



ADVANCEMENTS IN SELF-CONSISTENT MODELING OF TIME- AND SPACE- DEPENDENT PHENOMENA IN ECRIS PLASMA

Angelo Pidotella, David Mascali, Bharat Mishra, Eugenia Naselli, Giuseppe Torrisi, *INFN-LNS, Catania, Italy*
Alessio Galata, *INFN-LNL, Legnaro, Italy*

INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Globe-Geometry Heating in ECR Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR Plasma Ionization and Confinement Dynamics*

September 29, 2020

Angelo Pidotella - INFN-LNS

INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:

SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD

TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-frequency Heating in ECR'S Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR'S Plasma Turbulence and Confinement Dynamics*

September 29, 2020

Angelo Pidotella - INFN-LNS

INTRODUCTION

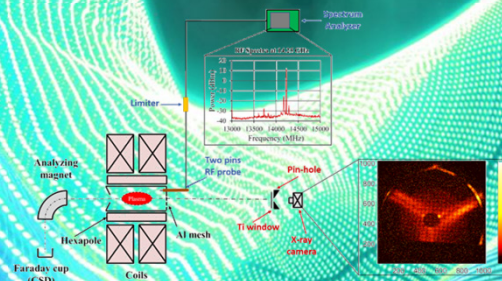
GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:

SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD

TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

THE EXPERIMENTAL SURVEY



See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-Geometry Heating in ECR'S Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR'S Plasma Ionization and Confinement Dynamics*

September 29, 2020

Angelo Pidatella - INFN-LNS

INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

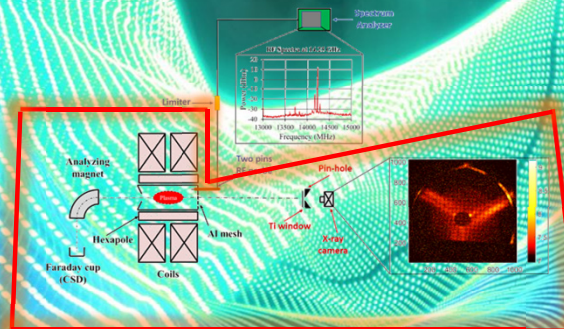
Examples:

SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD

TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

THE EXPERIMENTAL SURVEY

SPACE-RESOLVED PLASMA STUDY



See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-Transparency Heating in ECRS Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECRS Plasma Turbulence and Confinement Dynamics*

September 29, 2020

Angelo Pidatella - INFN-LNS

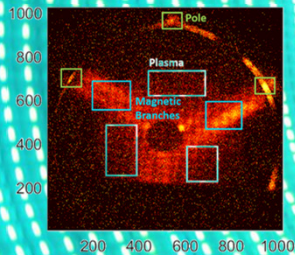
INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:
SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD
TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

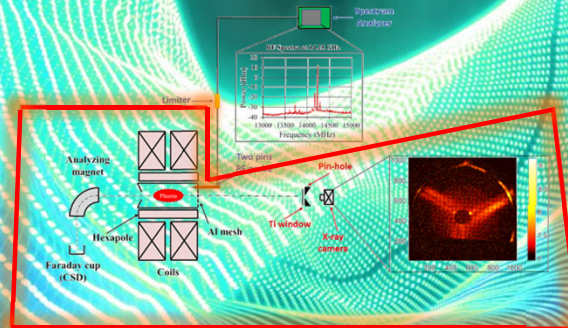
THE EXPERIMENTAL SURVEY

SPACE-RESOLVED PLASMA STUDY



Space-resolved soft X-Ray imaging: monitoring confinement dynamics and plasma structure

E. Naselli *et al.* (2019), 10.1088/1748-0221/14/10/C10008



See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-Geometry Heating in ECRS Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECRS Plasma Structure and Confinement Dynamics*

September 29, 2020

Angelo Pidotella - INFN-LNS

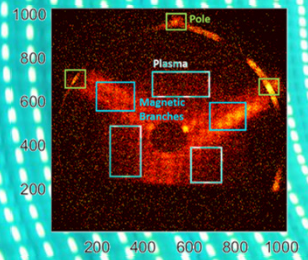
INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:
SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD
TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

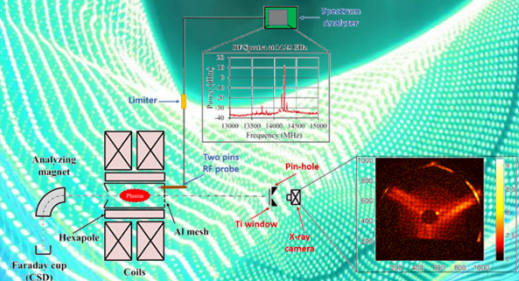
THE EXPERIMENTAL SURVEY

SPACE-RESOLVED PLASMA STUDY



Space-resolved soft X-Ray imaging: monitoring confinement dynamics and plasma structure

E. Naselli *et al.* (2019), 10.1088/1748-0221/14/10/C10008



- See talk from (September 28):
- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-Geometry Heating in ECR'S Plasmas*
 - E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR'S Plasma Structure and Confinement Dynamics*

September 29, 2020

Angelo Pidotella - INFN-LNS

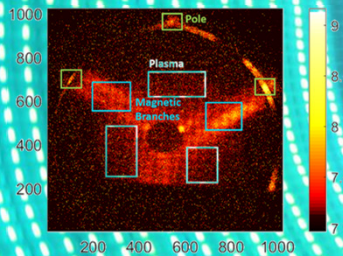
INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:
SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD
TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

THE EXPERIMENTAL SURVEY

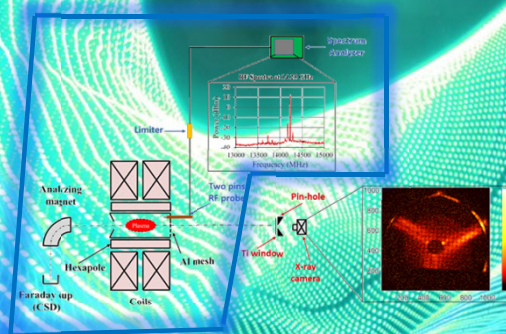
SPACE-RESOLVED PLASMA STUDY



Space-resolved soft X-Ray imaging: monitoring confinement dynamics and plasma structure

E. Naselli *et al.* (2019), 10.1088/1748-0221/14/10/C10008

TIME-RESOLVED PLASMA STUDY



See talk from (September 28):

- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Gloss-Transparency Heating in ECR'S Plasmas*
- E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR'S Plasma Turbulence and Confinement Dynamics*

September 29, 2020

Angelo Pidotella - INFN-LNS

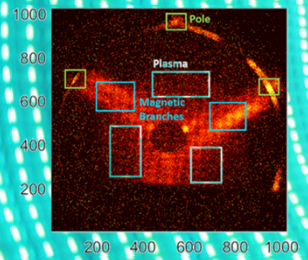
INTRODUCTION

GOAL: Bridge the gap between theoretical and experimental study of **space-dependent** and **time-dependent** electron/ion properties/**phenomena** in ECR plasmas

Examples:
SPACE: particles confinement dynamics, reaction rates for electromagnetic emissions, ion CSD
TIME: kinetic instabilities, plasma turbulences, fast transients, EC MASER instability

THE EXPERIMENTAL SURVEY

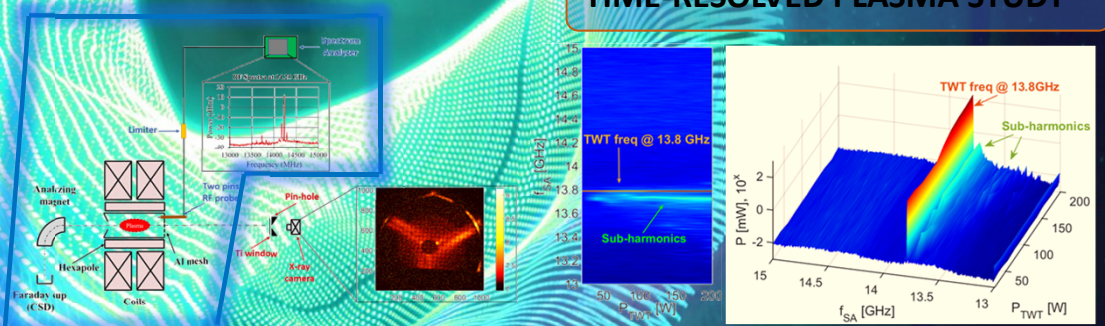
SPACE-RESOLVED PLASMA STUDY



Space-resolved soft X-Ray imaging: monitoring confinement dynamics and plasma structure

E. Naselli *et al.* (2019), 10.1088/1748-0221/14/10/C10008

TIME-RESOLVED PLASMA STUDY



Time-averaged single-frequency spectral evolution of the RF-probe detected signal vs. pumping wave power, with sub-harmonics generation already evident at around 40 W

E. Naselli *et al.*, PSST 28 (2019) 085021

- See talk from [September 28]:
- R. Rácz: *Imaging in X-ray Ranges to Locally Investigate the Effect of the Two-Globe-Geometry Heating in ECR'S Plasmas*
 - E. Naselli: *High Resolution X-ray Imaging as a Powerful Diagnostic Tool to Investigate ECR'S Plasma Structure and Confinement Dynamics*

September 29, 2020

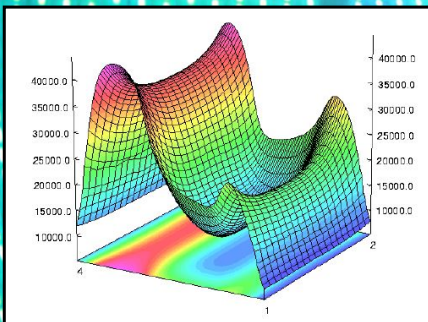
Angelo Pidatella - INFN-LNS

ECR PLASMA ANISOTROPICITY

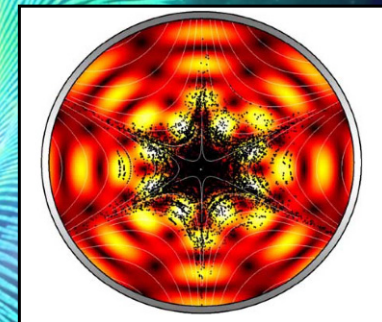
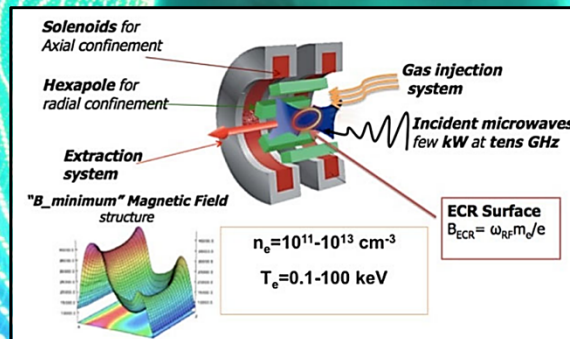
- Wave-to-particle coupling: heating process via electron cyclotron resonance (ECR)
 - RF guided wave + **multimodal structure**: highly **inhomogeneous**, **space-dependent** electric field
 - **Space-dependent** and **highly-inhomogeneous** magnetic field profile: plasma trapping + electron gyration
- **RESULTS**: closed ECR surface for wave-particle coupling and heating
 - **RESULTS** : space-dependent power transfer depending on electric field profile

$$\omega = \omega_c = \frac{eB(r)}{m}$$

$$P \propto E(r)^2$$



B-field profile as superposition of min-B and hexapole structures



Electromagnetic field pattern in the midplane for $f \sim 14 \text{ GHz}$

D. Mascali, *et al.*, Eur. Phys. J. A 53, 145 (2017).

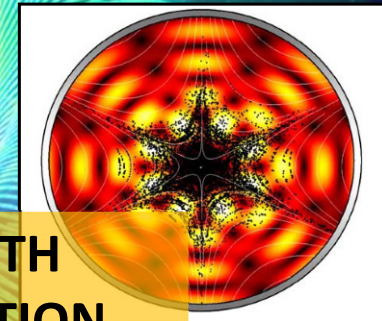
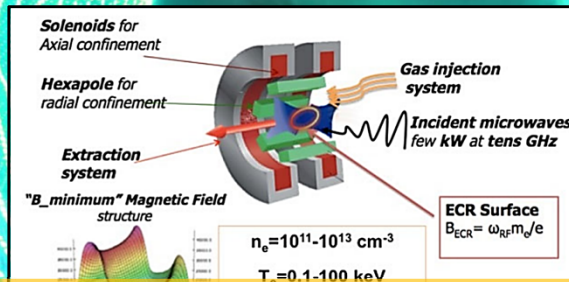
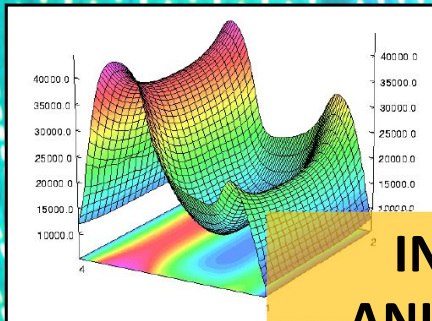
S. Gammino, *et al.*, IEEE transactions on plasma science 36, 4 (2008).

