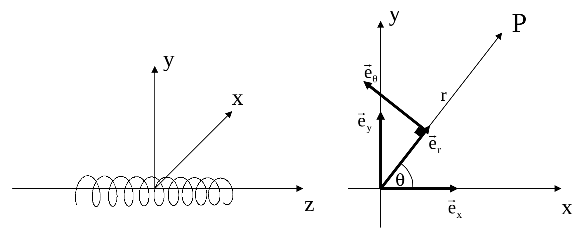

Figure 3: Cylindrical coordinate system and setup.

CONCLUSIONS

In this paper we discussed NHG in helical undulators, with particular emphasis on second harmonic generation. First we considered the NHG mechanism in helical undulators in all generality. Then we specialized our study to the case of second harmonic generation. Finally, to exemplify our results, we treated a simplified model where the beam modulation wavefront is orthogonal to the z axis, has a Gaussian transverse profile and is independent on the position inside the undulator. Our results show that on-axis harmonic generation from helical wigglers vanishes. This applies to any harmonic of interest with the exclusion of the fundamental, and independently of the form of the electron beam modulation (assuming that the electron beam as a whole propagates on-axis). Important consequences follow regarding the two mainstream development paths in FEL physics. First, as concerns short wavelength (x-ray) SASE FEL devices, vanishing on-axis harmonics make the option of a helical undulator less attractive as regards the exploitation of NHG radiation. Second, as concerns high average-power FEL oscillators, vanishing on-axis harmonics suggest that helical undulators carry relevant advantages over planar undulators, as potential for mirror damage is reduced. Previous studies reported non-vanishing on-axis power, due to the nature of a particular azimuthal resonance condition. We showed that this resonance condition is a misconception, arising from a kinematical mistake. This misconception was passed on to simulations, that confirmed the presence of on-axis power out of NHG from helical wigglers. Such result is incorrect.

For details on our work we refer the interested reader to [1].

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