Study of Microbunching Instability in MESA

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Motivation

The Institute for Nuclear Physics (KPH) at Mainz is building a multi-turn energy recovery linear accelerator known as the Mainz Energy-recovering Superconducting Accelerator (MESA) to deliver a CW beam at 105 MeV with short pulses, high current and small emittance for physics experiments with an internal target. Beam quality preservation is a general concern and Space Charge (SC) effects potentially cause beam quality degradation. We present a study on the Microbunching Instability (MBI) caused by longitudinal space charge (LSC) in MESA. This instability can lead to an increase in emittance and energy spread during recirculation. Our results demonstrate the impact of the MESA arc lattice design on the development of Microbunching Instability.

MESA Layout

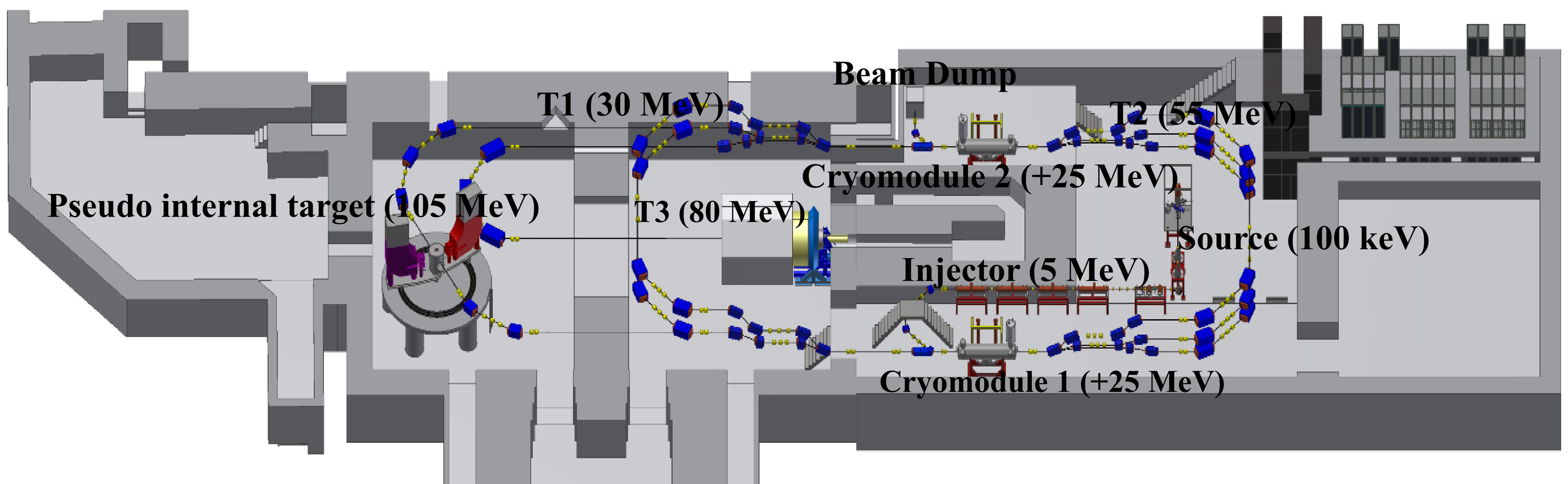


Fig 1: MESA Lattice

Table 1: MESA Beam parameters

Beam Energy	105 MeV
Operation Mode	1300 MHz, CW
Beam Current	10 mA
Bunch Charge	7.7 pC
Bunch Length	1.281 mm
Normalized Emittance	0.002 mm mrad