ECR PLASMA ANISOTROPICITY

- Wave-to-particle coupling: heating process via electron cyclotron resonance (ECR)
 - RF guided wave + **multimodal structure**: highly **inhomogeneous**, **space-dependent** electric field
- **Space-dependent** and **highly-inhomogeneous** magnetic field profile: plasma trapping + electron gyration
- **RESULTS**: closed ECR surface for wave-particle coupling and heating
- **RESULTS** : space-dependent power transfer depending on electric field profile

$$\omega = \omega_c = \frac{eB(r)}{m}$$

$$P \propto E(r)^2$$



INHOMOGENEOUS SYSTEM WITH ANISOTROPIC ENERGY DISTRIBUTION

B-field profile as superposition of min-B and hexapole structures

Electromagnetic field pattern in the midplane for $f \sim 14$ GHz

D. Mascali, *et al.*, *Eur. Phys. J. A* 53, 145 (2017).

S. Gammino, *et al.*, *IEEE transactions on plasma science* 36, 4 (2008).

3D MODELING OF ELECTRON KINETICS

**Approach to self-consistency: 3D particle mover + 3D FEM
Electromagnetic solver (INFN-LNL + INFN-LNS code)**

The **EM field** distribution and the **plasma medium** influence each other in a closed loop:

September 29, 2020

Angelo Pidotella - INFN-LNS

3D MODELING OF ELECTRON KINETICS

**Approach to self-consistency: 3D particle mover + 3D FEM
Electromagnetic solver (INFN-LNL + INFN-LNS code)**

The **EM field** distribution and the **plasma medium** influence each other in a closed loop:



3D MODELING OF ELECTRON KINETICS

Approach to self-consistency: 3D particle mover + 3D FEM
Electromagnetic solver (INFN-LNL + INFN-LNS code)

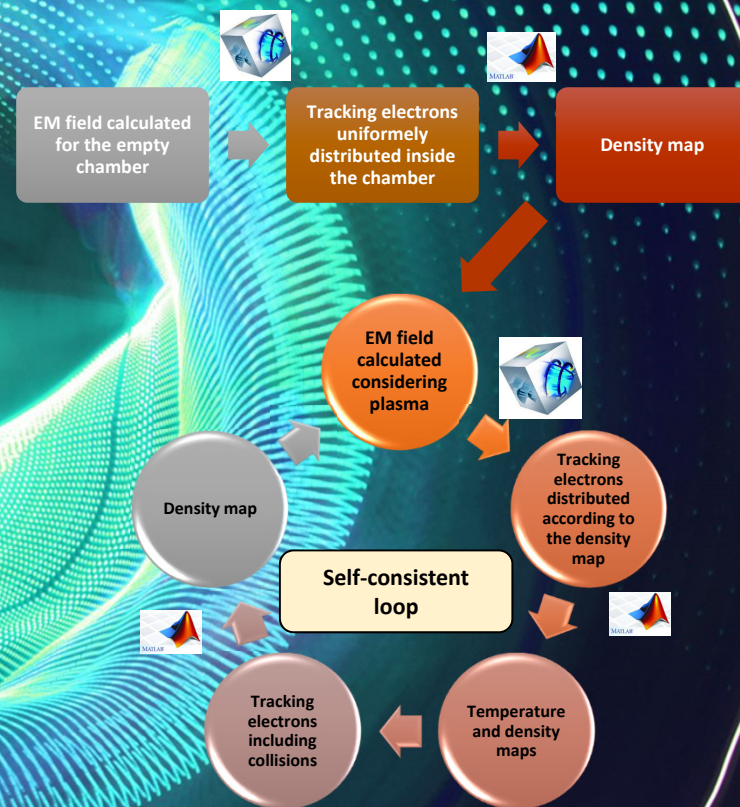
The EM field distribution and the plasma medium influence each other in a closed loop:



3D MODELING OF ELECTRON KINETICS

Approach to self-consistency: 3D particle mover + 3D FEM
Electromagnetic solver (INFN-LNL + INFN-LNS code)

The EM field distribution and the plasma medium influence each other in a closed loop:

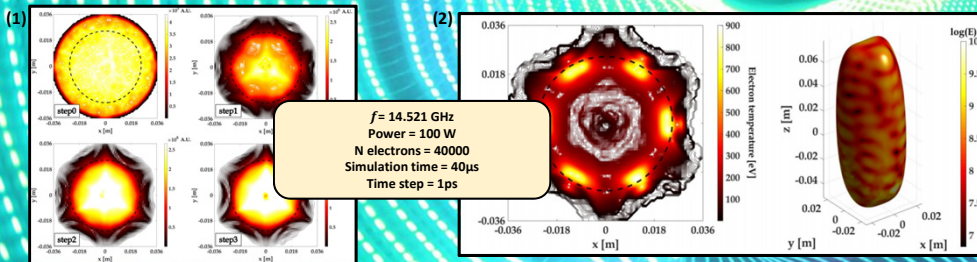


3D MODELING OF ELECTRON KINETICS

Approach to self-consistency: 3D particle mover + 3D FEM
Electromagnetic solver (INFN-LNL + INFN-LNS code)

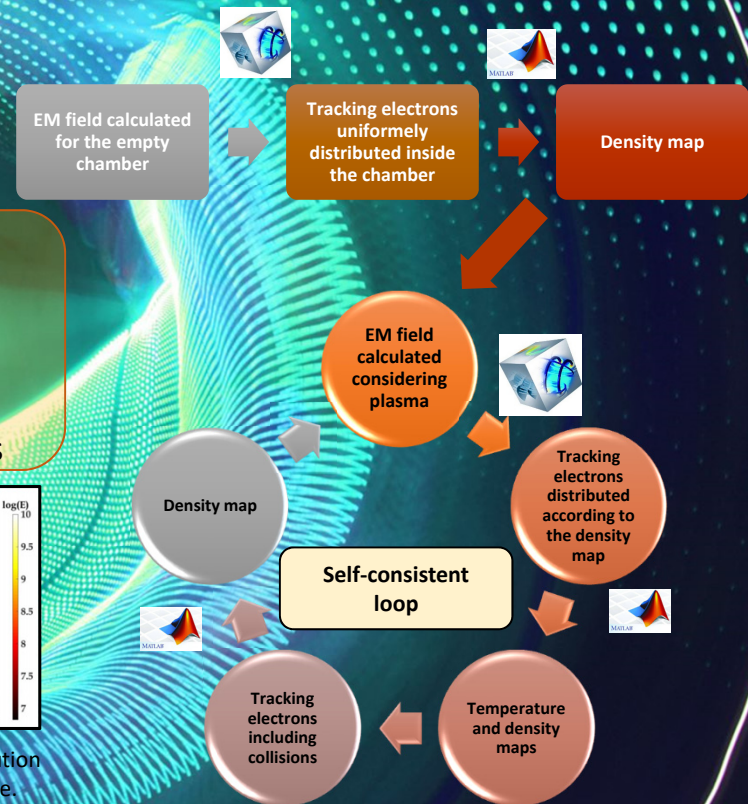
The EM field distribution and the plasma medium influence each other in a closed loop:

- Plasma concentrates in near resonance region: **dense plasmoid egg** surrounded by **rarefied halo**
- **Resonant cavity** and **mode structures** modify plasma density distribution
- **Electrons at different energies** distribute differently in the space: *cold* in the core, *hot* close to ECR boundaries

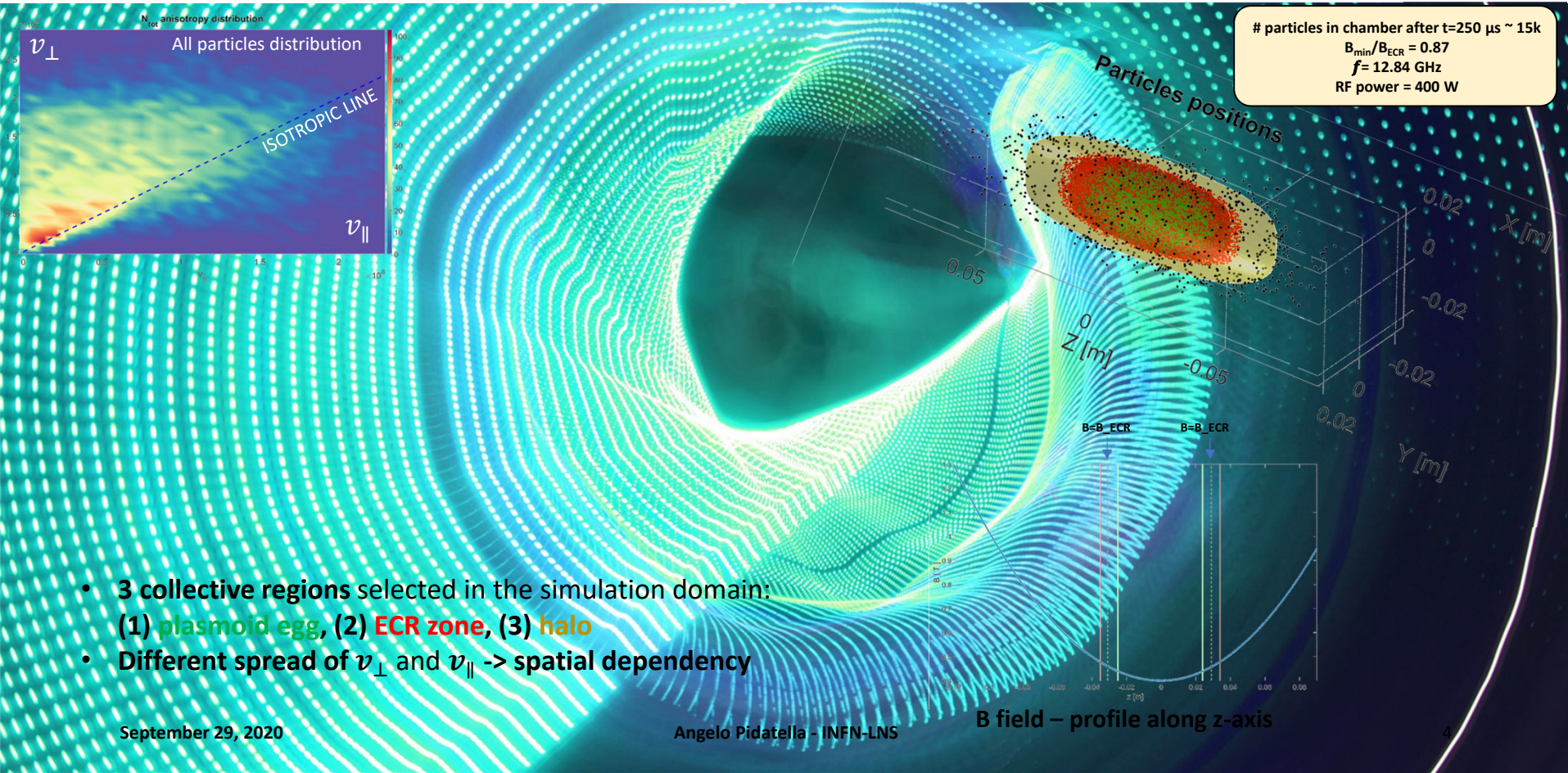


(1) z-axis projection of density maps during the calculation steps (ECR zone black dotted). (2-Left) Distribution of electron temperature in the midplane. (2-Right) Distribution of the EM field on the resonance surface.

