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TOWARDS BEAM-BEAM SIMULATIONS FOR FCC-ee*

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Abstract

The FCC-ee (Future Circular Collider) lepton collider is currently the most favored next generation research infrastructure project at CERN, aimed at studying properties of standard model particles with the highest precision ever. The chosen parameters of the machine yield unprecedented conditions which give rise to previously unseen dynamical effects during collisions. The exploration and understanding of these beam-beam effects is of crucial importance for the success of the FCC-ee feasibility study. To address this challenge, a new general purpose software framework for beam dynamics simulations is currently under development at CERN. This presentation will discuss the contributions to the software development related to beam-beam effects with benchmark studies and applications.

INTRODUCTION

The FCC-ee feasibility study [1] aims at verifying the possibility to build a near 100 km long circular collider in the Geneva area. The study would be the first stage towards a 100 TeV hadron collider, termed FCC-hh. These colliders aim notably to search for new physics beyond the standard model. During beam-beam collisions the particles in the two colliding beams experience an electromagnetic (EM) force by the presence of the opposite beam. This nonlinear beam-beam "kick" perturbs the particle trajectories resulting in long term changes in the dynamical behavior of the beams, collectively referred to as beam-beam effects [2]. Due to the nonlinear nature of the interaction, a purely analytical treatment of these effects is excluded. Instead, numerical multiparticle simulations are commonly used where the dynamical variables of the particles are tracked. The difficulty in simulating this dynamics lies in the complexity of the FCC-ee machine and the interplay of the different dynamical effects.

The collider infrastructure is designed to maximize achievable luminosity. To this end, a setup called the crab-waist scheme [3] has been proposed, which mitigates the nonlinear effect of beam-beam collisions and achieves extremely small, nanometer sized beams at the interaction points (IPs) by colliding beams with a crossing angle of 30 mrad and by using special purpose, so called crab-sextupoles. Another setup, commonly used in synchrotron light sources, is the top-up injection scheme [4], which means that new, low intensity beam bunches are injected with a high frequency to maintain high bunch intensities in the beams. This helps to maintain high luminosity, which decreases due to the reduced beam lifetime caused mainly by the emission of radiation during the collision.

Beamstrahlung

Arguably one of the most important beam-beam effects in the FCC-ee is beamstrahlung, i.e., the emission of high energy (up to GeV order) photons relative to the particle energy during collision. The photon emission happens due to the local bending of the particle trajectories in the collective EM field of the opposite bunch. Beamstrahlung has deteriorating impact on the bunch quality. The quantum nature of photon emission increases the energy spread of the beam, which is converted to an increase of the bunch length [5]. It also reduces the luminosity and leads to an increased loss rate of particles due to the reduction of the dynamic aperture [6].

SIMULATION OF FCC-ee BEAM-BEAM EFFECTS

The FCC-ee is a highly complex machine, where many dynamical effects interplay with each other. Therefore a simulation that aims to model the beam dynamics has to be self-consistent, i.e., not relying on any other external input or modification of intermediate variables during the simulation. Currently there exist several toolkits to model beam dynamics in high energy colliders. Some of the most well known codes are `MAD-X` [7], `SixTrack` [8], `PyHEADTAIL` [9] and `COMBIp` [10]. Each of these codes have been developed aiming for different studies, each having different features. There are other codes which were developed specifically for studying beam-beam effects in colliders. Some of the most well known are `BBWS` [11] and `BBSS` [12], `LIFETRAC` [13] and `GUINEA-PIG` [14]. Each of these codes uses different approximations to boost performance or numerical precision for certain types of studies. The main challenge that limits simulation capabilities is to interface such codes when we want to study the interplay of different mechanisms, crucial for the FCC-ee feasibility study. Hence the need for a single, self-consistent and open source simulation tool following mainstream computing paradigms, i.e., modern programming languages and compatibility with multiple platforms such as CPU or GPU from different vendors and which incorporates all elements of a complex accelerator, necessary for studying FCC-ee beam dynamics. A new simulation tool, called `xsuite` [15], targets the above outlined demanding

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criteria. This contribution will present recent progress on the development of this framework, related to beam-beam collision modelling, and first studies, performed using the tool.