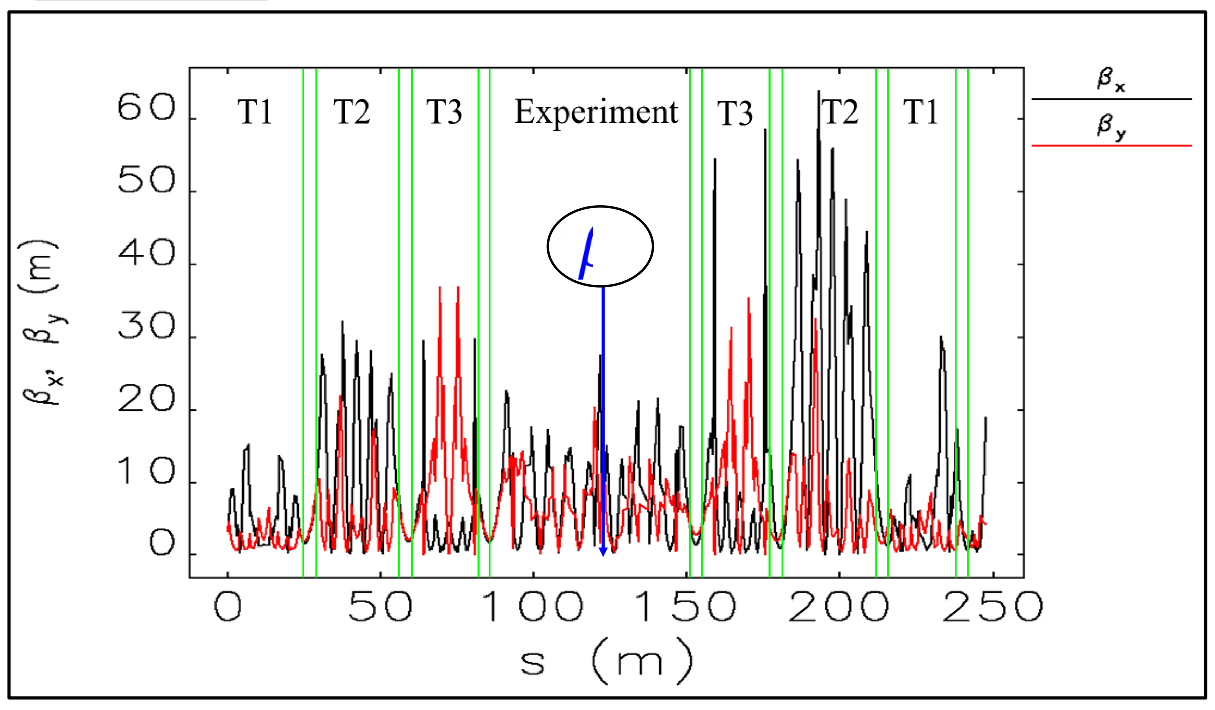
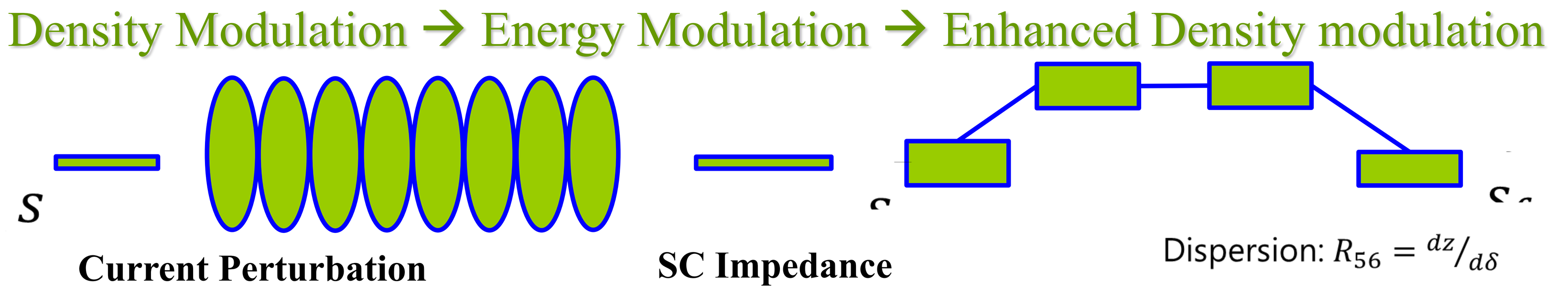


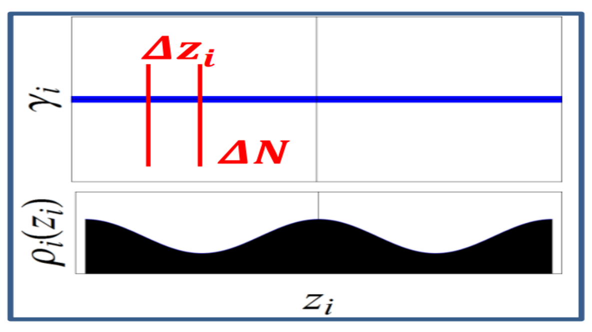
Fig 3: Beta functions of MESA without Space Charge