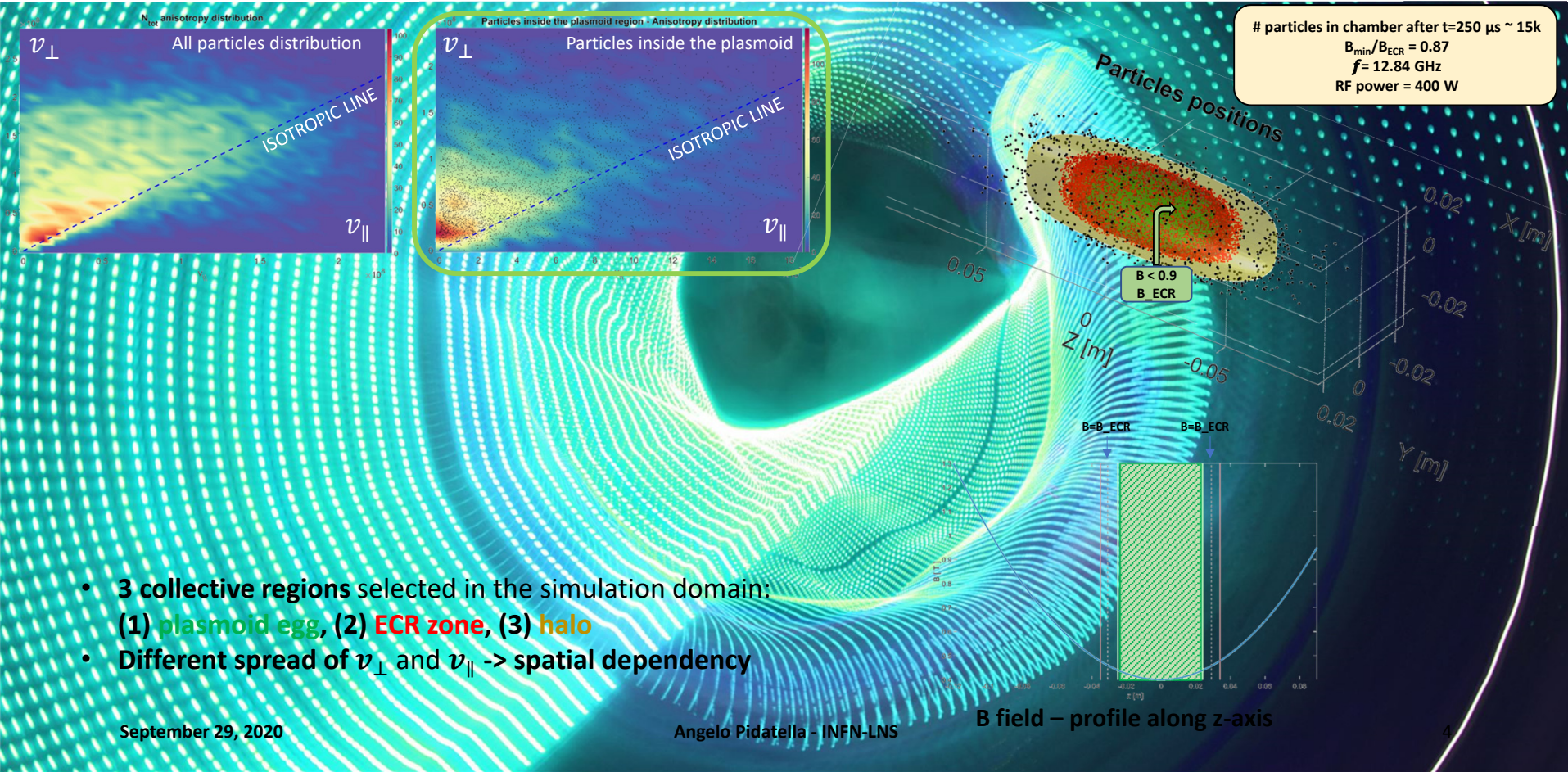
A. Galatà, et al., arXiv:1912.01988



VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}

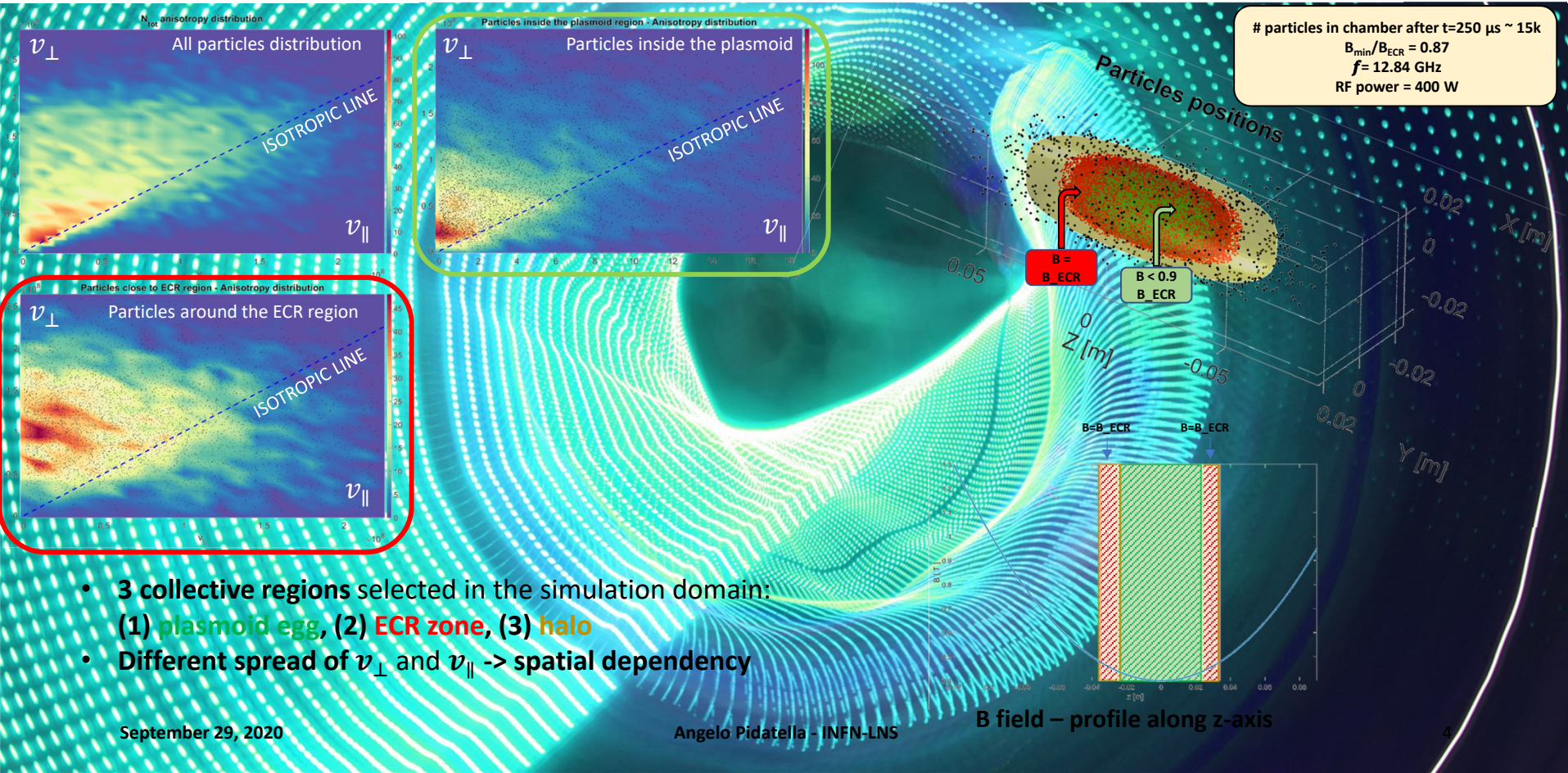


- **3 collective regions** selected in the simulation domain:
 (1) **plasmoid egg**, (2) **ECR zone**, (3) **halo**
- Different spread of v_{\perp} and v_{\parallel} -> **spatial dependency**

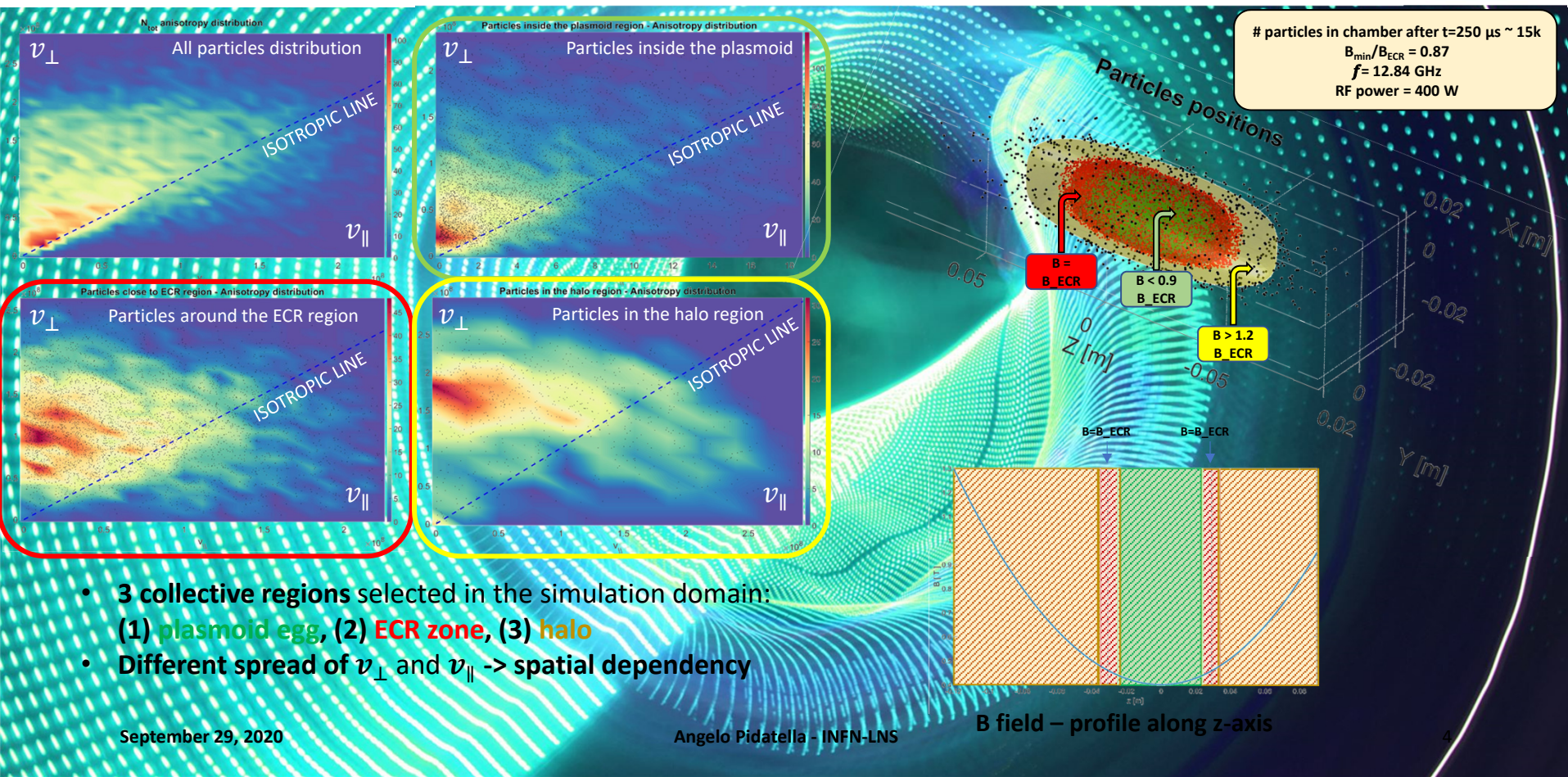
September 29, 2020

Angelo Pidotella - INFN-LNS

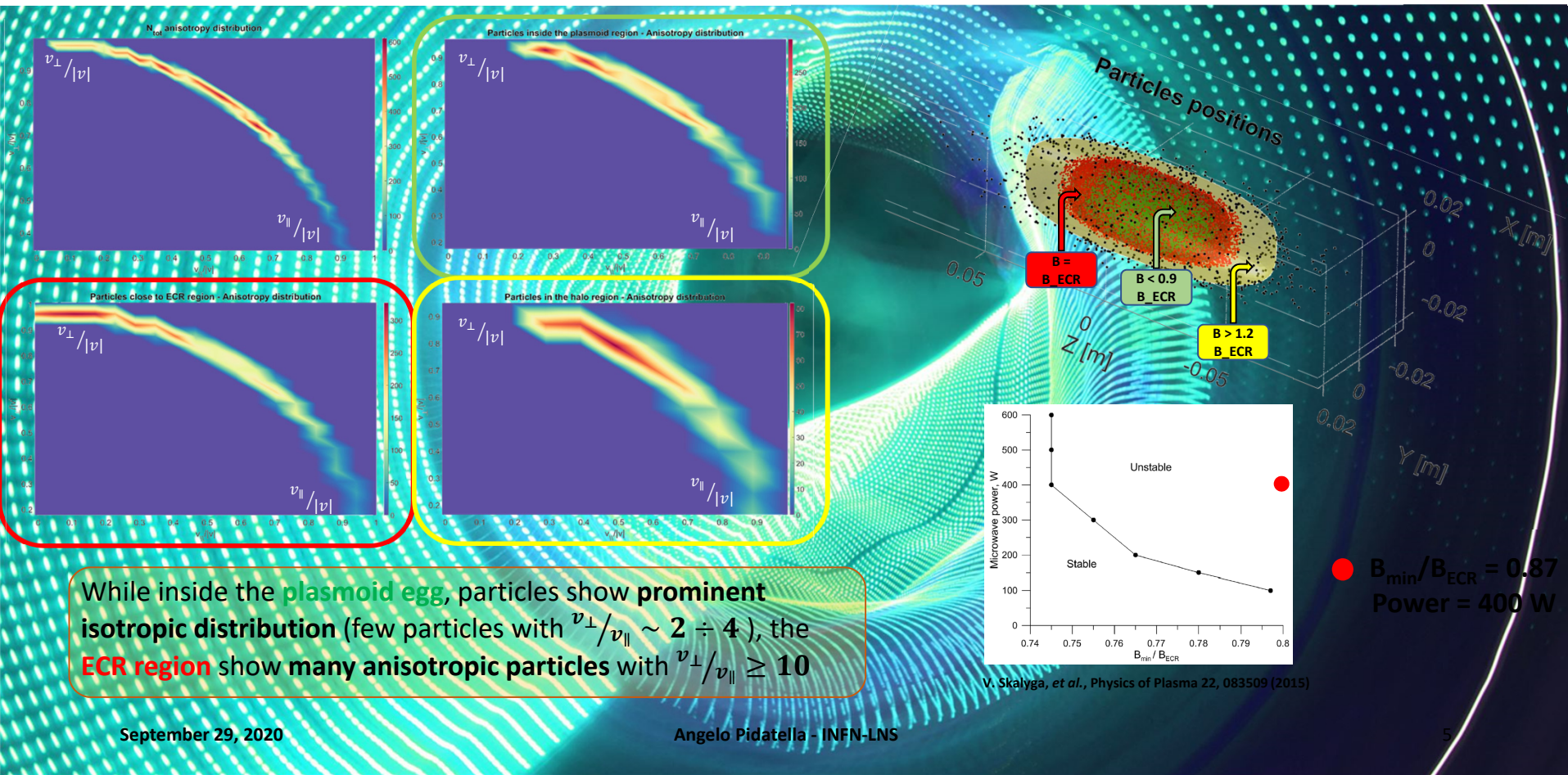
VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



