

SIMULATING FEM AMPLIFIERS: FEATURES IN VARIOUS REGIMES*

A.V. Elzhov⁺, A.K. Kaminsky, E.A. Perelstein, S.N. Sedykh, A.P. Sergeev
Joint Institute of Nuclear Research, Dubna, Russia

Abstract

Results of previous experimental investigations of Compton/Raman millimeter-wavelength FEL amplifiers are analyzed in numerical simulations. Compact quasi-one-dimensional models under helical-trajectory approach are employed. A tare on the stationary trajectories due to radiation is inserted. Reasonable agreement between simulation and experimental results is got in the regimes with reversed guide magnetic field.

1 INTRODUCTION

A full-scale three-dimensional simulation of an FEL amplifier allows one to get the best agreement between the simulation and experimental data. However the corresponding software possesses limited accessibility and requires considerable computing resource. Moreover not always one can distinguish principal physical factors defining features of particular FEL amplification regime. So use of compact approximate models seems to be attractive if they take account of dominating physical processes and allow one to determine parameters of amplifier operating with an acceptable accuracy.

In this paper quasi-one-dimensional models under helical-trajectory approach for various regimes of millimeter FEL (FEM) amplifier are considered, their applicability taking into account the experimental data [1–4] and problems of numerical simulation [5] are analyzed.

2 FEL REGIMES AND MODELS

A Raman regime of an FEL amplifier in which collective effects dominate is valid under 3 conditions [6]:

1°. The system is lengthy enough while the beam is high-dense so that several plasma oscillations get into the system length: $\lambda_p/L \leq 1$. (1)

Here $\lambda_p = 2\pi/\omega_p$, ω_p is the relativistic plasma frequency: $\omega_p^2 = 4\pi n_e e^2/m_0 \gamma_z^2 \gamma_0$ where n_e is the electron beam density, m_0 is the electron rest mass and γ_0 , γ_z are the total and longitudinal relativistic factors.

2°. Transverse velocities are much less than critical value [7] $\beta_w \ll \beta_{crit} \equiv F^{-1/2} (2\omega_p c^2 / v_{z0}^3 \gamma_z^2 k_w)^{1/2}$. (2)

Here $k_w = 2\pi/\lambda_w$ is the wiggler wavenumber (λ_w is the wiggler period), v_{z0} is the initial longitudinal electron velocity, F is the beam-waveguide fill factor.

This criterion is based on comparison of contribution of ponderomotive and Coulomb potentials and obtained in helical-trajectory approach. Analyzing real experiments one should use it with caution [6] for the complications

such as reduction of the effective plasma frequency in a waveguide, wiggler inhomogeneities, beam thermal effects and guide magnetic field. So the model may be insufficient for description of space-charge effects.

3°. Landau damping of the space-charge waves is not significant (the space-charge wavelength is greater than the Debye length) [6]. This restriction is imposed by the beam energy spread. For Gaussian distribution of electron velocities this requirement [8] transforms to

$$(\delta\gamma/\gamma)_z < (L/2\gamma)(\omega_p/2c) \equiv \pi L/2\gamma\lambda_p. \quad (3)$$

Let us study a self-consistent spatial problem of relativistic electron beam moving in the microwave field provided the regime is stationary, the wave is a single-mode one and its amplitude is slowly varied with length.

Assuming the electrons obeying helical trajectories with longitudinal velocity β_{\parallel} , we can write down a set of equations for a Raman FEL accounting [9,10,11]:

$$\frac{d\gamma_j}{dZ} = \frac{i\beta_{\perp j}}{2\sqrt{2}\beta_{\parallel j}} a_s e^{i\psi_j} + \frac{c}{4\omega_0} \frac{\omega_{p0}^2 \gamma_{z0}^2 \Phi_0 \beta_{\perp j} F_{s.c.}}{c^2 (k_z + k_w)} i e^{i\psi_j} \langle e^{-i\psi_j} \rangle + c.c. \quad (4)$$

$$\frac{d\theta_j}{dZ} = \frac{1}{\beta_{\perp j}} - \frac{1}{\beta_{ph}} \quad (5), \quad \frac{d\hat{a}}{dZ} = \eta_0 \gamma_0 \left\langle \frac{e^{-i\theta_j}}{\gamma_j} \right\rangle \quad (6)$$

Here γ_j is the j -th electron's relativistic factor; $Z = z\omega_0/c$ is the dimensionless longitudinal coordinate, ω_0 is the amplifier operating frequency, k_z is the longitudinal wavenumber. The phase variables are: θ_j is the electron-to-wave one; φ is that of the complex RF amplitude ($\hat{a} = a_s e^{i\varphi}$), $\psi_j = \varphi + \theta_j$ is the total ponderomotive phase. The variable $a_s = eE_g/m_0\omega_0 c$ is the dimensionless amplitude of RF electric field. The gain factor is $\eta_0 = (I_b/I_A) (2\kappa_0/N)$, $\kappa_0 = a_w/2\gamma_0$ [11], $a_w = eB_w/(k_w m c^2)$ is the dimensionless amplitude of the wiggler magnetic field, Alfvén constant is $I_A = m_0 c^3/e \approx 17$ kA; N is the wave norm; $\beta_{\perp j}$ and β_{ph} are longitudinal electron velocity and phase velocity of the microwave respectively. $F_{s.c.}$ is the attenuation factor of the space-charge wave, which is close to unity when the requirement (3) is satisfied. The constant in Eq.(4) is