Theoretical Formulation of MBI

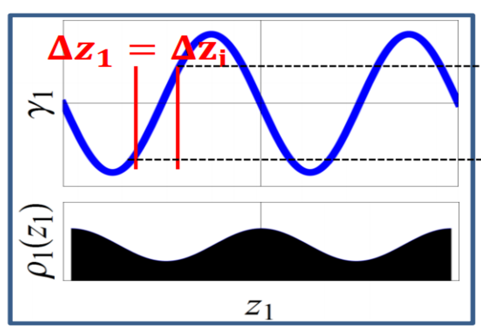
- **Microbunching Instability:**

Density Modulation → Energy Modulation → Enhanced Density modulation



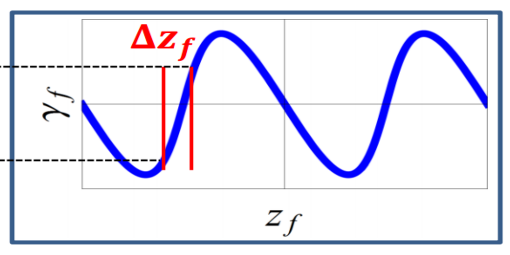


There are ΔN particles in interval Δz_i



$z_1 = z_i$

Dispersion: $R_{56} = {}^{dz}/_{d\delta}$



There ΔN particles are in shorter interval $\Delta z_f < \Delta z_i$

$z_f = z_i + R_{56}\delta$

- **Longitudinal Space Charge:**

Field in presence of LSC,

$$E_z(r,z) \approx -\frac{2qN}{4\pi\varepsilon_0\gamma^2}\frac{d\lambda(z)}{dz}\left(log\frac{r_p}{r_b} + \frac{{r_b}^2 - r^2}{2{r_b}^2}\right)$$

➢ Field is proportional to derivative of bunch profile and energy. (significant for very short bunches)

Impedance due to LSC, $Z(k) = \frac{iZ_0}{\pi r_b \gamma}\left[1 - \frac{kr_b}{\gamma}K_1\left(\frac{kr_b}{\gamma}\right)\right]$

Linear gain, $G = \left|\frac{\Delta \hat{I}_f}{\Delta \hat{I}_i}\right| = 4\pi\frac{I_0}{I_A}L_s\frac{|Z(k)|}{Z_0}k|R_{56}|$

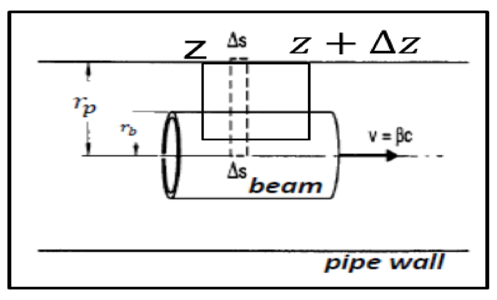


Fig 4: Ultra-relativistic beam of radius r_b

Beam radius, $r_b = 1.7(\sigma_x + \sigma_y)/2$

$k = 2\pi/modulation\_wavlenth$

Space Charge Envelope Solver

- **RMS beam envelopes with SC**

$x(s) = \bar{x} + \delta D_x \;\; p_x(s) = \bar{p}_x + \delta D'_x$

$\sigma_x = \sqrt{\langle\bar{x}^2\rangle}, \sigma_y = Y = \sqrt{\langle\bar{y}^2\rangle} \;\; X = \sqrt{\langle x^2\rangle} = \sqrt{\langle {\sigma_x}^2 + \sigma_\delta{}^2 {D_x}^2\rangle}$

$$\frac{d^2\sigma_x}{ds^2} + \left(\kappa_x(s) - \frac{K}{2X(X+Y)}\right)\sigma_x - \frac{{\epsilon_x}^2}{{\sigma_x}^3} = 0, \;\; \frac{d^2\sigma_y}{ds^2} + \left(\kappa_y(s) - \frac{K}{2Y(X+Y)}\right)\sigma_y - \frac{{\epsilon_y}^2}{{\sigma_y}^3} = 0$$