Beam-beam Models

In the following we describe how beam-beam interactions in high energy colliders are most commonly modelled in a multiparticle tracking code, such as `xsuite`. We model the interaction of two bunches at a time, each of which consists of a number of macroparticles, usually $10^4 - 10^6$. The macroparticles each have their own 6D dynamical variables (x, p_x, y, p_y, z, p_z). During a collision the two bunches move across each other and the particles receive a kick by the collective EM field of the opposite bunch, which corresponds to a change in their momentum variables. Our approach follows [16], in which the two bunches are first rotated and Lorentz-boosted into a frame where the initially large crossing angle is eliminated and the collision is head-on. In this new reference frame the EM fields are purely transversal due to the ultra-relativistic nature of the collision ($\gamma = \frac{E}{E_0} >> 1$), which makes the computation of the beam-beam kick easier. The bunches are longitudinally sliced to preserve symplecticity, to account for the transverse offset of the particles due to the rotation as well as the beam size variation due to the hourglass effect. Then they are moved across each other by one slice at a time, where each particle in each slice will now receive a separate kick from each slice of the opposite bunch. In general, the higher the number of slices, the more accurate is the model as more slices can better model the transverse geometry of the bunch, which is important for configurations with a large crossing angle, such as the FCC-ee. The sufficient amount of slices for a given beam can be estimated as

$$N_s = 10 \cdot \frac{\sigma_z}{\min(L_i, \beta_y^*)}, \qquad (1)$$

where N_s is the number of slices, σ_z is the equilibrium RMS bunch length, β_y^* is the optical beta function at the IP and L_i is the interaction length, i.e., the overlap area between the two bunches at collision, as described in [17]. The ratio of bunch length to waist or collision length is a measure for the variation of the bunch cross section during collision, with a high ratio indicating that more slices are required for accurate simulation.

In `xsuite` the beam-beam kick is computed in the soft-Gaussian approximation, using the Bassetti-Erskine formula [18], which is a computationally cheap approximation assuming and valid for Gaussian beam profiles. The formula computes the kick using the statistical moments of the slices of the opposite bunch, with which a given particle is interacting. The collision is simulated by sliding the sliced bunches across each other in discrete steps where in each step there are a number of slices of bunch 1 overlapping with slices of bunch 2. In each step the overlapping slice pairs are interacting, whereby the particles in one slice experience the kick from the opposite slice. This process is illustrated on Fig. 1.

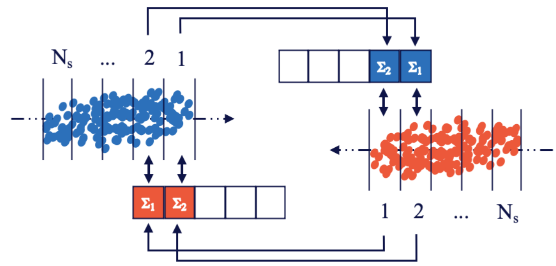

Figure 1: Modelling of beam-beam collision in a numerical tracking code, using the Lorentz-boost approach. Note the difference in the transverse offset between the head and the tail of the bunches.

In this model there is a trade-off between computational efficiency (speed) and accuracy, depending on the update frequency of the beam-beam kick strength. On one end, we can model the collision by only tracking one (called weak) bunch and freezing the other (called strong) bunch. This is called the weak-strong approximation. In this case the strong bunch slices represent a constant EM lens each of which the strength is precomputed and never changed. This model is not self-consistent because it does not follow the evolution of the strong bunch, but it is optimal for studying multi-turn single-particle effects, e.g., the evolution of the weak bunch sizes and emittances over many tracking turns. It is the computationally cheapest but the least accurate beam-beam model.

In the so called quasi strong-strong approximation both bunches are tracked and the beam-beam kicks are periodically recomputed using the up-to-date statistical moments of the bunch slices. This model is more accurate than the weak-strong but computationally more expensive because we have to recompute the statistical moments periodically. The quasi strong-strong approach is a good approximation if we want to study slow instabilities such as the 3D flip-flop instability [17], and configurations with a low disruption parameter, where the bunch profile does not change significantly within one collision. Recomputing the statistical moments every turn allows to simulate fast instabilities, e.g., the recently discovered coherent head-tail instability [19]

At the other end of the trade-off spectrum is the full strong-strong approach, where the statistical moments of each slice of both bunches are recomputed after each slice pair interaction. This is the computationally most expensive but the most realistic and the only self-consistent approach. With this we can more accurately simulate fast instabilities and the disruption of the bunch profile within the same collision (meaning a high disruption parameter).