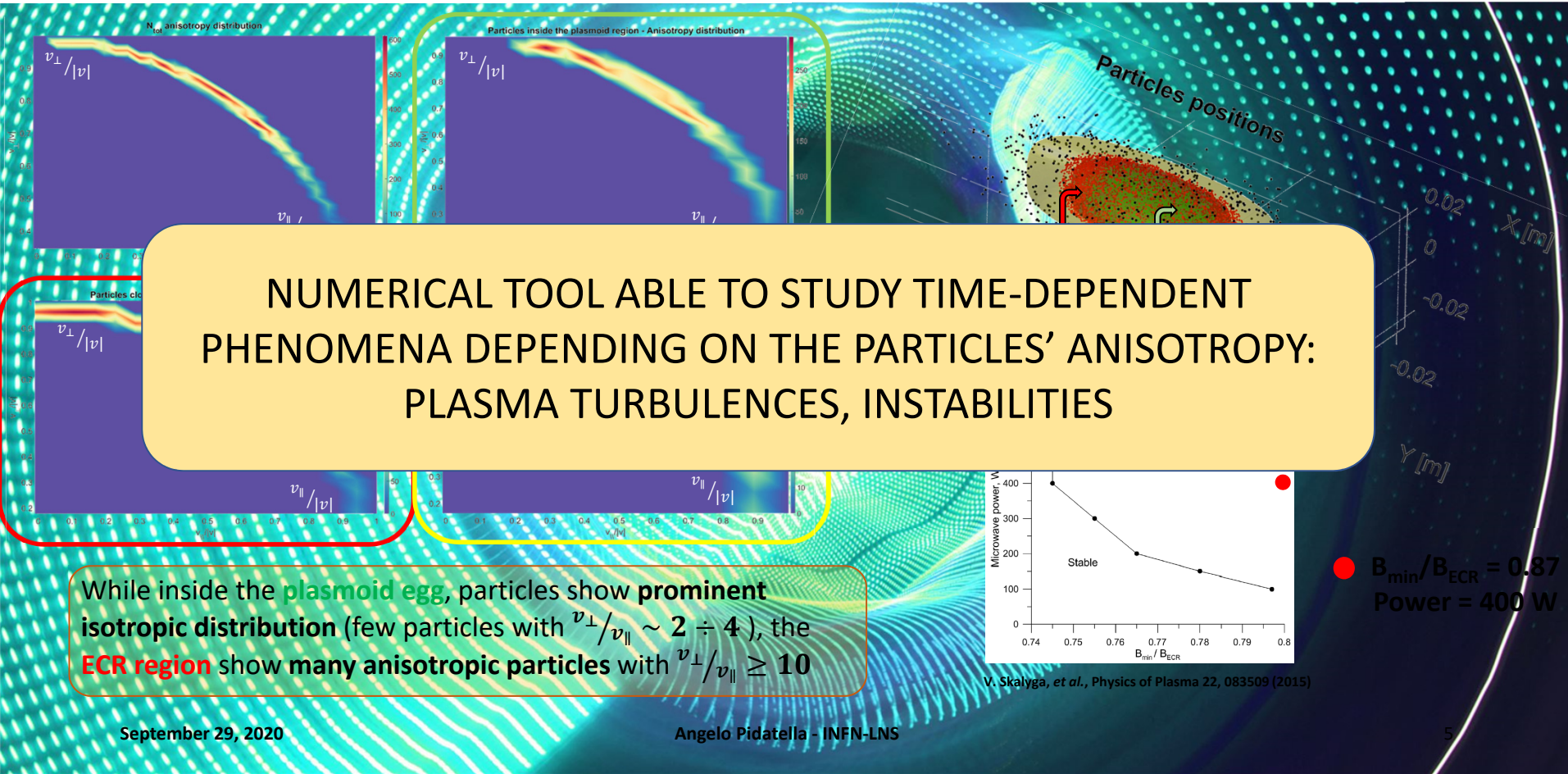
September 29, 2020

Angelo Pidatella - INFN-LNS

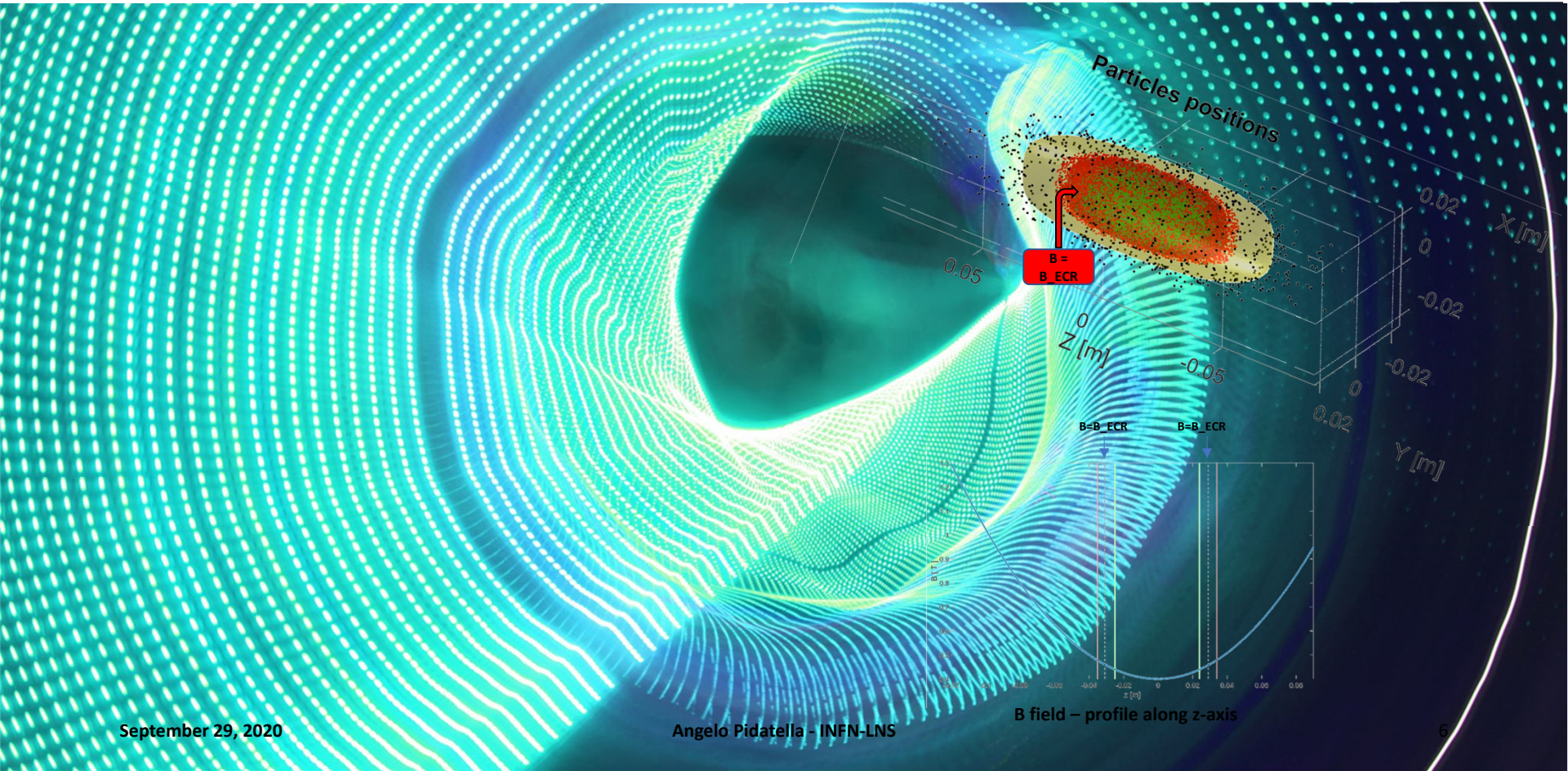
5

V. Skalyga, et al., Physics of Plasma 22, 083509 (2015)

VELOCITY-SPACE ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



ECR REGION ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}

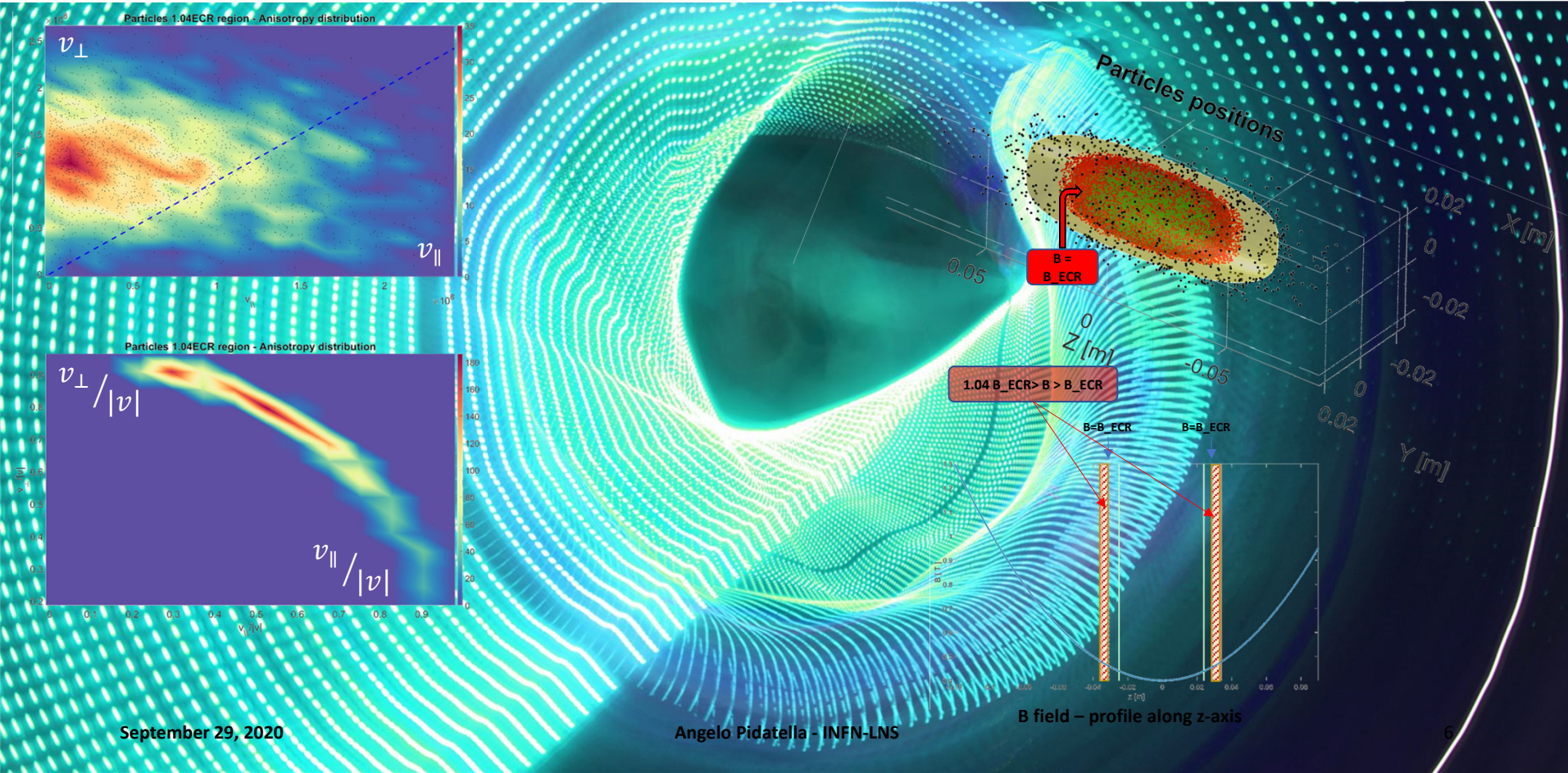


September 29, 2020

Angelo Pidatella - INFN-LNS

B field - profile along z-axis

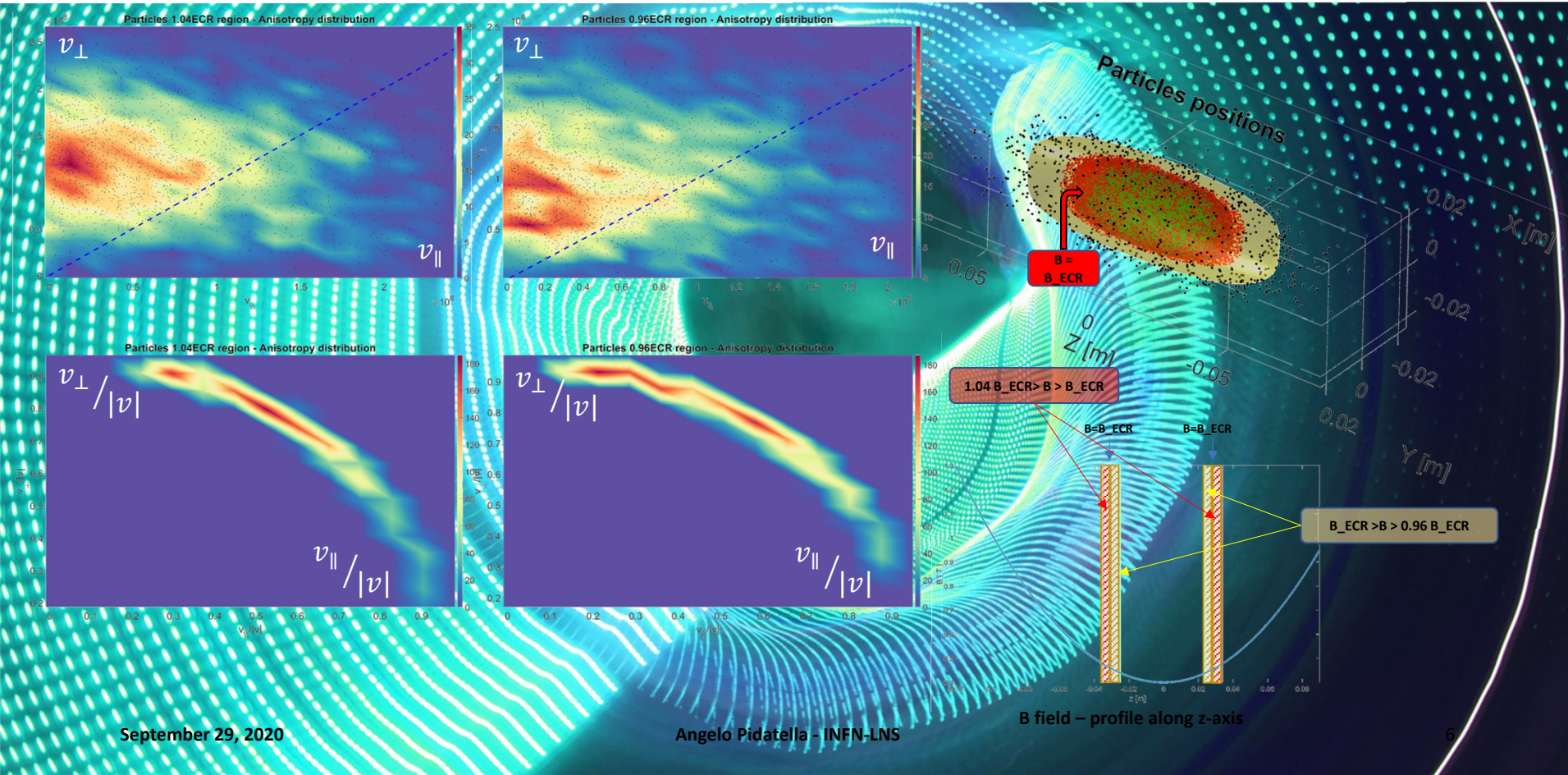
ECR REGION ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



September 29, 2020

Angelo Pidotella - INFN-LNS

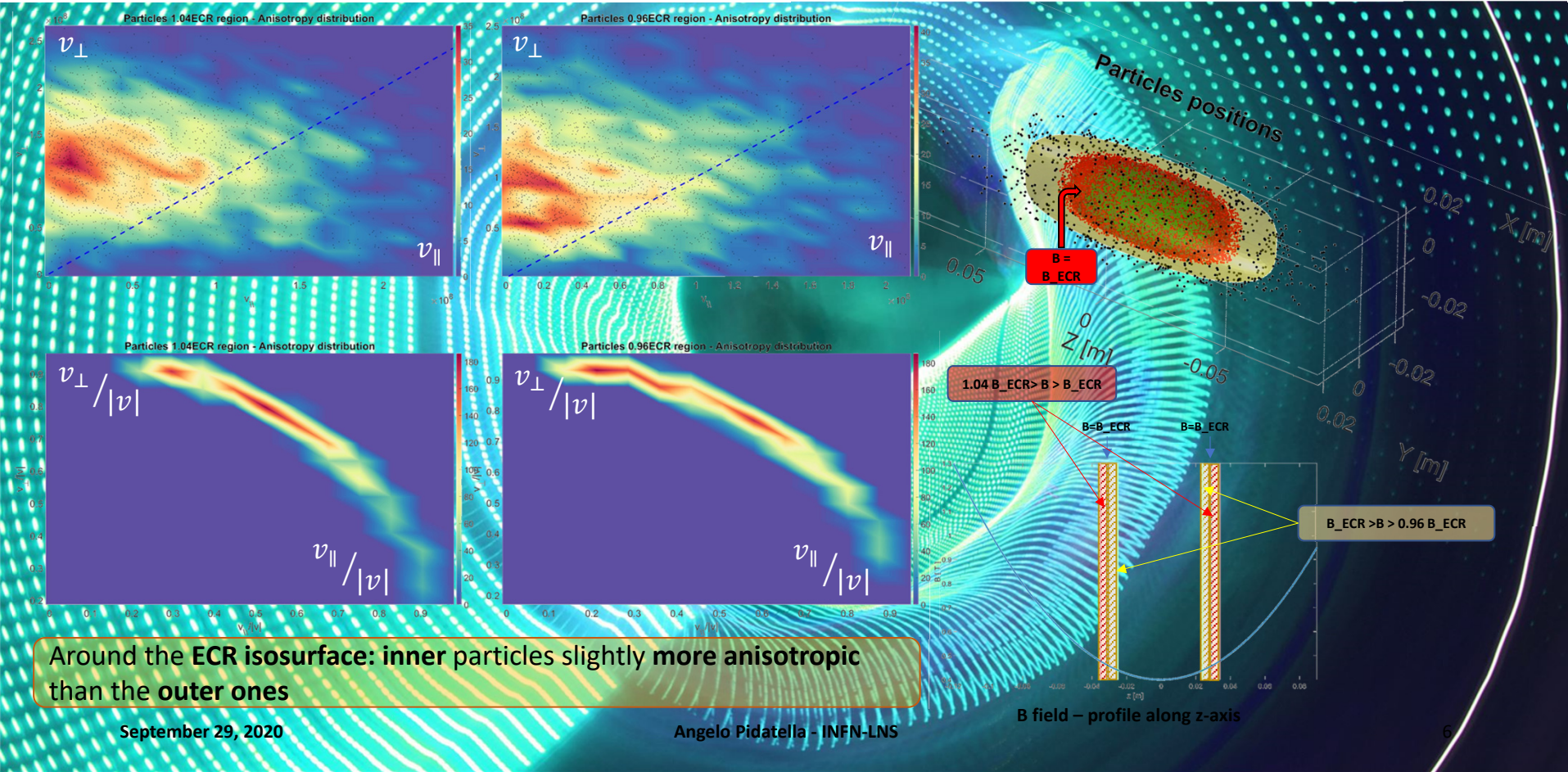
ECR REGION ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



