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MULTI-SPECIES CHILD-LANGMUIR LAW WITH APPLICATION TO ECR ION SOURCES * †

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Abstract

We generalize the classical single-species Child-Langmuir Law to analyze multi-species beams from ECR ion sources. The formulation assumes the relative weight of each species in the extracted beam is known. We apply the results to charge state distribution data from Artemis- and Venus-type sources at the NSCL and LBNL respectively. The total measured beam current is close to the maximum current predicted by the multi-species Child Langmuir law in each case, which indicates that beam extraction occurs close to space-charge-limited flow conditions. Prospects for application of the results and further studies on the topic are outlined.

INTRODUCTION

The classical Child-Langmuir law [1, 2] describes the maximum steady state current that can be transported within a diode given the following assumptions: 1) the diodes are infinite planes with uniform emission which renders the problem one-dimensional; 2) the charged particles are emitted with zero initial velocity (i.e. cold); 3) there is only one species of charged particles; and 4) the particles are non-relativistic. In such a system, space charge forces limit the current that can be transmitted and the maximum current density is given by:

$$J = \frac{4\sqrt{2}}{9}\epsilon_0\sqrt{\frac{q}{m}}\frac{V_0^{3/2}}{d^2} \tag{1}$$

where q and m are the charge and mass of the charged particle, V_0 and d are the potential difference and distance between the cathode and anode, and ϵ_0 is the permittivity of free space. Only ions are considered in this study so we take q to be always positive. If the current density exceeds the limit in Eq. (1), the E-field at the emitting surface would reverse its polarity and “choke” the flow. This space-charge-limited current, also called the Child-Langmuir (CL) current, scales as V_0 to the power 3/2.

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In this paper, we discuss how the classical Child-Langmuir law can be generalized to a multi-species case, i.e. when the beam consists of charged particles with multiple distinct charge-to-mass ratios. The generalized solution is applied to Electron Cyclotron Resonance (ECR) ion sources which typically extract ions with many different charge states.

MULTI-SPECIES CHILD-LANGMUIR LAW

To generalize the single-species Child-Langmuir law to multi-species, we first observe that the single-species solution is derived from a set of three equations:

$$\text{Poisson equation:} \quad \frac{d^2}{dz^2}\Phi = \frac{1}{\epsilon_0}\rho \tag{2}$$

$$\text{continuity equation:} \quad J = \rho v = \text{const} \tag{3}$$

$$\text{energy conservation:} \quad \Phi = \frac{1}{2}\frac{m}{q}v^2 \tag{4}$$

where z is the axial coordinate and $\Phi = V_0 - \phi$ with ϕ being the electric potential and V_0 being the potential difference between the two electrodes. Φ so defined gives $\Phi(z = 0) = 0$ at the cathode and $\Phi(z = d) = V_0$ at the anode, which simplify the equations when q is positive. ρ is the charge density, v is the beam velocity and the other quantities have the same definition as in Eq. (1). Note that Φ, ρ and v are all functions of z.

For the same diode system with n distinct species:

$$\text{Poisson equation:} \quad \frac{d^2}{dz^2}\Phi = \frac{1}{\epsilon_0}\sum_{i=1}^{n}\rho_i \tag{5}$$

$$\text{continuity equations:} \quad J_i = \rho_i v_i = \text{const} \tag{6}$$

$$\text{energy conservation:} \quad \Phi = \frac{1}{2}\frac{m_i}{q_i}v_i^2 \tag{7}$$

where all symbols with subscript i denote the respective variables of the i-th species. The Poisson equation couples all species, whereas each species has its own continuity equation and energy conservation equation. Hence the multi-species system is described by a total of $(2n + 1)$ equations.

Inspired by the two-species solution in the problem set of [3], one way to solve for the maximum current density in the multi-species case is to recast the $(2n + 1)$ equations into 3 equations that resemble the form of Eq. (2) - (4) as follows:

$$\text{Poisson equation:} \quad \frac{d^2}{dz^2}\Phi = \frac{1}{\epsilon_0}\rho \tag{8}$$

$$\text{continuity equation:} \quad J = \rho v_{\text{eff}} = \text{const} \tag{9}$$

$$\text{energy conservation:} \quad \Phi = \frac{1}{2}\left[\frac{m}{q}\right]_{\text{eff}} v_{\text{eff}}^2 \tag{10}$$

