# EXPERIMENTAL STUDY OF A VOLUME FREE ELECTRON LASER WITH A "GRID" RESONATOR 

V.G. Baryshevsky *, N.A.Belous, A.A.Gurinovich, A.S.Lobko, P.V.Molchanov, V.I.Stolyarsky, Research Institute for Nuclear Problems, 11 Bobryiskaya str., 220050, Minsk, Belarus

## Abstract

Operation of Volume Free Electron Laser with a "grid" photonic crystal, built from periodically strained metallic threads, was studied in the backward wave regime. Generation threshold was observed for different "grid" photonic crystals. Dependence of the generation threshold on the resonator length was investigated.

## INTRODUCTION

Generators using radiation from an electron beam in a periodic slow-wave circuit (travelling wave tubes, backward wave oscillators, free electron lasers) are now widespread [1].

Diffraction radiation [2] in periodical structures is in the basis of operation of travelling wave tubes (TWT) [3, 4], backward wave oscillators (BWO) and such devices as Smith-Purcell lasers [5, 6, 7] and volume FELs using twoor three-dimensional distributed feedback [8, 9, 10, 11].

A challenge of precise electron beam guiding over the slowing structure (the electron beam should pass at the distance $\delta \leq \frac{\lambda \beta \gamma}{4 \pi}$ over the diffraction grating, here $\delta$ is the so-called beam impact parameter, $\lambda$ is the radiation wavelength, $\beta=v / c, v$ is the electron beam velocity, $\gamma$ is the electron Lorentz-factor) restricts application of SmithPurcell lasers and the similar devices. Electrical endurance of resonator limits radiation power and current of acceptable electron beam. Conventional waveguide systems are essentially restricted by the requirement for transverse dimensions of resonator, which should not significantly exceed radiation wavelength.

The most of the above problems can be overcome in VFEL [8, 9, 10, 11, 12]. In VFEL the greater part of electron beam interacts with the electromagnetic wave due to volume distributed interaction. Transverse dimensions of VFEL resonator could significantly exceed radiation wavelength $D \gg \lambda$. In addition, electron beam and radiation power are distributed over the whole volume that is beneficial for electrical endurance of the system. One of the VFEL types uses a "grid" volume resonator ("grid" photonic crystal) that is formed by a periodically strained either dielectric or metallic threads.

The "grid" structure of dielectric threads was experimentally studied in [13], where it was shown that such "grid" photonic crystals have sufficiently high $Q$ factors $\left(10^{4}-10^{6}\right)$.
Theoretical analysis $[14,16]$ showed that periodic metal grid does not absorb electromagnetic radiation and the

[^0]"grid" photonic crystal of metal threads is almost transparent for the electromagnetic waves in the wavelength range, where the skin-depth is less then the thread radius. The conclusions of [14] declared possibility of development of VFEL with the "grid" photonic crystal of metal threads.

First lasing of the volume FEL with a "grid" volume resonator, which was formed by the periodic set of metal threads inside a rectangular waveguide, was observed in the proof-of-principle experiment [15] and completely confirmed conclusions of [14].

In the present paper dependence of the generation intensity as a function of the grid photonic crystal length is studied for the backward wave oscillation regime.

## THE CONCEPT

Waves propagation through photonic crystals is the subject for numerous studies both theoretical and experimental [17, 18, 19, 20].

Challenges, which appears when considering interaction of an electromagnetic wave with such a photonic crystal, are as follows: It is well known that a metal grid reflects electromagnetic waves perfectly. Therefore, the question arises as to whether a wave penetrate deep into resonator, especially since resonator contains a set of grids. Theoretical analysis $[14,16]$ showed that periodic metal grid does not absorb electromagnetic radiation and the "grid" photonic crystal, made of metal threads, is almost transparent for the electromagnetic waves in the frequency range from GHz to THz . In this range the skin depth $\delta$ is about $1 \mathrm{mi}-$ cron or less for the most of metals (for example, for 10 GHz $\delta_{C u}=0.66 \mu \mathrm{~m}, \delta_{A l}=0.8 \mu \mathrm{~m}, \delta_{W}=1.16 \mu \mathrm{~m}$ and so on). Thus, in this frequency range the metallic threads can be considered as perfect conducting.