September 29, 2020

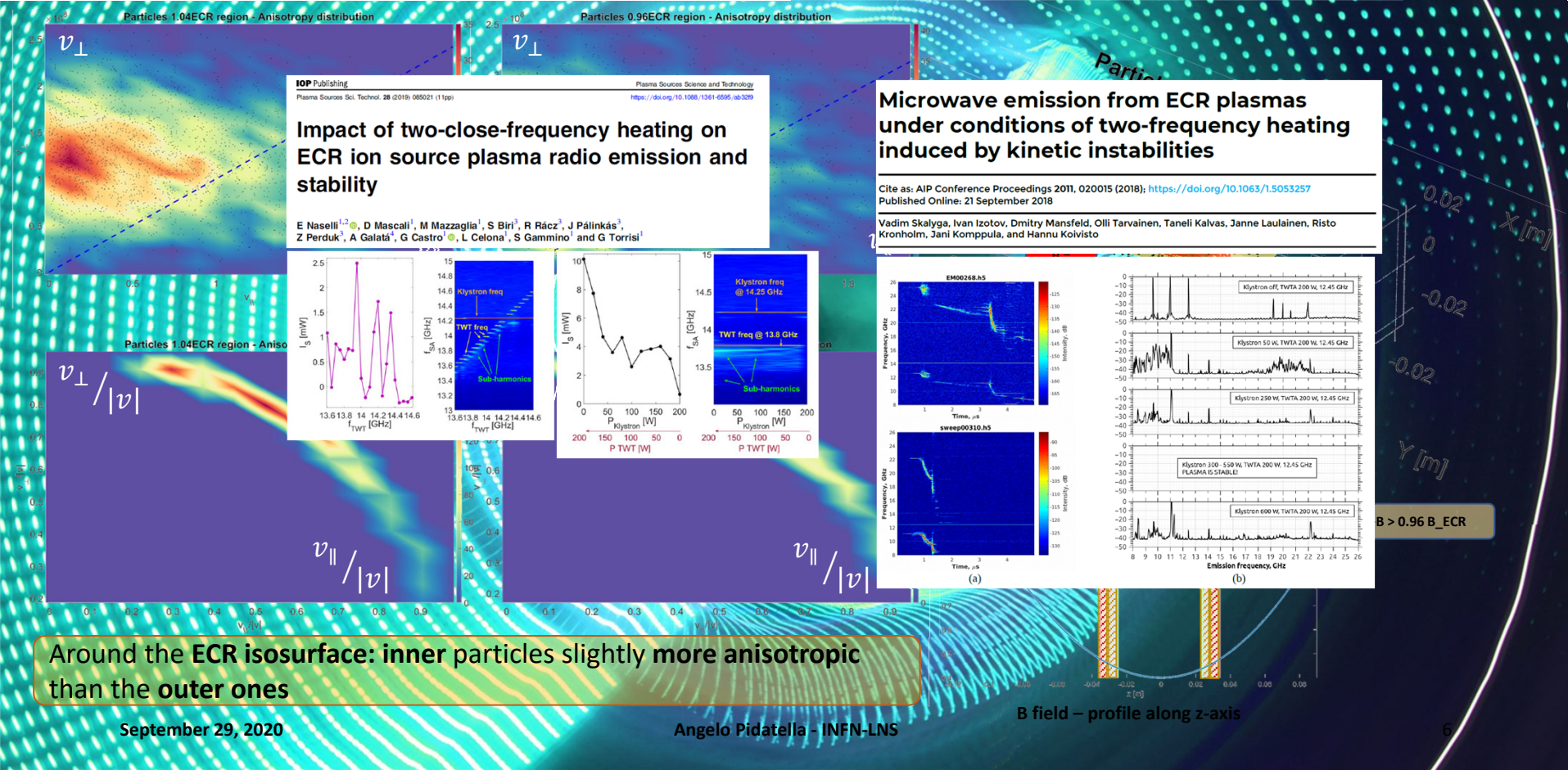
Angelo Pidotella - INFN-LNS

ECR REGION ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



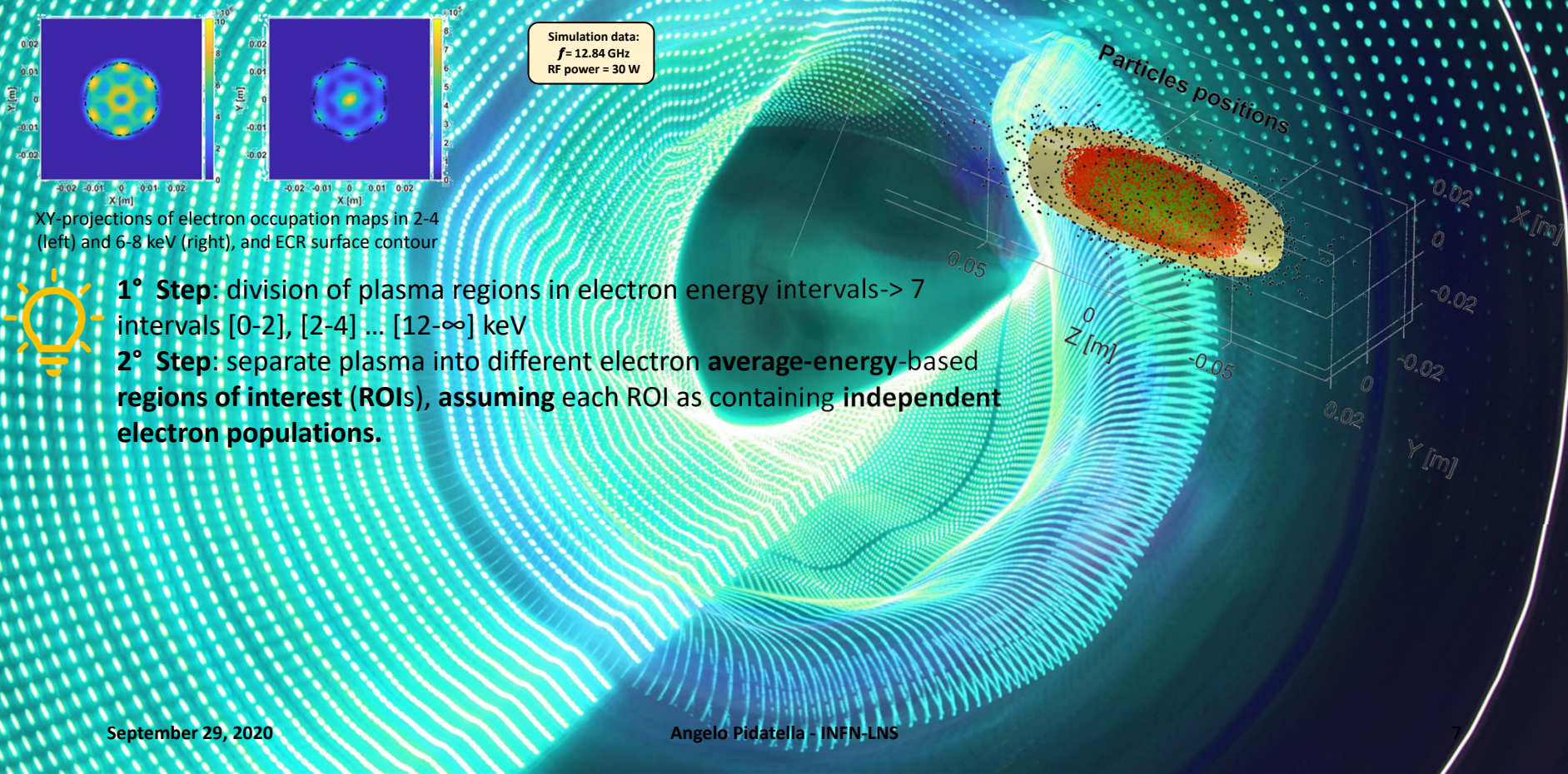
Around the ECR isosurface: inner particles slightly more anisotropic than the outer ones

ECR REGION ANISOTROPY DISTRIBUTION v_{\perp}/v_{\parallel}



SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

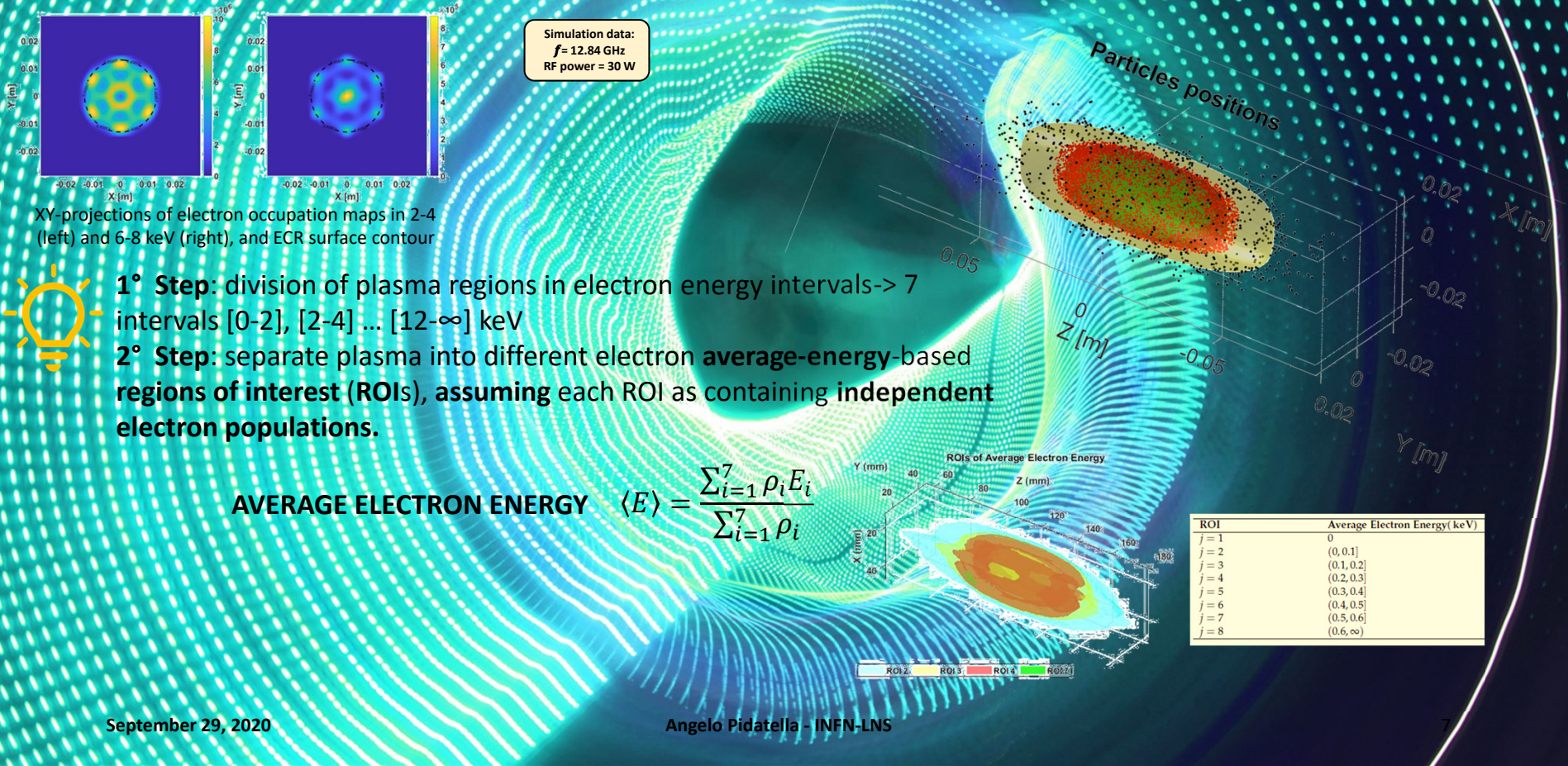
(Submission paper in progress)



- 1° Step:** division of plasma regions in electron energy intervals \rightarrow 7 intervals [0-2], [2-4] ... [12- ∞] keV
- 2° Step:** separate plasma into different electron **average-energy-based regions of interest (ROIs)**, assuming each ROI as containing **independent electron populations**.