The beam-beam model of `xsuite` is being developed in a way that the choice of the approximation is flexible and uses the same code. The model is planned to be extended by the capability to simulate background generating pro-

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cesses, such as beamstrahlung (first implementation already exists, benchmark shown in Sec. 4.) and Bhabha scattering (implementation ongoing), as well as to use a numerical field-solver for general, non-Gaussian profiles, which is to be tested in the future. Such field solvers are already implemented in xsuite for other purposes but not yet linked to the beam-beam model.

PERFORMANCE OF THE XSUITE BEAM-BEAM MODEL

We have chosen to benchmark the performance of the xsuite beam-beam model in the strong-strong approach, that being the computationally heaviest. We have chosen COMBIp as our benchmark code, which is a well established tracking tool optimized for strong-strong simulations at the LHC. In the study we have performed a single beam-beam collision followed by a linear tracking through the LHC arc and we tracked for 10 turns with both codes in the exact same setting. The computation time needed for the tracking with the linear transfer map (being a simple matrix multiplication) is negligible compared to that needed for the simulation of the strong-strong beam-beam collision, therefore the measured wall times are characteristic of the beam-beam model. We have opted for a configuration featuring the HL-LHC, with no crossing angle and round Gaussian beams for simplicity. We have initialised 10^6 macroparticles and performed a scan in the number of longitudinal slices in the beam-beam model. Figure 2 shows a comparison of the wall times averaged per turn for COMBIp (blue) and for xsuite (red).

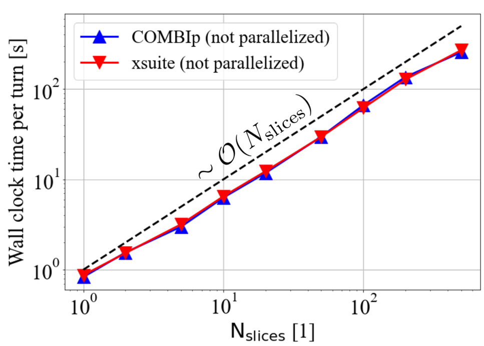

Figure 2: Simulated average wall clock time per turn of the strong-strong beam-beam model as a function of the number of longitudinal slices for xsuite (red) and the reference code COMBIp (blue).

The study shows that the runtimes scale approximately linear to the number of slices. In addition, it can be seen that xsuite could be optimised to have similar runtimes to COMBIp. Note that these simulations did not use any parallelisation. In xsuite it is possible to use OpenMP for multi-threading, which has been tested with an example study, scanning the number of threads and measuring the wall time, using the FCC-ee Z parameters, and tracking for 100 collisions corresponding to 100 half turns, with 10^6 macroparticles and 300 slices, which is the optimal setting for this configuration. The parallelisation is done on the loop over the macroparticles inside the beam-beam model. The obtained scaling is shown on Fig. 3. The displayed wall times are normalised to that with only one thread requested. The scaling up to 4 threads is ideal, with a factor 4 speedup. Afterwards it saturates at about a factor 5 speedup compared to the sequential case. The saturation is likely caused by the relatively low number of macroparticles per slice (3333). After a given thread count, the time needed to communicate between the C kernel and the python interface becomes comparable to the time spent, per thread, looping over the particles. This could be improved by using a higher number of macroparticles.

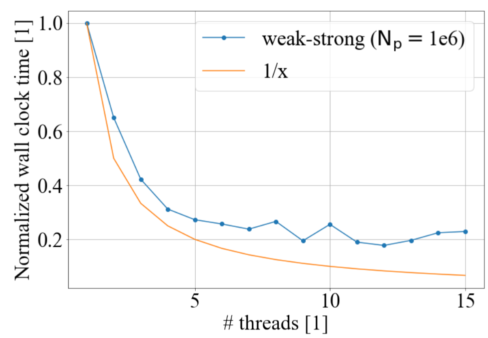

Figure 3: Integrated wall clock time as a function of the number of compute threads for a set of weak-strong simulations (including a linear half-arc) with the FCC-ee Z parameters, each tracked for 100 half turns.

Full scale simulations using an element by element model of the collider ring and the strong-strong collision model with many macroparticles will likely require a better parallelisation scheme. xsuite is designed to be a multiplatform software, and for beam-beam simulations the performance on GPUs is planned to be tested in the near future.