$$\Phi_0 = 1 - \frac{\tilde{\beta}_w^2 \gamma_z^2 \Omega_0}{(1 + \tilde{\beta}_w^2) \Omega_0 - k_w c \beta_{z0}} \equiv 1 - \frac{\tilde{\beta}_w^2 \gamma_z^2}{1 + \tilde{\beta}_w^2 - \gamma_0 \beta_{z0} / a_g}$$

where $a_g = eB_g/m_0 c^2 k_w = \Omega_0 \gamma_0 / c k_w$ is the dimensionless amplitude of guide magnetic field with induction value of B_g , and $\tilde{\beta}_w = \beta_{\perp}/\beta_{z0}$ is the extended wiggling parameter.

The same system is suitable for simulation of high-gain Compton FEL if the $F_{s.c.}$ term in (4) is removed.

A uniform longitudinal guide magnetic field allows one to rise the electron beam stability relatively to

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+ Corresponding author: A. Elzhov, E-mail: artel@sunse.jinr.ru

transverse perturbations. But the trajectories became more complicated. Particularly for the wiggler field of helically symmetric distribution generated by a sinusoidal surface current in a bifilar helix [12], one can get a transcendental equation for stationary trajectories for a narrow beam.

The electron trajectories are stable with respect to the excitation of betatron oscillations, in two areas of beam parameters and external fields. These areas are separated by the vicinity of the cyclotron resonance where a perturbation of transversal oscillations results to their exponential growth, so the helical trajectory approach assuming $\beta_{\perp}/\beta_z \ll 1$ becomes non-applicable.

The stationary trajectories were derived from external magnetic fields (wiggler and guide ones) only. In a real FEL an RF wave appears that the relations for the velocity components should be derived more accurately. Assuming $a_s \ll a_w, a_g$ we consider the effect of the RF wave as a tare for the motion over non-perturbed stationary trajectories. The velocity vector of each particle

is resolved to "wiggler" and "radiation" parts: $\mathbf{v} = \mathbf{v}^w + \mathbf{v}^R$. The former is $\mathbf{v}^w = (c\beta^w \cos k_w z, c\beta^w \sin k_w z, c\beta_z^w)$, the latter is $\mathbf{v}^R = (c\beta_{\perp}^R \cos \theta, c\beta_{\perp}^R \sin \theta, c\beta_z^R)$. The equation for longitudinal momentum of the particle results in

$$\gamma \frac{d\beta_z^R}{dZ} = \frac{\beta_w}{\beta_{z0}} \left(\frac{ck_s}{\omega_0} + \frac{d\varphi}{dZ} \right) a_s \sin \psi - \frac{\beta_w}{\beta_{z0}} \frac{da_s}{dZ} \cos \psi + \frac{1}{\beta_{z0}} \left(\frac{d\gamma}{dZ} \right)_p - \beta_z^R \frac{d\gamma}{dZ} \quad (7)$$

where $(d\gamma/dZ)_p$ is the potential term of Eq.(4) (i.e. the second one on the right side).

3 FEL AMPLIFIER EXPERIMENTS AND SIMULATION RESULTS

Basic parameters of FELs and electron beams in experiments [1–4] are presented in Table 1.

Table 1. Parameters of millimeter-wave FEL amplifiers [1–4].

	CESTA	JINR	MIT		
			I	II	III
Electron beam energy E_b , MeV	2.2	1.5	0.75	0.75	0.75
Electron current in wiggler I_b , A	500	50–70	90–119	300	300
Electron beam radius r_b , cm	0.5	0.3–0.5	0.25	0.25	0.25
Initial energy spread of the electron beam, %	< 1.5	1–2			
Wiggler period λ_w , cm	12	7.2	3.18	3.18	3.18
Wiggler magnetic field amplitude B_w , kGs	1.1	2.1	0.63	1.47	0.63
Guide magnetic field amplitude B_g , kGs	no	–1.4	4.06	10.92	–10.92
RF wave frequency $f_0 = \omega_0/2\pi$, GHz	35	35	33.9	33.9	33.9
Radiation spectral band (FWHM), GHz	0.16	0.2–0.3	<0.16	<0.16	<0.16
Operating mode (cylindrical waveguide)	H_{11}	H_{11}	H_{11}	H_{11}	H_{11}
RF power at the wiggler entrance, kW	8–10	6–20	8.5	8.5	8.5
RF power at the wiggler exit, MW	15	2–3	5.8	4.2	61
FEL amplifier efficiency, %	1.5	2–3	9	2	27
Spatial increment, dB/m	33	23	44	38	41
Saturation length, cm	~ 150	~ 120	~ 120	~ 150	~ 170

Numerical simulation of microwave amplification and electron beam bunching [9] for CESTA experiments [1,2] under Compton high-gain regime model agrees with the experiment well. It is in conformity with the above criteria for space-charge effects. The plasma wavelength ~ 11 m much exceeds the interaction length. The relation (2) is not satisfied at least by an order of magnitude.