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where v_{eff} is an effective velocity and $[m/q]_{\text{eff}}$ is an effective mass-to-charge ratio. The total charge and current densities ρ and J are given by:

$$\rho = \sum_{i=1}^{n} \rho_i \tag{11}$$

$$J = \sum_{i=1}^{n} J_i \tag{12}$$

If such a transformation exists, the multi-species problem is reduced to single-species and the classical Child-Langmuir solution immediately follows:

$$J = \frac{4\sqrt{2}}{9}\epsilon_0 \sqrt{\left[\frac{q}{m}\right]_{\text{eff}}} \frac{V_0^{3/2}}{d^2} \tag{13}$$

where

$$\left[\frac{q}{m}\right]_{\text{eff}} \equiv \left(\left[\frac{m}{q}\right]_{\text{eff}}\right)^{-1} \tag{14}$$

The effective velocity v_{eff} in Eq. (10) is fixed by Eq. (9) and is just the proportionality constant between J and ρ. Therefore, the key to solving the multi-species Child-Langmuir law is finding the correct effective mass-to-charge ratio $[m/q]_{\text{eff}}$ such that Eq. (10) holds. We prove in the Appendix that the effective mass-to-charge ratio is:

$$\left[\frac{m}{q}\right]_{\text{eff}} = \left(\sum_{i=1}^{n} c_i \sqrt{\frac{m_i}{q_i}}\right)^2 \tag{15}$$

where

$$c_i \equiv J_i/J \tag{16}$$

denotes the current weight of the i-th species which we presume to be specified.

A comparison between the multi-species solution in Eq. (13), and the single-species solution in Eq. (1), of the Child-Langmuir law shows that the only difference lies in the charge-to-mass ratio. This means that all information relevant to the multi-species nature of the problem is encapsulated in the effective charge-to-mass ratio given by Eq. (15), and the dependence of the maximum current density on all other parameters remains identical. This is also precisely the reason it was possible to reduce the multi-species problem to single-species in the first place.

APPLICATION TO ECR ION SOURCES

Beams from ECR ion sources typically contain $\gtrsim 10$ species whose respective currents are routinely measured to obtain the charge state distribution (CSD). With knowledge of the current weights c_i in Eq. (15) via the CSD, the multi-species Child-Langmuir law is readily applicable to the beam extraction system to determine the maximum transmittable current and to analyze whether the extraction process is consistent with space-charge-limited flow.

In terms of atomic number A and charge state Q, the maximum current density can be expressed as:

$$J = 5.47 \times 10^{-8} \sqrt{\left[\frac{Q}{A}\right]_{\text{eff}} \frac{V_0^{3/2}}{d^2}} \; [\text{A/m}^2] \tag{17}$$

where

$$\left[\frac{Q}{A}\right]_{\text{eff}} = \left(\sum_{i=1}^{n} c_i \sqrt{\frac{A_i}{Q_i}}\right)^{-2} \tag{18}$$

For a circular emitting surface with radius R, ignoring 2D effects, the multi-species Child-Langmuir current is:

$$I = \pi R^2 J = 5.43 \left[\frac{Q}{A}\right]_{\text{eff}}^{1/2} \left(\frac{R}{d}\right)^2 (V_0[\text{kV}])^{3/2} \; [\text{mA}] \tag{19}$$

We applied Eq. (19) to CSD measurements of the ECRs ARTEMIS-A [4] from the NSCL and VENUS [5] from LBNL. The CSDs are plotted in Fig. 1 with a summary of the results in Table 1. The total measured current is the sum of the measured currents of all species. Due to beam losses between the source and the Faraday cup, the total measured current should be ≤ the drain current supplied to the source. Ideally, the drain current is expected to be a better estimate of the current that is extracted from the ECR. Table 1 also lists extraction system parameters and the associated maximum currents predicted by the single- and multi-species Child-Langmuir law. The target species is used in the single-species case as is conventional.

Table 1: Measurement results, extraction parameters and predictions of single- and multi-species Child-Langmuir (CL) law for the data sets shown in Fig. 1.

Parameter	ARTEMIS-A	VENUS
Number of species meas.	8	20
Target species	$^{16}O^{4+}$	$^{238}U^{33+}$
Total measured current	0.74 mA	2.97 mA
Drain current	1 to 2 mA	no data
Extraction aperture R	4 mm	4 mm
Extraction gap d	40 mm	40 mm
Extraction voltage V_0	20 kV	30 kV
$[Q/A]_{\text{target-species}}$	0.250	0.139
$[Q/A]_{\text{effective}}$	0.191	0.131
Target species CL current	2.43 mA	3.32 mA
Multi-species CL current	2.12 mA	3.23 mA

In both data sets, the effective charge-to-mass ratio calculated from all measured species is smaller than that of the target species. The difference can be over 20%. The total measured beam current or drain current is close to the multi-species Child-Langmuir current which indicates that the ECR extraction region may be operating at, or close to, space-charge-limited flow.