BEAMSTRAHLUNG BENCHMARK

As mentioned in the previous section, a first model of the beamstrahlung photon emission has been implemented in xsuite. The implementation is based on GUINEA-PIG, which is considered to be a state of the art tool for beamstrahlung simulation. It is capable of modelling a single beam-beam collision and generating background radiation of different kinds. It uses a particle in cell (PIC) solver, which corresponds to a fully self-consistent strong-strong model. In the following benchmark study we have compared the energy spectrum of the emitted beamstrahlung photons in a flat beam configuration ($\varepsilon_x = 2.7 \cdot 10^{-10}$ m, $\varepsilon_x = 2.7 \cdot 10^{-12}$ m, $\beta_x = \beta_y = 0.15$ m) with the nominal FCC-ee crossing angle (30 mrad) between GUINEA-PIG and xsuite, using the weak-strong approximation in the latter, with 100 slices having a uniform bin width. With both codes,

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we have performed a single collision (without tracking in the arc) using 10^5 macroparticles and recorded the photon spectrum, which is shown in Fig. 4. The plot shows the absolute energy spectrum of the emitted beamstrahlung photons against the normalised photon count, which shows a good qualitative agreement between the two codes.

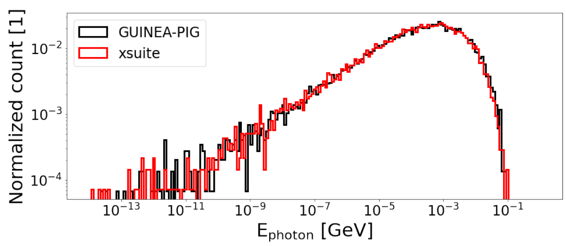

Figure 4: Energy spectrum of emitted beamstrahlung photons using `GUINEA-PIG` (black) and `xsuite` (red). Photon counts are normalised to 1.

A next step in this direction is to implement an event generator for the Bhabha-scattering process, which is useful for simulating the beam lifetime, beam losses as well as photons used for luminosity calibration.

SIMPLIFIED TRACKING SIMULATIONS

After benchmarking the beam-beam element's performance and the beamstrahlung photon generation, the next step is to perform simplified tracking simulations with `xsuite`. For these studies we exploit the superperiodicity of the FCC-ee ring, namely we only simulate half a turn in one iteration, using the half tunes. Our simulations consist of an IP, including beamstrahlung, plus a simplified tracking over the half arc with a linear transfer matrix. Furthermore, the arc is split into 3 segments and we insert 2 crab-sextupoles between them to implement the crab waist scheme. We start each (half) turn in front of the right sextupole, where our observation point for the emittances is located. Our observation point for the RMS beam sizes is located in front of the IP. We implement an effective model for synchrotron radiation, by using a simplified exponential damping and Gaussian noise excitation. In the following studies we use 300 bins for the longitudinal slicing of the bunches, each containing an equal amount of charge. Our setup is sketched on Fig. 5.

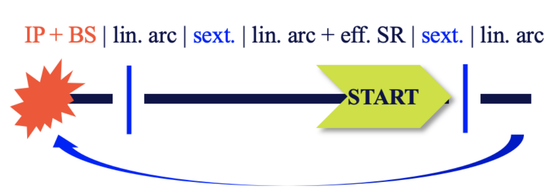

Figure 5: Simplified tracking model used for our simulations presented in this contribution.

Equilibrium Bunch Length

First we have looked at the evolution of the weak bunch length, which blows up as a direct consequence of beamstrahlung, in the weak-strong approximation. We initialise the length of the weak bunch to the equilibrium value without beamstrahlung, but with synchrotron radiation. The length of the strong bunch, a constant EM lens in this case, but computed from an actual Gaussian distribution of 10^6 macroparticles, is initialised with the equilibrium bunch length with beamstrahlung. We have performed tracking for 10^4 turns in all FCC-ee configurations using 10^4 macroparticles in the weak bunch. Figure 6 shows the bunch length evolution in units of the equilibrium length with beamstrahlung.