For the JINR FEL amplifier with reversed guide magnetic field [3] the parameters are in the intermediate range where no any interaction type dominates. (Parts of the inequalities (1), (2) are comparable in pairs.) From (3), the Landau damping is negligible in this case.

The dispersion relation at non-zero guide magnetic field stated in [7] at $\beta_{\perp}/\beta_z \ll 1$, $a_s \ll a_w, a_g$ allows one to transform Eq. (5) to

$$\frac{d\theta_j}{dZ} = -\frac{c(k_s + k_w)}{\omega_0} + \frac{1}{\beta_{zj}} + \frac{\omega_p \sqrt{\Phi_0}}{\omega_0 \beta_{z0}} \quad (8)$$

The previous simulation [13] for the parameters of experiment [3] had been performed under Compton

model with phase equation as (8). In this paper we allow space charge using Eqs. (4), (8), (6) for the simulation.

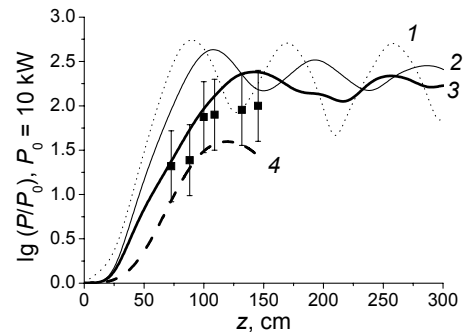


Figure 1: RF power of FEL amplifier [3] versus length. The experimental data are shown with dots. Curves 1,2,3 – Compton model simulation [13] at the energy spread of: 1) 0; 2) 3%; 3) 5%; curve 4 – Raman model simulation for cold beam.

Fig. 1 presents the results of RF power calculations [13] for different values of the initial energy spread of the beam, and the spatial distribution of RF power obtained in Raman model for cold beam (zero energy spread). The experimental results [3] are depicted by dots. They took an intermediate position between simulation results from extreme models. This fact is in agreement with the idea of continuity in the parameter space.

A detailed investigation of FEL amplifier with guide magnetic field was carried out at MIT [4]. Three basic regimes were studied: 1) positive (conventional-direction) guide magnetic field with Larmor frequency less than the cyclotron resonance frequency; 2) positive guide field at the other side from the cyclotron resonance; 3) negative (reversed) field (see Table 1). The experimental results indicated that the reversed field regime turned out to be most preferable by the beam current, spatial increment of RF power, saturation level and energetic efficiency.

According to the estimation over all criteria (1), (2), (3), the space charge effect is sufficient for each regime. The plasma wavelength ($\sim 20\text{--}30$ cm) gets into the interaction length several times. The inequality (2) is satisfied by over than one order of magnitude. The limitation over thermal spread is practically absent: the right side of (3) amounts to several units.

So we have a definite Raman FEL amplifier. We simulated it over Eqs. (4), (5), (6), (7).

At the small positive guide field (I) the operating point is relatively near the cyclotron resonance. Transversal oscillations of the particles grow much so that helical-trajectory approach is not applicable, the solutions become nonconvergent. In the regime of large guide field (II) the simulation yields an excessive level of the output power. Though this discrepancy persists in the three-dimensional simulation [5] also. According to the paper [14], an amplification depression in the regimes with positive guide field may be caused by the competition between the operating wave and high-frequency modes excited from the noise level.

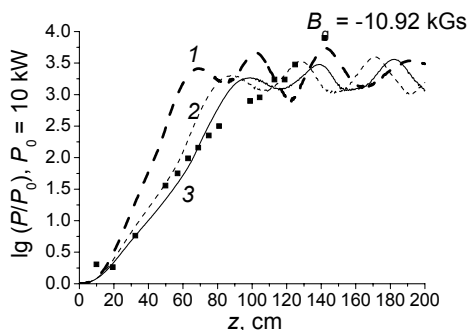


Figure 2: RF power of FEL amplifier [4] versus length: experimental data (dots) and simulation results for different values of the energy spread: 1) 2%; 2) 8%; 3) 10%.

For the reversed field case an acceptable agreement with the experiment has been obtained in the distribution of the radiation power over the length (Fig. 2).

The difficulties in the simulation of the positive-field regimes, especially approaching the cyclotron resonance, confirm the limited possibilities of quasi-one-dimensional helical-trajectory models for description of the particle motion. For more adequate simulation of the FEL amplification it is expedient to employ full-scale three-dimensional models. One should foresee a possibility of excitation of RF parasite modes, take into the account the features of beam delivery to the interaction region and wiggler&solenoid inhomogeneties.

4 CONCLUSIONS

Series of numerical simulation of experiments [1–4] in helical-trajectory approach in various FEL regimes have been carried out. The used models are compact and don't demand much computing resource. Simultaneously they are suitable for simulation of regimes with reversed guide field in order to define basic amplification parameters. This may be employed during preparation of an experiment for selection of amplifier operating parameters.

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