Multi-Species Child-Langmuir Law with Application to ECR Ion Sources

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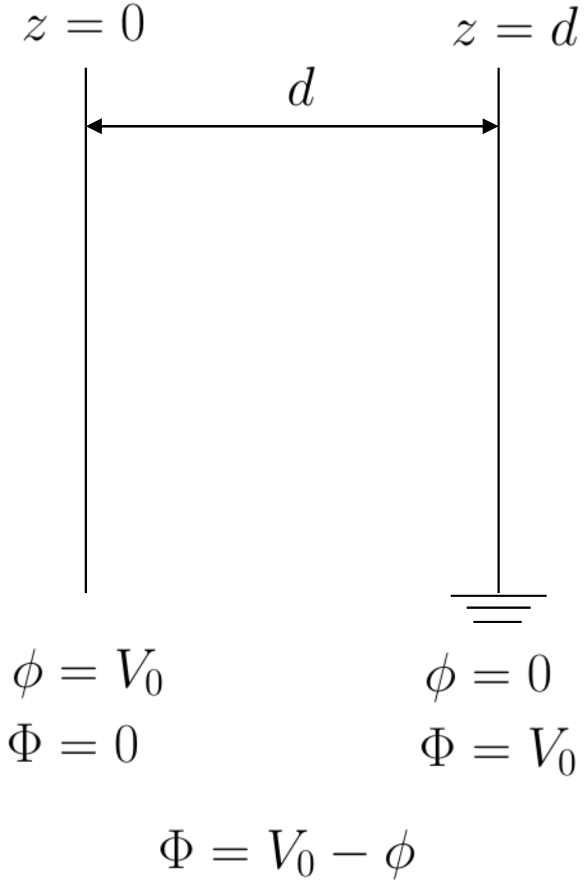
*Previously from NSCL/MSU which provided significant support to this work

Classical Single-Species Child-Langmuir (CL) Law

- Maximum extractable ion current at space-charged limited flow
  - Infinite planar diodes
  - Cold beam



- Solution: current density $J = \frac{4}{9}\epsilon_0\sqrt{2\frac{q}{m}}\frac{V_0^{3/2}}{d^2}$

Generalize to Multi-Species Beam

	Single-Species (3 equations)	Multi-Species (2n+1 equations)
Poisson Equation	$\nabla^2\Phi = \frac{1}{\epsilon_0}\rho$	$\nabla^2\Phi = \frac{1}{\epsilon_0}\sum_{i=1}^{n}\rho_i$
Continuity Equation	$J = \rho v = \mathrm{const}$	$J_i = \rho_i v_i = \mathrm{const}$
Energy Conservation	$\Phi = \frac{1}{2}\frac{m}{q}v^2$	$\Phi = \frac{1}{2}\frac{m_i}{q_i}v_i^2$

Possible to Recast into Single-Species Form?

$$\rho = \sum_{i=1}^{n} \rho_i \qquad J = \sum_{i=1}^{n} J_i$$

Classical CL solution immediately follows:

$$J = \frac{4}{9}\epsilon_0 \sqrt{2\left(\frac{q}{m}\right)_{\text{eff}}} \frac{V_0^{3/2}}{d^2}$$

Effective Single-Species (3 equations)

$$\nabla^2\Phi = \frac{1}{\epsilon_0}\rho$$

$$J = \rho v_{\text{eff}}$$

$$\Phi = \frac{1}{2}\left(\frac{m}{q}\right)_{\text{eff}} v_{\text{eff}}^2$$

Multi-Species (2n+1 equations)

$$\nabla^2\Phi = \frac{1}{\epsilon_0}\sum_{i=1}^{n}\rho_i$$

$$J_i = \rho_i v_i = \text{const}$$

$$\Phi = \frac{1}{2}\frac{m_i}{q_i}v_i^2$$

Oak Ridge National Laboratory | Spallation Neutron Source

Solution to Multi-Species Child-Langmuir Equation

- Effective charge-to-mass ratio $\left(\frac{m}{q}\right)_{\text{eff}} = \left(\sum_{i=1}^{n} c_i \sqrt{\frac{m_i}{q_i}}\right)^2$

  $c_i = \frac{J_i}{J}$ is current weight of the i-th species with $\sum_{i=1}^{n} c_i = 1$

- For ECR, current weights c_i can be obtained from measured CSD currents

$$\left(\frac{Q}{A}\right)_{\text{eff}} = \left(\frac{q[\text{e}]}{m[\text{amu}]}\right)_{\text{eff}}$$

- Multi-species solution: $J = 5.47 \times 10^{-8} \left(\frac{Q}{A}\right)_{\text{eff}}^{1/2} \frac{V_0^{3/2}}{d^2} \; [\text{A/m}^2]$