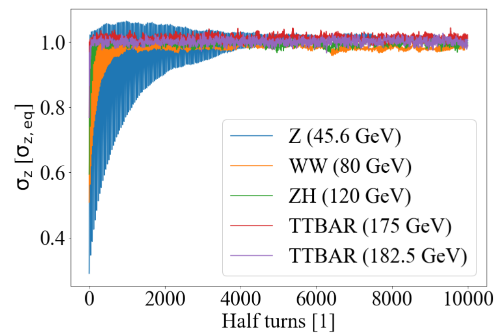

Figure 6: Evolution of weak bunch length for all FCC-ee energies. The values are always normalised to the nominal equilibrium bunch length, taken from [1].

It can be seen that the bunch length converges to the equilibrium value in all configurations. The rate of damping increases with increasing energy which corresponds to our expectations.

Crab Waist and Transverse Blowup

In the following study, using the same tracking model as outlined earlier, we have investigated the equilibrium transverse bunch sizes. These blow up due to the nonlinear kick received from the beam-beam interaction, even without beamstrahlung. In general the crab-waist scheme improves the nonlinear dynamics at the collision and mitigates this transverse blowup. With the crab-sextupoles implemented in our model, we expect no blowup in either transverse size. Since the geometrical magnet strength k_2 of the crab-sextupoles is a free parameter which affects the final blowup, we have performed an optimisation study where we scanned this parameter and observed the equilibrium bunch sizes. In each setting we have performed tracking for $3 \cdot 10^4$ turns, otherwise identical parameters to the previous study. Note that the previous study has been performed using the optimal crab-sextupole strength. Figure 7 shows the equilibrium bunch sizes (of the weak bunch) as a function of the k_2 geometrical sextupole strength. The values on the y axis are normalised to the initial bunch size, which is also the expected final size since we expect no blowup.

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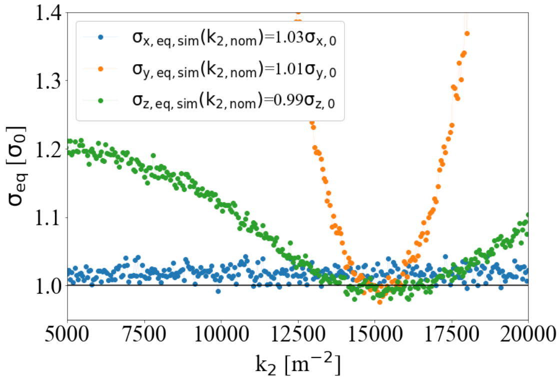

Figure 7: Simulated equilibrium weak bunch sizes as a function of the geometrical crab-sextupole strength k_2 at the FCC-ee Z resonance. The values are always normalised to the nominal equilibrium (=initial in this case) bunch sizes.

The statistical uncertainty on the presented values is around 1%. The results verify that the minimum of the blowup occurs with the sextupole strength set to its nominal value, reported in [1].

In case of strong-strong simulations with the same settings, we observed a transverse blowup. In this case, both the horizontal and vertical bunch size blows up for both bunches. The reason for this could be an insufficient statistics in the bunch slices to compute the beam-beam kick. Since we have used 10^4 macroparticles per bunch with 300 slices, it equals to about 33-34 macroparticles per slice. Alternatively, the blowup could be a sign of the recently observed coherent head-tail instability [19]. The understanding of this blowup in strong-strong simulations requires further investigation, which is currently ongoing.

SUMMARY AND CONCLUSIONS

We have developed a flexible beam-beam collision model in the `xsuite` beam dynamics simulation framework and performed several benchmark studies. A first implementation of Beamstrahlung is available [15] for further studies, such as collimation. We have experienced a rapid transverse blowup in strong-strong simulations, which is likely linked to insufficient statistics. There is ongoing work to investigate the source of this blowup using further parameter scans (e.g., tune scans) as well as frequency map analysis (FMA) [20].

After a sufficient benchmark of the `xsuite` beam-beam model, we are planning to perform studies related to the 3D flip-flop instability which can result from an initial asymmetry in the colliding bunch intensities. This scenario will be relevant during the FCC-ee top-up injection and can efficiently be simulated using `xsuite`, since the injection and the beam-beam collision can be treated self-consistently within the same framework. Another priority is to implement an efficient event generator for the Bhabha scattering process which will enable us to estimate luminosity, study beam lifetime and better understand the consequences of beam background on the infrastructure.

Once the necessary ingredients are finalised, `xsuite` will have a large potential for complex beam-dynamics studies in the context of the FCC-ee, such as the study of lattice imperfections, the interplay with a real lattice model or with wakefields, multiple IP configurations, monochromation and much more.

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