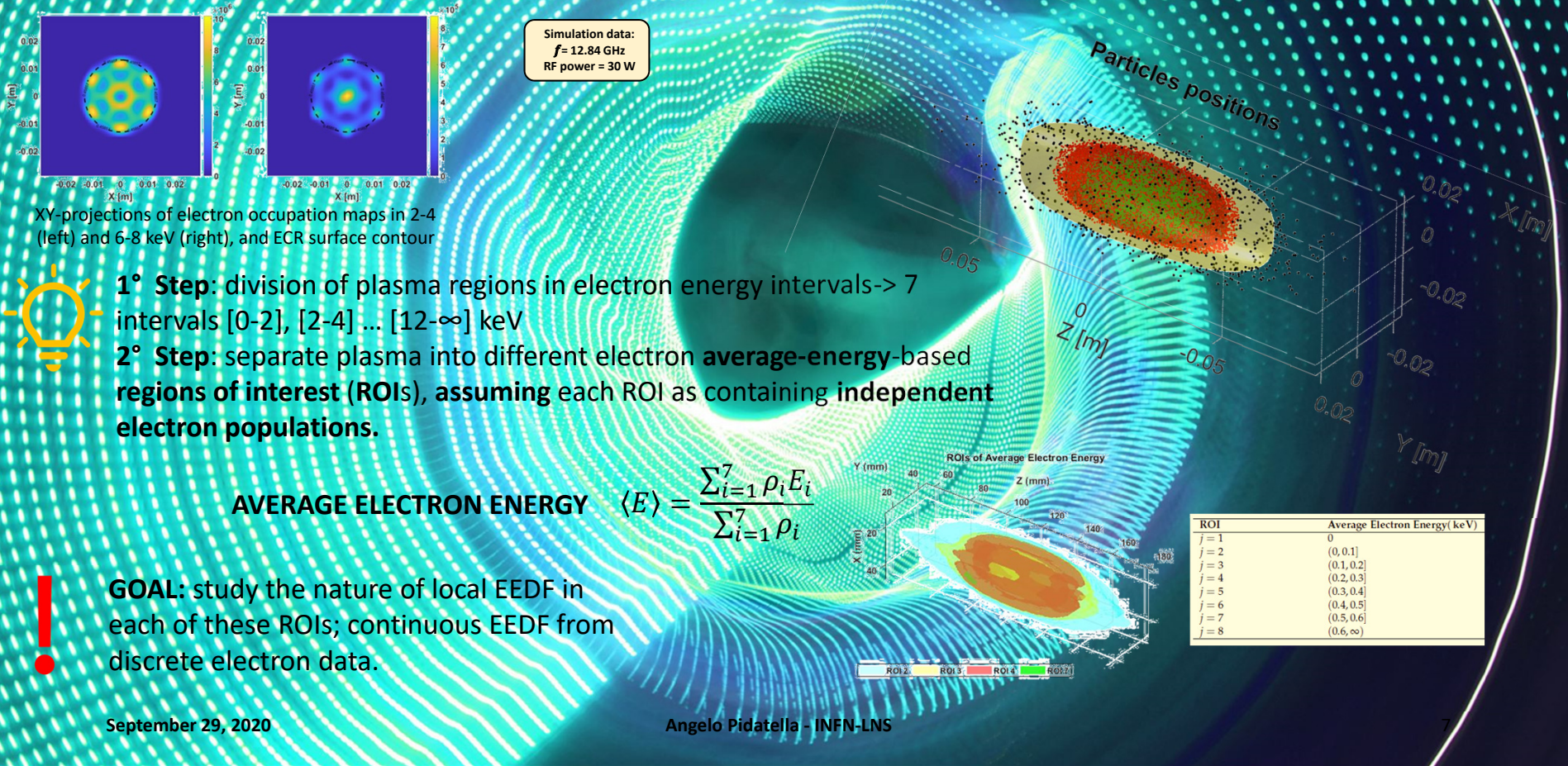
SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)



SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)



SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (**EEDF1** and **EEDF2**)

3-components test EEDFs: low-T + mid-T + high-T properties (**EEDF3** and **EEDF 4**)

Ref. case name	Type
EEDF1	Low- $E f_M$ + High- $E f_M$
EEDF2	Low- $E f_M$ + High- $E f_D$
EEDF3	Low- $E f_M$ + Medium- $E f_M$ + High- $E f_M$
EEDF4	Low- $E f_M$ + Medium- $E f_D$ + High- $E f_M$

September 29, 2020

Angelo Pidotella - INFN-LNS

8

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (**EEDF1** and **EEDF2**)

3-components test EEDFs: low-T + mid-T + high-T properties (**EEDF3** and **EEDF 4**)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Ref. case name	Type
EEDF1	Low- $E f_M$ + High- $E f_M$
EEDF2	Low- $E f_M$ + High- $E f_D$
EEDF3	Low- $E f_M$ + Medium- $E f_M$ + High- $E f_M$
EEDF4	Low- $E f_M$ + Medium- $E f_D$ + High- $E f_M$

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Ref. case name	Type
EEDF1	Low- $E f_M$ + High- $E f_M$
EEDF2	Low- $E f_M$ + High- $E f_D$
EEDF3	Low- $E f_M$ + Medium- $E f_M$ + High- $E f_M$
EEDF4	Low- $E f_M$ + Medium- $E f_D$ + High- $E f_M$

Fit parameters

$$(k_B T_e)_f = \frac{2}{3} C_f E_{fj}, \quad (k_B T_e)_f = S_f \frac{\sum_{j=1}^{N(f)} \rho_{fj} E_{fj}}{\sum_{j=1}^{N(f)} \rho_{fj}}$$

Fit analysis

$$\langle MSE \rangle_f = \frac{1}{N(f)} \sum_{k(f)=1}^{N(f)} MSE_{k(f)}$$

$$\sigma_{MSE_f} = \frac{1}{\sqrt{N(f)-1}} \sqrt{\sum_{k(f)=1}^{N(f)} (MSE_{k(f)} - \langle MSE \rangle_f)^2}$$

$$\langle \rho^2 \rangle_f = \frac{1}{N(f)} \sum_{k(f)=1}^{N(f)} \rho_{k(f)}^2$$

$$\sigma_{\rho^2_f} = \frac{1}{\sqrt{N(f)-1}} \sqrt{\sum_{k(f)=1}^{N(f)} (\rho_{k(f)}^2 - \langle \rho^2 \rangle_f)^2}$$

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval

Fit parameters

$$(k_B T_1)_j = \frac{2}{3} C_j E_{ij}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^j \rho_{ij} E_{ij}}{\sum_{i=1}^j \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

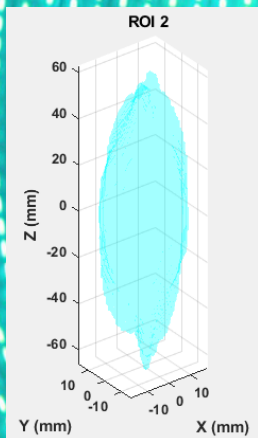
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



Fit parameters

$$(k_B T_e)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_h)_j = S_j \frac{\sum_{i=1}^{N(i)} \rho_{ij}^2 E_{ij}}{\sum_{i=1}^{N(i)} \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k,j}$$

$$\sigma_{MSE,j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k,j} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{kj}^2$$

$$\sigma_{\rho^2,j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\rho_{kj}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

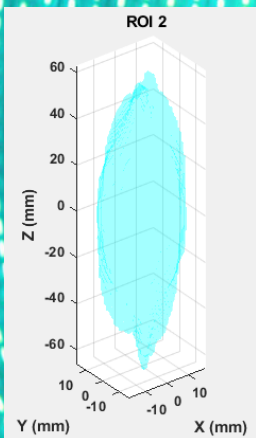
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{ij} = \sum_{k=1}^K \rho_{ik} \int_0^{\infty} f(E, k_B T_{ik}) \delta(k_B T_{ij} - E) dE$$

Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k,j}$$

$$\sigma_{MSE,j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k,j} - \langle MSE \rangle_j)^2}$$

$$\langle \sigma^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \langle \sigma^2 \rangle_{k,j}$$

$$\sigma_{\sigma^2,j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\langle \sigma^2 \rangle_{k,j} - \langle \sigma^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

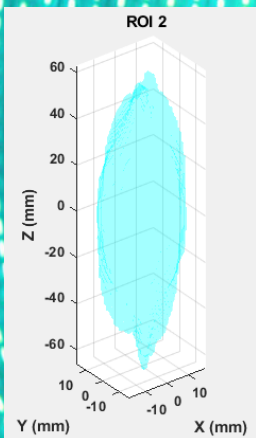
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

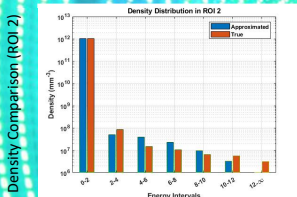
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(i),est} = \sum_{j=1}^J \rho_{(j)} \int_0^{\infty} f(E, k_B T_{1j}, k_B T_{2j}) dE$$



Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k,j}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k,j} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{k,j}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\rho_{k,j}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

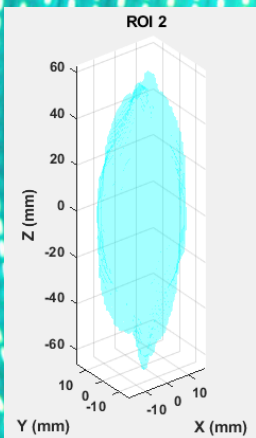
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

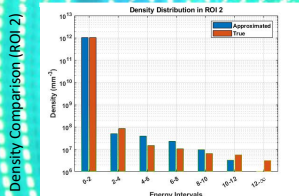
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(j)}^{est} = \sum_{i=1}^I \rho_{(i)} \int_{E_i}^0 f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$



$$\langle E_{(j)} \rangle_{est} = \frac{\int_{E_i}^0 E f(E; (k_B T_1)_j; (k_B T_2)_j) dE}{\int_{E_i}^0 f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$

Fit parameters

$$(k_B T_2)_j = \frac{2}{5} C_j E_{(j)}, \quad (k_B T_1)_j = S_j \frac{\sum_{i=1}^I \rho_{(i)} E_{(j)}}{\sum_{i=1}^I \rho_{(i)}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle T^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \langle T^2 \rangle_{k(j)}$$

$$\sigma_{T^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\langle T^2 \rangle_{k(j)} - \langle T^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

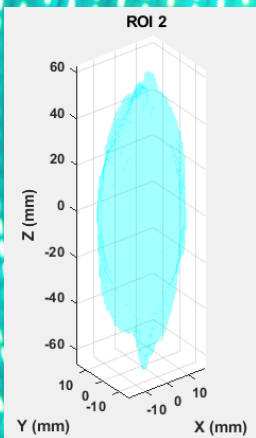
3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

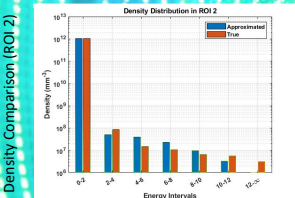
$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

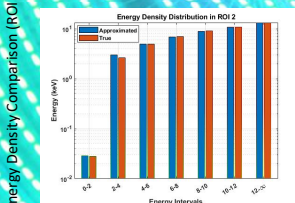
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(j)}^{est} = \sum_{i=1}^I \rho_{(i)} \int_0^{\infty} f(E; (k_B T_1)_i; (k_B T_2)_i) dE$$



$$\langle E_{(j)} \rangle_{est} = \frac{\int_0^{\infty} E f(E; (k_B T_1)_j; (k_B T_2)_j) dE}{\int_0^{\infty} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$



Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{(j)}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^I \rho_{(i)} E_{(i)}}{\sum_{i=1}^I \rho_{(i)}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval

$$\rho_{ij}^{est} = \sum_{n=1}^N \rho_{in} \int_{E_i}^{E_{i+1}} f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$

Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_2)_j = S_j \frac{\sum_{n=1}^N \rho_{in} E_{ij}}{\sum_{n=1}^N \rho_{in}}$$

Fit analysis

$$\langle E_i \rangle_{est} = \frac{\int_{E_i}^{E_{i+1}} f(E; (k_B T_1)_j; (k_B T_2)_j) E dE}{\int_{E_i}^{E_{i+1}} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \sigma^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \langle \sigma^2 \rangle_{k(j)}$$

$$\sigma_{\sigma^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\langle \sigma^2 \rangle_{k(j)} - \langle \sigma^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

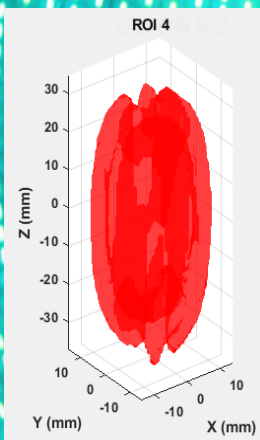
3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(i)}^{est} = \sum_{j=1}^J \rho_{(i)} \int_0^{\infty} f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$

Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle E_{ij} \rangle_{est} = \frac{\int_0^{\infty} E f(E; (k_B T_1)_j; (k_B T_2)_j) dE}{\int_0^{\infty} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$

$$\langle \sigma^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \langle \sigma^2 \rangle_{k(j)}$$

$$\sigma_{\sigma^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\langle \sigma^2 \rangle_{k(j)} - \langle \sigma^2 \rangle_j)^2}$$

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

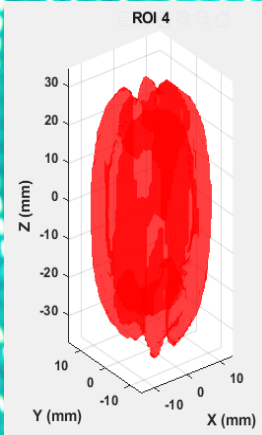
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

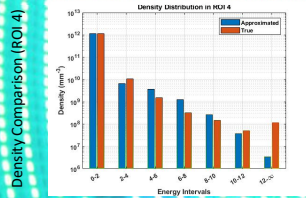
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(i)}^{est} = \sum_{j=1}^J \rho_{(i)} \int_{E_i}^0 f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$



$$\langle E_i \rangle_{est} = \frac{\int_{E_i}^0 f(E; (k_B T_1)_j; (k_B T_2)_j) E dE}{\int_{E_i}^0 f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$

Fit parameters

$$(k_B T_2)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_1)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k(j)=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k(j)=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

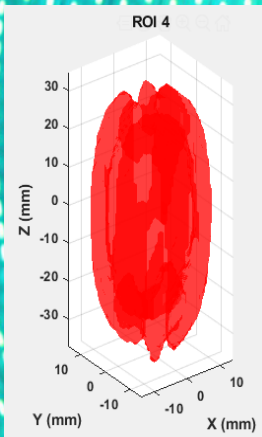
3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

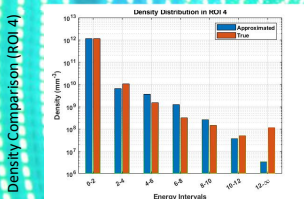
$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

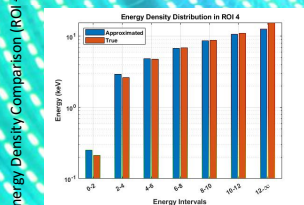
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(j)} = \sum_{i=1}^N \rho_{(i)} \int_0^{\infty} f(E; (k_B T_1)_i; (k_B T_2)_i) dE$$



$$\langle E_{(j)} \rangle_{est} = \frac{\int_0^{\infty} f(E; (k_B T_1)_j; (k_B T_2)_j) E dE}{\int_0^{\infty} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$



Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{(j)}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^N \rho_{(i)} E_{(i)}}{\sum_{i=1}^N \rho_{(i)}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

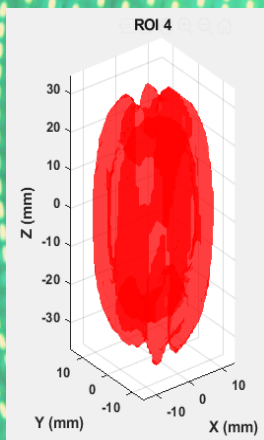
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