According to $[14,16]$ the refraction index for the "grid" photonic crystal, can be expressed as:

$$
\begin{equation*}
n_{\|(\perp)}^{2}=1+\frac{\eta_{\|(\perp)}}{k^{2}} \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\eta_{\|(\perp)}=\frac{4 \pi}{\Omega_{2}} \frac{A_{0}}{1+i \pi A_{0}-2 C A_{0}} \tag{2}
\end{equation*}
$$

$n_{\|}$and $n_{\perp}$ are the refraction indices for the waves with polarization parallel and perpendicular to the thread axis, respectively, $k=2 \pi / \lambda$ is the wave number, $R$ is the thread radius, $\Omega_{2}=d_{y} \cdot d_{z}$, where $d_{y}$ and $d_{z}$ are the photonic crystal periods along the axis $y$ and $z$, respectively, $C=$ 0.5772 is the Eiler constant. The values $A_{0(\|)}$ and $A_{0(\perp)}$ for perfectly conducting threads are defined as [15, 16]:
$A_{0(\|)}=\frac{1}{\pi} \frac{J_{0}(k R) N_{0}(k R)}{J_{0}^{2}(k R)+N_{0}^{2}(k R)}+\frac{i}{\pi} \frac{J_{0}^{2}(k R)}{J_{0}^{2}(k R)+N_{0}^{2}(k R)}$
$A_{0(\perp)}=\frac{1}{\pi} \frac{J_{0}^{\prime}(k R) N_{0}^{\prime}(k R)}{J_{0}^{\prime 2}(k R)+N_{0}^{\prime 2}(k R)}+\frac{i}{\pi} \frac{J_{0}^{\prime 2}(k R) \quad \text { (3) }}{J_{0}^{\prime 2}(k R)+N_{0}^{\prime 2}(k R)}$,
where $J_{0}, N_{0}, J_{0}^{\prime}$ and $N_{0}^{\prime}$ are the Bessel and Neumann functions and their derivatives, respectively.

To consider threads with finite conductivity one should use the following expressions:
$A_{0(\|)}=\frac{i}{\pi} \frac{J_{0}\left(k_{t} R\right) J_{0}^{\prime}(k R)-\sqrt{\varepsilon_{t}} J_{0}^{\prime}\left(k_{t} R\right) J_{0}(k R)}{J_{0}\left(k_{t} R\right) H_{0}^{(1) \prime}(k R)-\sqrt{\varepsilon_{t}} J_{0}^{\prime}\left(k_{t} R\right) H_{0}^{(1)}(k R)}$,
$A_{0(\perp)}=\frac{i}{\pi} \frac{J_{0}\left(k_{t} R\right) J_{0}^{\prime}(k R)-\frac{1}{\sqrt{\varepsilon_{t}}} J_{0}^{\prime}\left(k_{t} R\right) J_{0}(k R)}{J_{0}\left(k_{t} R\right) H_{0}^{(1) \prime}(k R)-\frac{1}{\sqrt{\varepsilon_{t}}} J_{0}^{\prime}\left(k_{t} R\right) H_{0}^{(1)}(k R)}$,
where $\varepsilon_{t}$ is the dielectric permittivity of the thread material, $k_{t}=\sqrt{\varepsilon_{t}} k, H_{0}^{(1)}$ is the Hankel function of the zero order. The expressions (3) can be obtained from (4) considering $\varepsilon_{t} \rightarrow \infty$.

Difference in the refraction indices for different wave polarizations ( $n_{\|} \neq n_{\perp}$ ) indicates that the system owns optical anisotropy (i.e. possesses birefringence and dichroism). To escape this anisotropy we can alternate the treads position in the grid: threads in each layer are orthogonal to those in the previous and following layers.

Smith - Purcell (diffraction) radiation in photonic crystal for an electron beam with the velocity $\vec{v}$ passing through the "grid"arises when the radiation condition is fulfilled:

$$
\begin{equation*}
\omega-\vec{k} n(k) \vec{v}=\vec{\tau} \vec{v}, \tag{5}
\end{equation*}
$$

where $\vec{\tau}$ is the reciprocal lattice vector and $n(k)$ is the refraction index (see (1)). Suppose the electron beam velocity is directed along the axis $O Z$, then (5) can be presented in the form:

$$
\begin{equation*}
k-\tau_{z} \beta=k n(k) \beta \cos \theta \tag{6}
\end{equation*}
$$

where $\beta=\frac{v}{c}$, the angle between $\vec{k}$ and the electron beam velocity is denoted by $\theta$ and $\tau_{z}=\frac{2 \pi m_{h}}{d_{z}}$, where $m_{h}=$ $1,2, \ldots$ is the number of harmonic. The roots of this equation give the spectrum of frequencies of diffraction (SmithPurcell) radiation, which is induced by a particle moving in the above "grid" photonic crystal.