- **Numerical transformation of the lattice functions with SC**

$B(s) = \varepsilon^2\begin{pmatrix}\beta & -\alpha \\ -\alpha & \gamma\end{pmatrix}$ (Beam matrix)

$B_{s_1} = M.B_{s_0}.M^T$ (Transformation of beam matrix)

$\begin{pmatrix}D \\ D' \\ 1\end{pmatrix}_{s_1} = M.\begin{pmatrix}D \\ D' \\ 1\end{pmatrix}_{s_0}$ (Transformation of dispersion matrix)

Transport matrix with linear space charge (kick): $M(s_0, s_0 + \Delta s) = M_{\Delta s/_2} M^{sc}_{\Delta s} M_{\Delta s/_2}$

$M^{sc}_{\Delta s} = \begin{pmatrix}1 & 0 \\ f_{sc}\Delta s & 1\end{pmatrix} \quad f_x = \frac{K}{a_x(a_x + a_y)} \quad {a_x}^2 = \beta_x\varepsilon_x + (D_x\delta)^2, \; {a_y}^2 = \beta_y\varepsilon_y$

Simulation Results

- **Space Charge Analysis along the beamline**

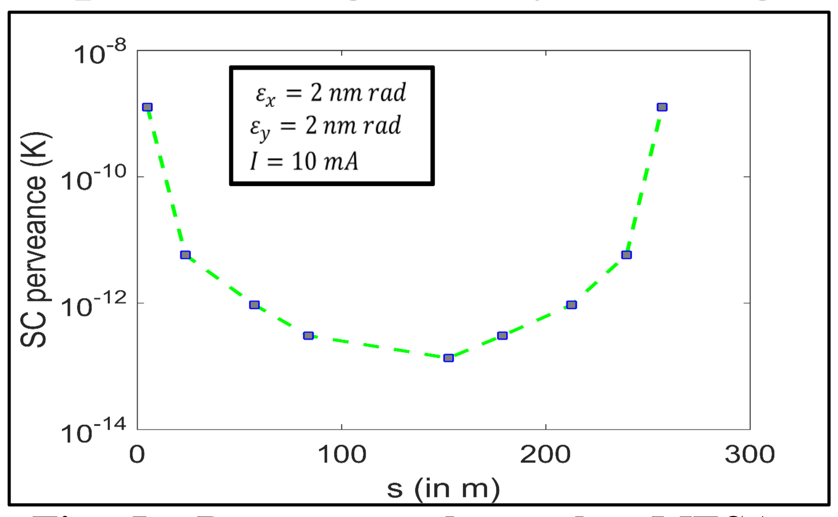


Fig 5: Perveance along the MESA beamline. When the beam energy is low during injection and extraction effect of SC is very high.

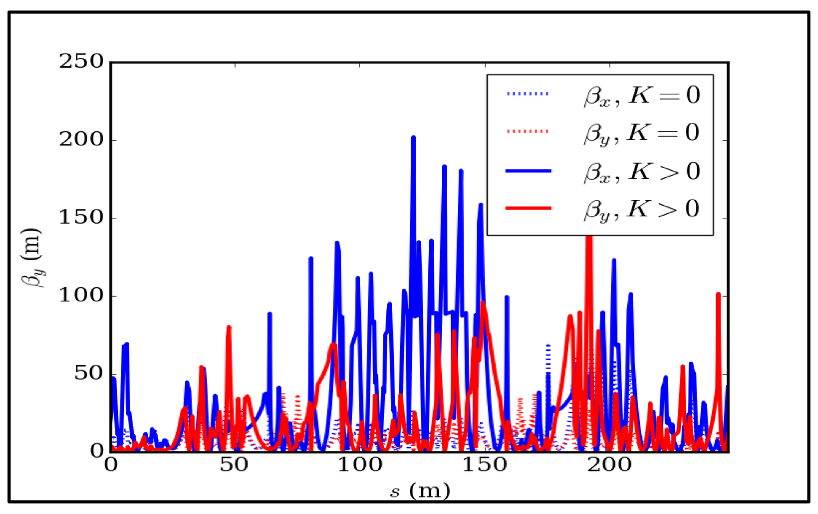


Fig 6: Beta functions with space charge. Due to SC there is mismatch and beam envelope expands unsustainably.