- For circular aperture: $I = \pi R^2 J = 5.43 \left(\frac{Q}{A}\right)_{\text{eff}}^{1/2} \left(\frac{R}{d}\right)^2 (V_0[\text{kV}])^{3/2} \; [\text{mA}]$

$$R = \text{source radius}$$

Example: Artemis Source at NSCL/MSU

- Artemis A ^{16}O

$$\left(\frac{Q}{A}\right)_{\text{eff}} = \left(\sum_{i=1}^{n} c_i \sqrt{\frac{A_i}{Q_i}}\right)^{-2} \Bigg|_{8 \text{ species meas.}} \approx 0.191$$

Ion	A	Q	Q/A	I(emA)	$c_i = I/I_{total}(\%)$
O	16	1	0.0625	0.107	14.41
O	16	2	0.125	0.092	12.35
O	16	3	0.1875	0.065	8.80
O	16	4	0.25	0.164	22.07
O	16	5	0.3125	0.143	19.19
O	16	6	0.375	0.139	18.74
O	16	7	0.4375	0.031	4.17
O	16	8	0.5	0.002	0.28
Total:				0.743	100

(Courtesy Bryan Isherwood)

Take $\frac{R}{d} = \frac{4\text{ mm}}{40\text{ mm}} = 0.1$ $\quad V_0 = 20\text{ kV}$

Multi-species CL current: $I = 2.12\text{ mA}$

Compare $I_{\text{drain}} \approx 1 \text{ to } 2\text{ mA}$

(drain current also has contribution from e$^-$)

Example: VENUS Source at LBNL

- VENUS ^{238}U with ^{16}O support gas

$$\left(\frac{Q}{A}\right)_{\text{eff}} = \left(\sum_{i=1}^{n} c_i \sqrt{\frac{A_i}{Q_i}}\right)^{-2}\Bigg|_{20 \text{ species meas.}} \approx 0.131$$

Take $\frac{R}{d} = \frac{4\text{ mm}}{40\text{ mm}} = 0.1$ $\quad V_0 = 30\text{ kV}$

Multi-species CL current: $I = 3.23\text{ mA}$

Compare $I_{\text{total}} = 2.97\text{ mA}$

$I_{\text{drain}} \approx I_{\text{total}}$? (data not available)

Ion	A	Q	Q/A	I(emA)	$c_i = I/I_{total}(\%)$
U	238	25	0.105	0.035	1.18
U	238	26	0.109	0.051	1.72
U	238	27	0.113	0.068	2.29
U	238	28	0.118	0.088	2.97
U	238	29	0.122	0.115	3.88
U	238	30	0.126	0.15	5.06
U	238	31	0.130	0.175	5.90
U	238	32	0.134	0.193	6.51
U	238	33	0.139	0.21	7.08
U	238	34	0.143	0.205	6.91
U	238	35	0.147	0.178	6.00
U	238	36	0.151	0.142	4.79
U	238	37	0.155	0.11	3.71
U	238	38	0.160	0.072	2.43
U	238	39	0.164	0.043	1.45
U	238	40	0.168	0.031	1.05
O	16	1	0.063	0.3	10.11
O	16	2	0.125	0.3	10.11
O	16	3	0.188	0.3	10.11
O	16	4	0.250	0.2	6.74
Total				2.97	100

(Eduard Pozdeyev; measured at LBNL)

Single- and Multi-Species Child-Langmuir Law Predictions vs Measurements

- Use charge and mass of target species in single-species case
- Single- and Multi-species predictions can differ by $\approx 15\%$
  - Production modes where the difference is even larger?

	Artemis ^{16}O	**Venus ^{238}U**
Target Species	$^{16}O^{4+}$	$^{238}U^{33+}$
I singles-species CL Law	2.43 mA	3.32 mA
I multi-species CL Law	2.12 mA	3.23 mA
$\sum_n I_n$ measured at Faraday Cup	0.743 mA	2.97 mA
I_{drain} measured	1 to 2 mA	??

- Operations appear close to space-charge limited flow

Conclusion

- Multi-species Child-Langmuir Law gives more accurate limit on maximum extractable current from ECR
  - Further corrections: initial temperature, e^- neutralization, 2D geometry

- Artemis and VENUS seem close to idealized Child-Langmuir limit

- Does total extracted current scale as $V_0^{3/2}$?
  - Current weights may be function of V_0 which changes effective ratio
  - What does it mean if individual species do not scale as $V_0^{3/2}$?
  - Use ECR plasma scaling relations to constrain c_i and enhance utility?