Diffraction radiation in a metal waveguide of the rectangular cross-section with the "grid" photonic crystal placed inside it is shown in $[14,16]$ to be described by the equation similar to (6):

$$
\begin{equation*}
\left(\frac{\omega-\tau_{z} v}{v}\right)^{2}=\left(\frac{\omega}{c}\right)^{2}-\left(\kappa_{m n}^{2}-\eta\right) \tag{7}
\end{equation*}
$$

where $\eta$ is determined by (2) and the eigenvalues $\kappa_{m n}$ are determined by the waveguide transverse dimensions (width $a$ and height $b$ ):

$$
\begin{equation*}
\kappa_{m n}^{2}=\left(\frac{\pi m}{a}\right)^{2}+\left(\frac{\pi n}{b}\right)^{2} \tag{8}
\end{equation*}
$$

The roots of equation (7) for $\frac{\eta}{\tau_{\varepsilon}^{2} \beta^{2}} \ll 1$ can be obtained as:
$\omega_{1}(m, n)=\frac{\tau_{z} v}{1-\beta^{2}}\left(1-\beta \sqrt{1-\frac{\left(\kappa_{m n}^{2}-\eta\right)}{\tau_{z}^{2}} \frac{1-\beta^{2}}{\beta^{2}}}\right)$,
$\omega_{2}(m, n)=\frac{\tau_{z} v}{1-\beta^{2}}\left(1+\beta \sqrt{1-\frac{\left(\kappa_{m n}^{2}-\eta\right)}{\tau_{z}^{2}} \frac{1-\beta^{2}}{\beta^{2}}}\right)$.

It should be reminded here that $\tau_{z}=\frac{2 \pi m_{h}}{d_{z}}$, where $m_{h}=$ $1,2, \ldots$ is the number of harmonic. From (9) it follows that higher harmonics provide for getting radiation with higher frequencies. For example, for the electron beam with the energy 200 keV , considering $\theta \sim 20^{\circ}$ and $d_{z}=1.6 \mathrm{~cm}$, the first harmonic ( $m_{h}=1$ ) gives radiation frequencies $\sim 10$ GHz and $\sim 40 \mathrm{GHz}$ for the roots 1 and 2 of the equation (7), respectively, the 30-th harmonic ( $m_{h}=30$ ) provides $\sim 230 \mathrm{GHz}$ and $\sim 1 \mathrm{THz}$.

## EXPERIMENTAL SETUP

The "grid" photonic crystal is built from tungsten threads with the diameter 0.1 mm strained inside the rectangular waveguide with the transversal dimensions $a=35 \mathrm{~mm}$, $b=35 \mathrm{~mm}$ and length 300 mm (see Fig.1). The distance between the threads along the axis $O Z$ is $d_{z}=12.5 \mathrm{~mm}$. A pencil-like electron beam with the diameter 32 mm , energy $\sim 200 \mathrm{keV}$ and current $\sim 2 \mathrm{kA}$ passes through the above structure. The magnetic field guiding the electron beam is $\sim 1.55-1.6$ tesla. Period of grating is chosen to provide radiation frequency $\sim 8.4 \mathrm{GHz}$. The "grid" structure


Figure 1: The "grid" diffraction grating placed inside the waveguide.
is made of separate frames each containing the layer of 1 , 3 or 5 parallel threads with the distance between the next threads $\left.d_{y}=6 \mathrm{~mm}\right)$. Frames are joined to get the "grid" structure with the distance $d_{z}$ between layers.

Frequency range was evaluated by means of tunable waveguide filters, which were tuned in the band 7.8-12.4 GHz with passbands $0.25 \mathrm{GHz}, 0.5 \mathrm{GHz}$ and 1 GHz . Attenuation of radiation in the suppressed band of this filter is $\sim-25 \mathrm{~dB}$.

## EXPERIMENTAL RESULTS

The objective of the experiment is to study dependence of the generated radiation intensity on the "grid" photonic crystal length.

The maximal radiation power of VFEL generator in the this experiment was about 1.5 kiloWatt for one thread in the frame, 5 kiloWatts for three threads in the frame and 10 kiloWatt for five threads in the frame.

The sample oscillogram is shown in Fig.2, where signals marked 1 and 2 are the signals obtained from microwave detectors. Other two curves are the electron gun voltage and electron beam current. Time scale is 80 ns .


Figure 2: The sample oscillogram.