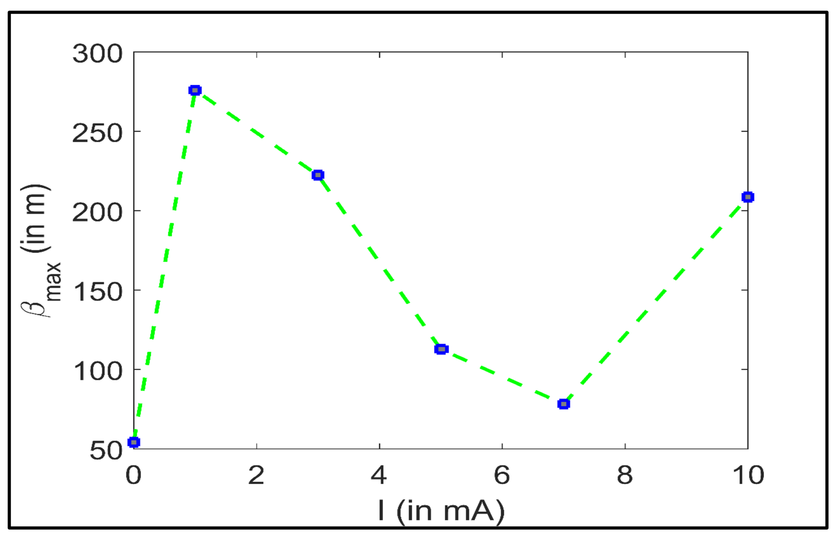


Fig 7: Maximum beta as a function of current. Lattice is not robust against small changes. There is a strong mismatch of envelopes with SC.

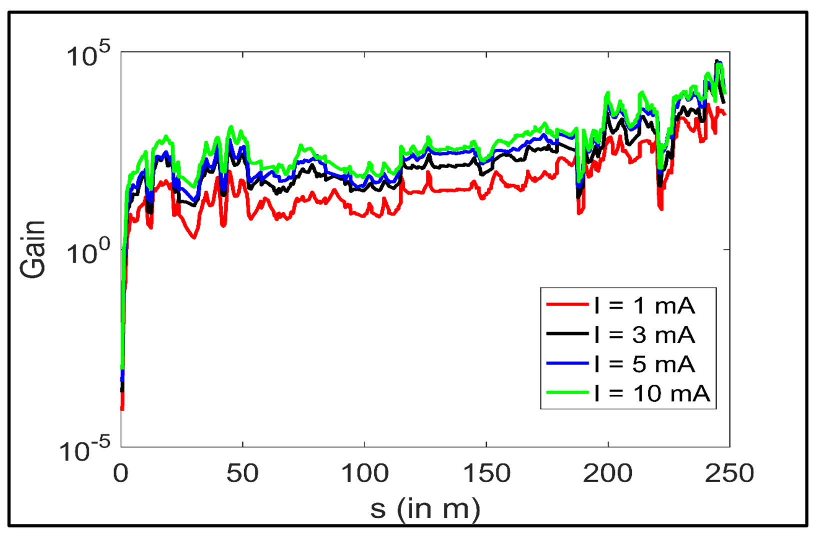


Fig 8: Gain along the beamline at different values of current.

Conclusion and Outlook

Conclusion:

- Gain value is high after 1 mA (fig 8).
- MESA main goal is fixed beta function at internal target experiment can't be fulfilled by current MESA lattice. We have to optimize the arcs.

Challenges:

- Matching with SC in a non-periodic structure like MESA.
- Beam stability during recirculation.
- Fixed beta functions at internal target positions (fig. 1).

Future Plans:

- Improve the lattice and work on the injection arc.
- Dispersion study with SC.
- Benchmarking with Particle tracking code.

Acknowledgment

We are grateful for the financial support provided by DFG through GRK 2128 Accelence project D-3 to study effect of space charge and wake-fields in Energy Recovery Linac MESA.

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