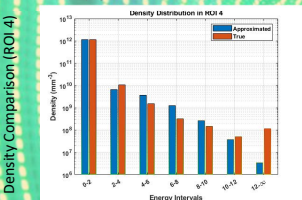
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

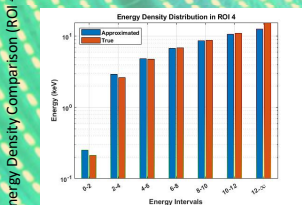
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(j)}^{est} = \sum_{i=1}^N \rho_{(i)} \int_{E_i}^{E_{i+1}} f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$



$$\langle E_{(j)} \rangle_{est} = \frac{\int_{E_1}^{E_N} f(E; (k_B T_1)_j; (k_B T_2)_j) E dE}{\int_{E_1}^{E_N} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$



Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{(j)}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^N \rho_{(i)} E_{(i)}}{\sum_{i=1}^N \rho_{(i)}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\rho^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

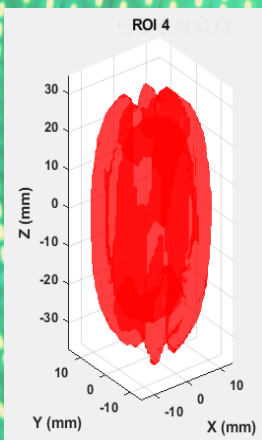
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

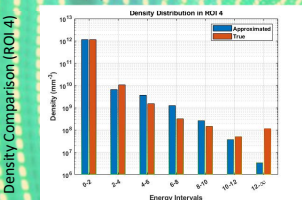
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

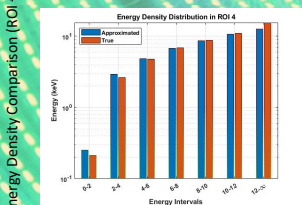
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(j)}^{est} = \sum_{i=1}^N \rho_{(i)} \int_{E_i}^{E_{i+1}} f(E; (k_B T_1)_j; (k_B T_2)_j) dE$$



$$\langle E_{(j)} \rangle_{est} = \frac{\int_{E_1}^{E_N} f(E; (k_B T_1)_j; (k_B T_2)_j) E dE}{\int_{E_1}^{E_N} f(E; (k_B T_1)_j; (k_B T_2)_j) dE}$$



Fit parameters

$$(k_B T_1)_j = \frac{2}{5} C_j E_{(j)}, \quad (k_B T_2)_j = S_j \frac{\sum_{i=1}^N \rho_{(i)} E_{(i)}}{\sum_{i=1}^N \rho_{(i)}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} MSE_{k(j)}$$

$$\sigma_{\langle MSE \rangle_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (MSE_{k(j)} - \langle MSE \rangle_j)^2}$$

$$\langle \rho^2 \rangle_j = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{k(j)}^2$$

$$\sigma_{\langle \rho^2 \rangle_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} (\rho_{k(j)}^2 - \langle \rho^2 \rangle_j)^2}$$

Ref. case name	Type
EEDF1	Low-E f_M + High-E f_M
EEDF2	Low-E f_M + High-E f_D
EEDF3	Low-E f_M + Medium-E f_M + High-E f_M
EEDF4	Low-E f_M + Medium-E f_D + High-E f_M

Quantitative analysis cell-by-cell

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

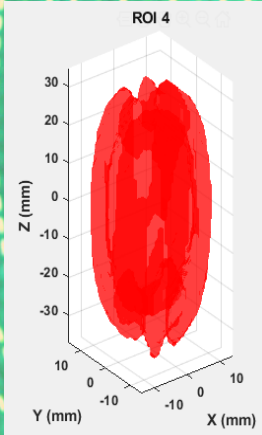
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

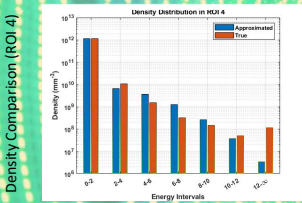
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

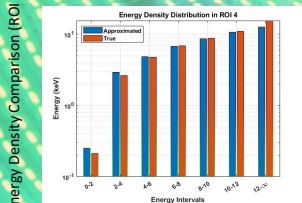
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(i)}^{est} = \sum_j \rho_{(j)} \int_0^{\Delta E} f(E; (k_B T_{ij}), (k_B T_{ij})) dE$$



$$\langle E_{ij} \rangle_{est} = \frac{\int_0^{\Delta E} E f(E; (k_B T_{ij}), (k_B T_{ij})) dE}{\int_0^{\Delta E} f(E; (k_B T_{ij}), (k_B T_{ij})) dE}$$



Fit parameters

$$(k_B T_2)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_1)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{i=1}^{N(j)} MSE_{k,i,j}$$

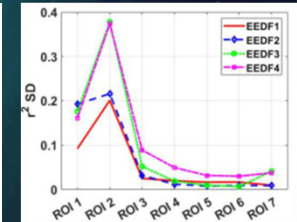
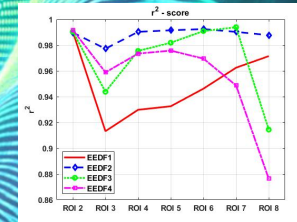
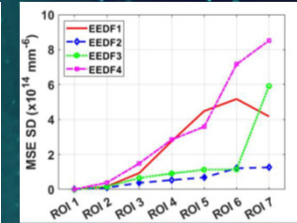
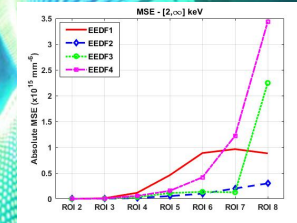
$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{i=1}^{N(j)} (MSE_{k,i,j} - \langle MSE \rangle_j)^2}$$

$$r^2_{j,i} = \frac{1}{N(j)} \sum_{k=1}^{N(j)} \rho_{k,i,j}^2 / \langle \rho \rangle_j^2$$

$$\sigma_{r^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} ((r^2_{k,i,j}) - (r^2)_{j,i})^2}$$

Ref. case name	Type
EEDF1	Low- $E f_M$ + High- $E f_M$
EEDF2	Low- $E f_M$ + High- $E f_D$
EEDF3	Low- $E f_M$ + Medium- $E f_M$ + High- $E f_M$
EEDF4	Low- $E f_M$ + Medium- $E f_D$ + High- $E f_M$

Quantitative analysis cell-by-cell



Mean Squared Error and r^2 Values for different EEDFs in each ROI

SPACE-RESOLVED ELECTRON ENERGY DISTRIBUTION FUNCTION (EEDF)

(Submission paper in progress)

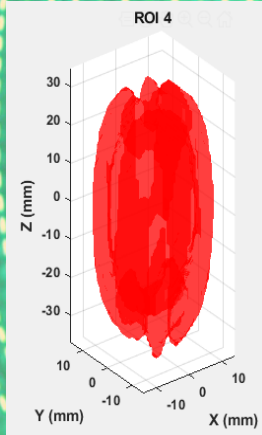
2-components test EEDFs: capturing low-T and high-T properties (EEDF1 and EEDF2)

3-components test EEDFs: low-T + mid-T + high-T properties (EEDF3 and EEDF 4)

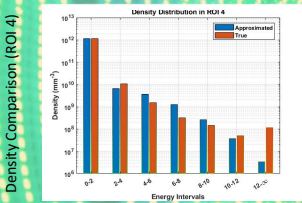
BEST EEDF2: LOW-MAXWELLIAN + HIGH-DRUYVESTEYN

$$f(E, kT_1, kT_2) = N_1 \left(\frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\sqrt{(kT_1)^3}} e^{-E/kT_1} \right) + N_2 \left(1.04 \frac{\sqrt{E}}{\sqrt{(kT_2)^3}} e^{-0.55E^2/(kT_2)^2} \right)$$

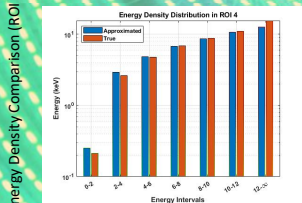
Qualitative analysis on j-th ROI-aggregated collective data for the i-th energy interval



$$\rho_{(i)est} = \sum_j \rho_{(j)} \int_0^b f(E; (k_B T_{ij}); (k_B T_{ij})) dE$$



$$\langle E_{ij} \rangle_{est} = \frac{\int_0^b f(E; (k_B T_{ij}); (k_B T_{ij})) E dE}{\int_0^b f(E; (k_B T_{ij}); (k_B T_{ij})) dE}$$



Fit parameters

$$(k_B T_2)_j = \frac{2}{5} C_j E_{ij}, \quad (k_B T_1)_j = S_j \frac{\sum_{i=1}^I \rho_{ij} E_{ij}}{\sum_{i=1}^I \rho_{ij}}$$

Fit analysis

$$\langle MSE \rangle_j = \frac{1}{N(j)} \sum_{i=1}^{N(j)} MSE_{k,i,j}$$

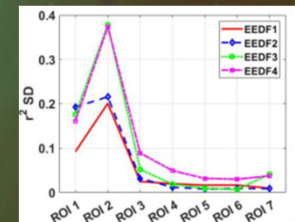
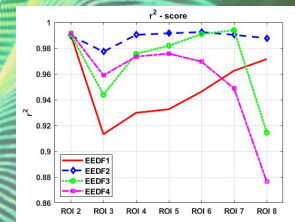
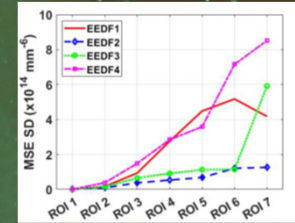
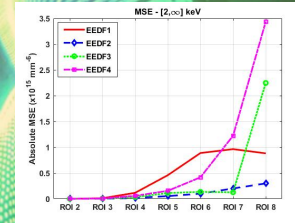
$$\sigma_{MSE_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{i=1}^{N(j)} (MSE_{k,i,j} - \langle MSE \rangle_j)^2}$$

$$r^2_{j,i} = \frac{1}{N(j)} \sum_{k=1}^{N(j)} (r^2)_{k,i,j}$$

$$\sigma_{r^2_j} = \frac{1}{\sqrt{N(j)-1}} \sqrt{\sum_{k=1}^{N(j)} ((r^2)_{k,i,j} - (r^2)_{j,i})^2}$$

Ref. case name	Type
EEDF1	Low- $E f_M$ + High- $E f_M$
EEDF2	Low- $E f_M$ + High- $E f_D$
EEDF3	Low- $E f_M$ + Medium- $E f_M$ + High- $E f_M$
EEDF4	Low- $E f_M$ + Medium- $E f_D$ + High- $E f_M$

Quantitative analysis cell-by-cell

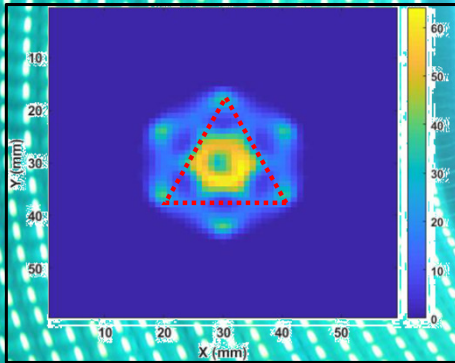


Mean Squared Error and r^2 Values for different EEDFs in each ROI

1st EEDF TEST BENCH: ESTIMATE OF Ar FLUORESCENCE K- α EMISSION

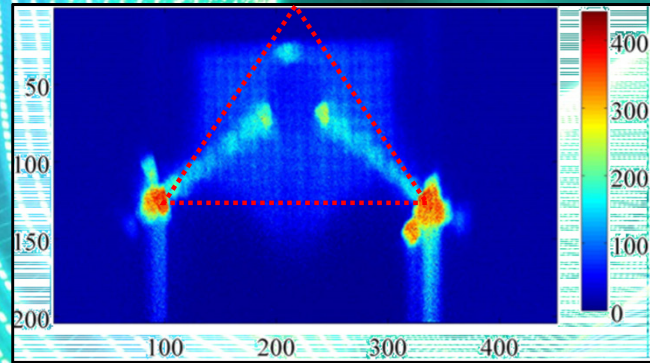
Argon plasma emission rate for the K-shell X-ray fluorescence at 2.96 keV supporting the **experimental campaign** carried out jointly by the ATOMKI (Debrecen) and INFN-LNS/LNL groups in 2017 on **space-resolved X-ray spectroscopy**

- **Electron data** from simulations used to **evaluate a 3D K- α emission rate map : using the analytical EEDF**
- Geometrical efficiency of detecting system, quantum efficiency of CCD camera and exposure time considered for the emission map



Longitudinally integrated estimated K- α map at the CCD, considering Total Emission, LGE and Quantum Efficiency