Two types of experiments are reported.

1. The radiation power was measured for photonic crystal with $4,8,10$ and 24 frames each containing one thread equidistant from waveguide top and bottom walls (see Fig.3). The result of these measurements is presented in Fig.4, where the radiation power is normalized to the maximal detected power ( 1.5 kiloWatt).
2. The radiation power is measured for photonic crystal with $4,6,10,12,14$ and 22 frames each containing five threads distant $d_{y}=6 \mathrm{~mm}$ each from other (see Fig.5).

The result of these measurements is presented in Fig.6, where the radiation power is also normalized to the maximal detected power ( 10 kiloWatt). The solid curve in this figure shows the numerically simulated radiation power, which also normalized.

## CONCLUSION

Operation of Volume Free Electron Laser with a "grid" photonic crystal, built from periodically strained metallic threads, was studied in the backward wave regime. Generation threshold was observed for different "grid" photonic crystals. Dependence of the generation threshold on the resonator length was investigated. Use of volume resonators of the described type provides to weaken requirements for the electron beam shape and guiding precision,


Figure 3: Photonic crystal with frames each containing one thread equidistant from waveguide top and bottom walls.


Figure 4: Dependence of the generation intensity on the length of the "grid" photonic crystal with one thread in the frame, the upper scale show the resonator length in the number of wavelength $L / \lambda_{0}$, where $\lambda_{0}=3.6 \mathrm{~cm}$.
because the electron beam passes directly through the photonic crystal.

## REFERENCES

[1] V.L.Granatstein, R.K.Parker and C.M.Armstrong, Proceedings of the IEEE 87, no. 5 (1999).
[2] B.M.Bolotovskii and G.V.Voskresenskii, Usp. Fiz. Nauk. 88, 209 (1966) (Sov. Phys. Usp. 9, 73 (1966)).
[3] R.Kompfner, Wireless World 52, 369 (1946).
[4] R.Pierce, Proc. IRE 35, 111 (1947).
[5] S.J.Smith and E.M.Purcell, Phys. Rev. 92, 1069 (1953).
[6] W.W.Salisbury, US Patent 2,634,372 (1953); J.Opt. Soc.Am. 60, 1279 (1970).
[7] G.Doucas, J.H.Mulvey, M.Omori, J.Walsh and M.F.Kimmit, Phys.Rev.Lett. 69, 1761 (1992); John E. Walsh US Patent 5,790,585 (1996).


Figure 5: Photonic crystal with frames each containing five threads distant $d_{y}=6 \mathrm{~mm}$ each from other


Figure 6: Dependence of the generation intensity on the length of the "grid" photonic crystal with 5 threads in the frame marked with squares and numerically simulated dependence of the wave amplitude on the "grid" photonic crystal length for the electron beam with the energy $\sim 200$ keV and current density $\sim 2 \mathrm{kA} / \mathrm{cm}^{2}$
[8] V.G.Baryshevsky, NIM 445A, 281 (2000); LANL e-print archive physics/9806039.
[9] V.G.Baryshevsky, K.G.Batrakov, A.A.Gurinovich et al., NIM 483A, 21 (2002).
[10] V.G.Baryshevsky, K.G.Batrakov, A.A.Gurinovich et al., NIM 507A, 137 (2003).
[11] V.G.Baryshevsky et al., Eurasian Patent no. 004665.
[12] V.G.Baryshevsky, I.D.Feranchuk, Phys.Lett. 102A, 141 (1984).
[13] V.G.Baryshevsky,K.G.Batrakov,I.Ya.Dubovskaya,V.A.Karpovich, V.M.Rodionova, NIM 393A, 71 (1997).
[14] V.G.Baryshevsky, A.A.Gurinovich LANL e-print archive: physics/0409107
[15] V.G. Baryshevsky, K.G. Batrakov, N.A. Belous, A.A. Gurinovich, A.S. Lobko, P.V. Molchanov, P.F. Sofronov, V.I. Stolyarsky, LANL e-print archive: physics/0409125.
[16] V.G.Baryshevsky, A.A.Gurinovich, to be published in NIM B, Topical Issue RC2005.
[17] A. L. Pokrovsky and A. L. Efros, Phys. Rev. B 65, 045110 (2002).
[18] A. L. Pokrovsky, Phys. Rev. B 69, 195108 (2004).
[19] E. I. Smirnova, C. Chen, M. A. Shapiro et al., J. Appl.Phys. 91(3), 960 (2002).


[^0]:    * bar@inp.minsk.by