B. Mishra, *Master Thesis work (2020)*

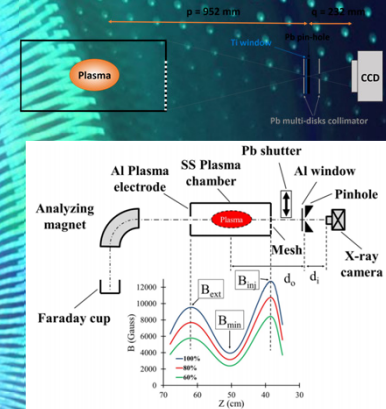


Plot of experimental photon counted images

R. Rácz *et al.*, PSST 26 (2017) 075011

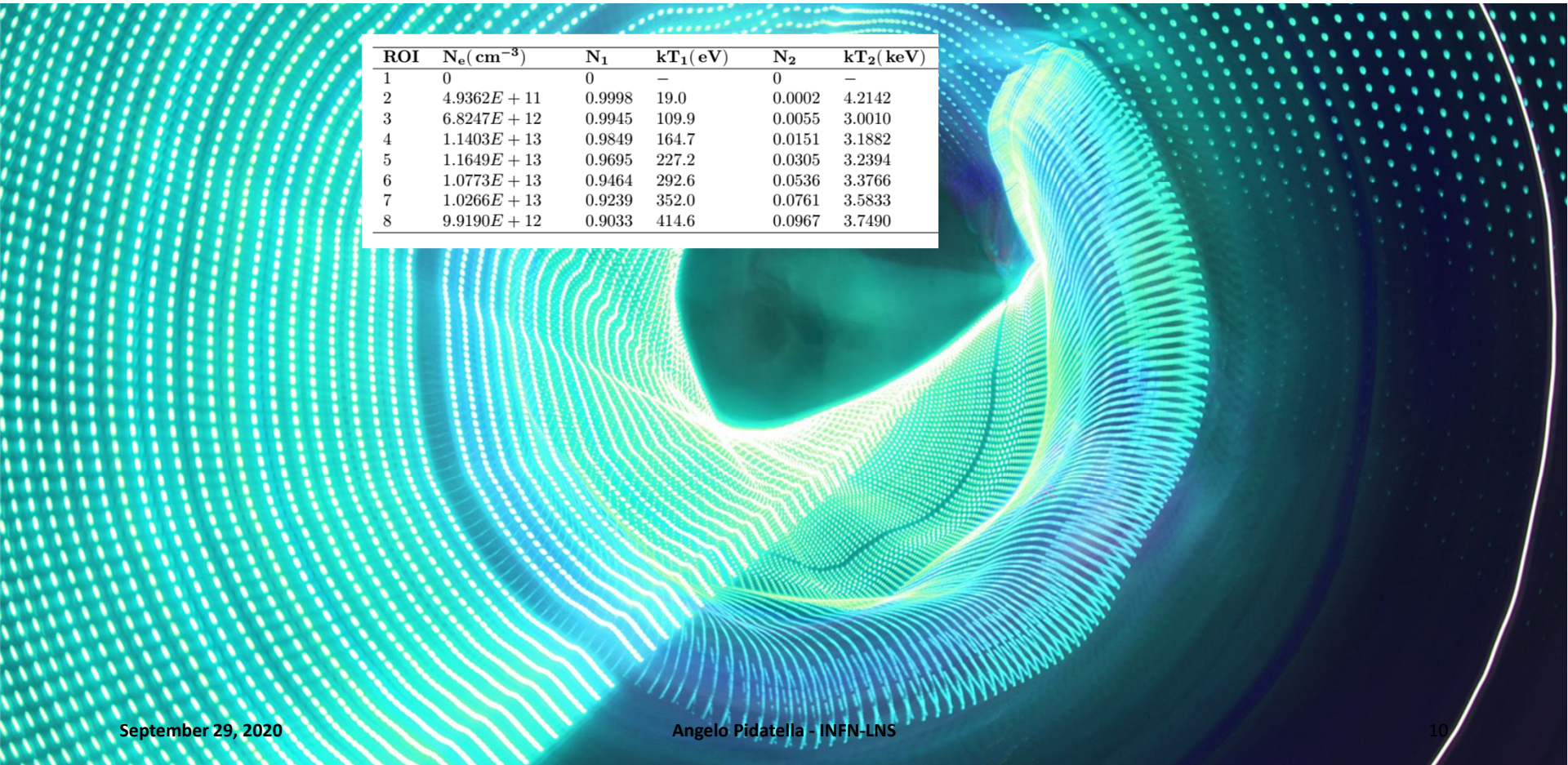
$$R = \rho_e \rho_{Ar} \int_0^{\infty} \sigma(E) v(E) f(E) dE$$

EXPERIMENTAL SETUP



Numerical emission map not too far from capturing the true physical experimental emission image: **robustness of the evaluated EEDF**

2nd EEDF TEST BENCH: ESTIMATE OF ION CHARGE STATE DISTRIBUTION



ROI	$N_e(\text{cm}^{-3})$	N_1	$kT_1(\text{eV})$	N_2	$kT_2(\text{keV})$
1	0	0	—	0	—
2	$4.9362E + 11$	0.9998	19.0	0.0002	4.2142
3	$6.8247E + 12$	0.9945	109.9	0.0055	3.0010
4	$1.1403E + 13$	0.9849	164.7	0.0151	3.1882
5	$1.1649E + 13$	0.9695	227.2	0.0305	3.2394
6	$1.0773E + 13$	0.9464	292.6	0.0536	3.3766
7	$1.0266E + 13$	0.9239	352.0	0.0761	3.5833
8	$9.9190E + 12$	0.9033	414.6	0.0967	3.7490

2nd EEDF TEST BENCH: ESTIMATE OF ION CHARGE STATE DISTRIBUTION

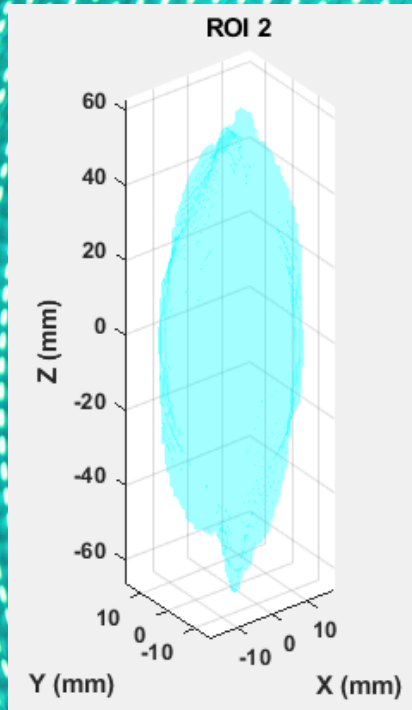
ROI	$N_e(\text{cm}^{-3})$	N_1	$kT_1(\text{eV})$	N_2	$kT_2(\text{keV})$
1	0	0	—	0	—
2	$4.9362E + 11$	0.9998	19.0	0.0002	4.2142
3	$6.8247E + 12$	0.9945	109.9	0.0055	3.0010
4	$1.1403E + 13$	0.9849	164.7	0.0151	3.1882
5	$1.1649E + 13$	0.9695	227.2	0.0305	3.2394
6	$1.0773E + 13$	0.9464	292.6	0.0536	3.3766
7	$1.0266E + 13$	0.9239	352.0	0.0761	3.5833
8	$9.9190E + 12$	0.9033	414.6	0.0967	3.7490

- Ion CSD calculated in spatially-aggregated ROI cells treated as 0D plasma cell: 0D code suite FLYCHK*

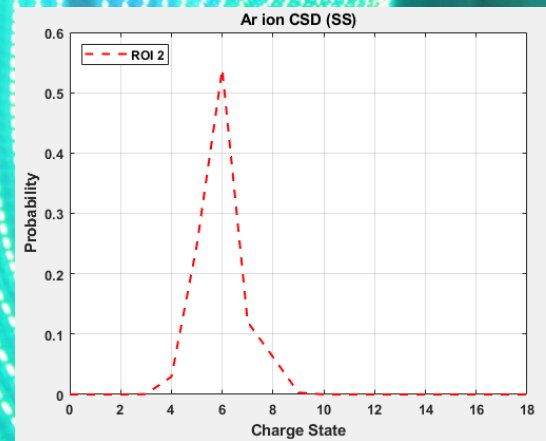
*H.K. Chung, R.W. Lee, M.H. Chen and Y. Raizerchenko, The How To For FLYCHK @ NIST (2008)

- Data input: plasma density + EEDF

2nd EEDF TEST BENCH: ESTIMATE OF ION CHARGE STATE DISTRIBUTION

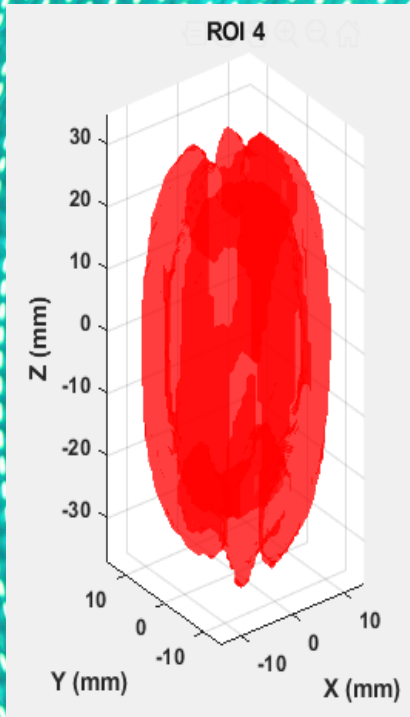


ROI	N_e (cm ⁻³)	N_1	kT_1 (eV)	N_2	kT_2 (keV)
1	0	0	—	0	—
2	4.9362E + 11	0.9998	19.0	0.0002	4.2142
3	6.8247E + 12	0.9945	109.9	0.0055	3.0010
4	1.1403E + 13	0.9849	164.7	0.0151	3.1882
5	1.1649E + 13	0.9695	227.2	0.0305	3.2394
6	1.0773E + 13	0.9464	292.6	0.0536	3.3766
7	1.0266E + 13	0.9239	352.0	0.0761	3.5833
8	9.9190E + 12	0.9033	414.6	0.0967	3.7490

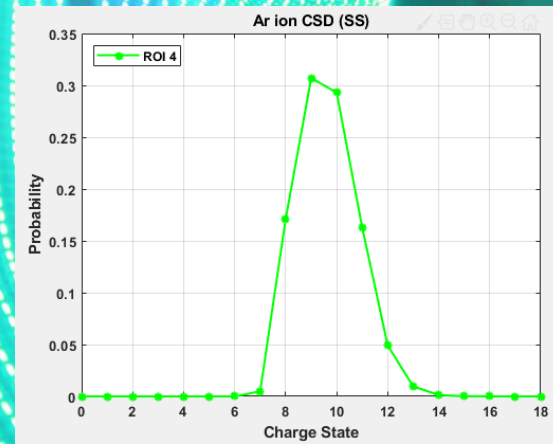


- **Ion CSD** calculated in spatially-aggregated ROI cells treated as **0D plasma cell**: 0D code suite **FLYCHK***
*H.K. Chung, R.W. Lee, M.H. Chen and Y. Raizerchenko, The How To For FLYCHK @ NIST (2008)
- **Data input**: plasma density + EEDF

2nd EEDF TEST BENCH: ESTIMATE OF ION CHARGE STATE DISTRIBUTION



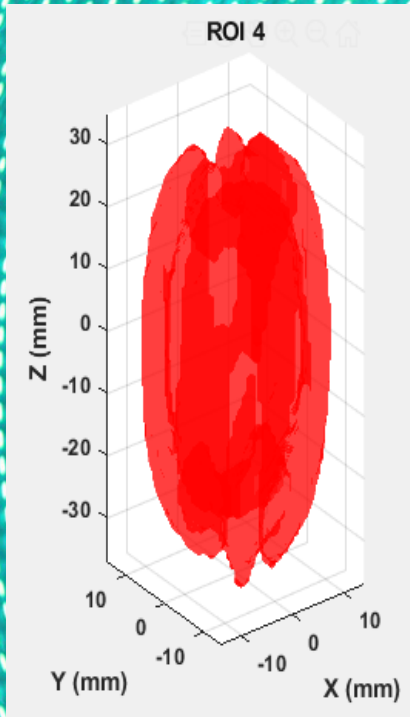
ROI	N_e (cm ⁻³)	N_1	kT_1 (eV)	N_2	kT_2 (keV)
1	0	0	—	0	—
2	4.9362E + 11	0.9998	19.0	0.0002	4.2142
3	6.8247E + 12	0.9945	109.9	0.0055	3.0010
4	1.1403E + 13	0.9849	164.7	0.0151	3.1882
5	1.1649E + 13	0.9695	227.2	0.0305	3.2394
6	1.0773E + 13	0.9464	292.6	0.0536	3.3766
7	1.0266E + 13	0.9239	352.0	0.0761	3.5833
8	9.9190E + 12	0.9033	414.6	0.0967	3.7490



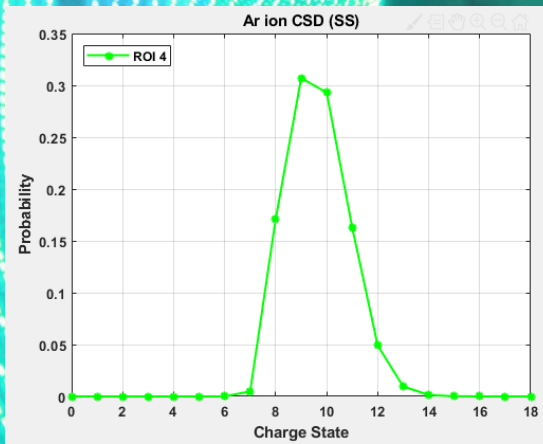
- Ion CSD calculated in spatially-aggregated ROI cells treated as 0D plasma cell: 0D code suite FLYCHK^{*}
- *H.K. Chung, R.W. Lee, M.H. Chen and Y. Raizerchenko, The How To For FLYCHK @ NIST (2008)
- Data input: plasma density + EEDF

B. Mishra, Master Thesis work (2020)

2nd EEDF TEST BENCH: ESTIMATE OF ION CHARGE STATE DISTRIBUTION



ROI	N_e (cm ⁻³)	N_1	kT_1 (eV)	N_2	kT_2 (keV)
1	0	0	—	0	—
2	4.9362E + 11	0.9998	19.0	0.0002	4.2142
3	6.8247E + 12	0.9945	109.9	0.0055	3.0010
4	1.1403E + 13	0.9849	164.7	0.0151	3.1882
5	1.1649E + 13	0.9695	227.2	0.0305	3.2394
6	1.0773E + 13	0.9464	292.6	0.0536	3.3766
7	1.0266E + 13	0.9239	352.0	0.0761	3.5833
8	9.9190E + 12	0.9033	414.6	0.0967	3.7490



- Ion CSD calculated in spatially-aggregated ROI cells treated as 0D plasma cell: 0D code suite FLYCHK^{*}
- *H.K. Chung, R.W. Lee, M.H. Chen and Y. Raizerchenko, The How To For FLYCHK @ NIST (2008)
- Data input: plasma density + EEDF

- Higher electron energy content shifts the ion CSD to higher charge states
- Possibility to use the 0D CSD for 3D modeling ion-electron dynamics

B. Mishra, Master Thesis work (2020)



CONCLUSION

- Self-consistent numerical modeling of ECRIS plasma to study space-resolved and time-resolved phenomena
- Space-resolved particles' anisotropy in the the velocity space reflects the inhomogenous and space-dependent properties of ECRIS plasma
- Space-resolved EEDF provides a local information on the electron distribution
- 2 test bench for the analytical EEDF: 3D map of the fluorescence X-ray emission rate compared with experimental results, evaluation of local ion charge state distribution

ACKNOWLEDGMENTS



David Mascali
Bharat Mishra
Eugenia Naselli
Giuseppe Torrisi



Alessio Galatà



CONCLUSION

- Self-consistent numerical modeling of ECRIS plasma to study space-resolved and time-resolved phenomena
- Space-resolved particles' anisotropy in the the velocity space reflects the inhomogenous and space-dependent properties of ECRIS plasma
- Space-resolved EEDF provides a local information on the electron distribution
- 2 test bench for the analytical EEDF: 3D map of the fluorescence X-ray emission rate compared with experimental results, evaluation of local ion charge state distribution

ACKNOWLEDGMENTS



David Mascali
Bharat Mishra
Eugenia Naselli
Giuseppe Torrisi



Alessio Galatà

THANK YOU ALL!