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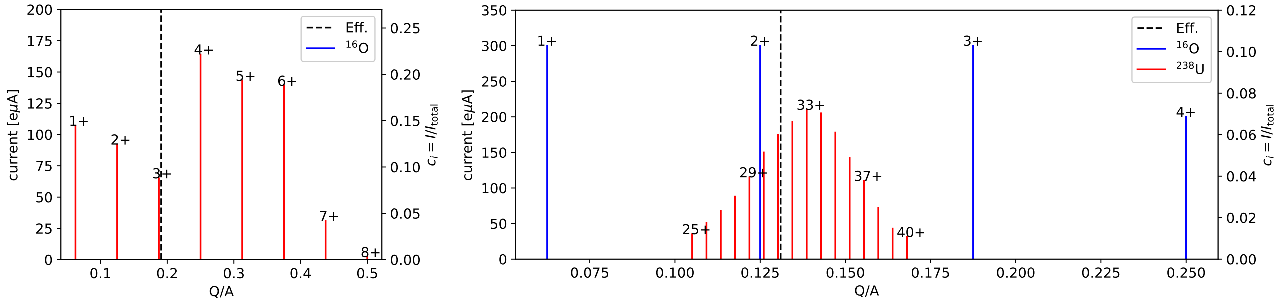

(a) ARTEMIS-A from NSCL (b) VENUS from LBNL

Figure 1: Measured charge state distribution and the corresponding effective charge-to-mass ratio (dashed line) for: a) ARTEMIS-A producing ^{16}O [6] ;and b) VENUS producing ^{238}U with ^{16}O as the support gas [7]. Each plot shows both the species current (left scale) and its proportion in the total current (right scale).

CONCLUSION

We generalized the classical single-species Child-Langmuir law to study the maximum transportable current in a multi-species system. The solution is found to depend on the charge-to-mass ratio of each component species and its relative weight in the total current. When applied to data sets from ARTEMIS-A and VENUS, the maximum current predicted by the multi-species Child-Langmuir law is close to the total extracted current. These results suggest that the extraction region of ECR ion sources are operating near space-charge-limited flow. This phenomenon may have interesting implications - at fixed extraction parameters, desired increase in the current of high charge state target species must be accompanied by a decrease in currents of other species.

Further studies are required to investigate whether the multi-species Child-Langmuir law can be extended to include the effects of initial ion temperature, electron neutralization and finite emitting region which renders the problem two-dimensional. In particular, escaping electrons not immediately swept back into the plasma chamber may alter the singular charge density at the emission plane associated with CL-type solutions. Progress in extending the model should enhance physical insight on space-charge-limited transport and provide a more accurate limit on the maximum current in an ECR extraction system.

While preliminary comparisons have shown that the multi-species Child-Langmuir current is close to the total extracted current for one extraction configuration, performing experiments to check whether the extracted current varies as $V_0^{3/2}$ will constitute a stronger test on whether ECR beam extraction is space charge limited. During such studies, it will also be interesting to observe whether the effective charge-to-mass ratio changes with extraction voltage, and, if it does, one can explore the implications of individual species not following the $V_0^{3/2}$ scaling. One can also investigate whether ECR scaling relations can be used to constrain the current weights c_i and hence the effective charge-to-mass ratio. This would enable the multi-species Child-Langmuir law to make predictions in the absence of CSD measurement results and enhance its utility in planning and design.

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APPENDIX: VERIFICATION OF EQ. (15)

To verify that Eq. (15) is the correct effective mass-to-charge ratio that leads to a solution of the multi-species Child-Langmuir law, we have to prove Eq. (10). Referring to the class of energy conservation equations Eq. (7), it suffices to prove that:

$$\frac{1}{2}\left[\frac{m}{q}\right]_{\mathrm{eff}} v_{\mathrm{eff}}^2 = \frac{1}{2}\frac{m_i}{q_i}v_i^2 \tag{20}$$

for any i. Without loss of generality, we show this is true for the last species, i.e. for $i = n$.

The class of energy conservation equations in Eq. (7) imply:

$$\frac{m_i}{q_i}v_i^2 = \frac{m_n}{q_n}v_n^2 \tag{21}$$

for any i. This enables us to derive the following relation

$$\begin{aligned}\sum_{i=1}^{n} c_i\sqrt{\frac{m_i}{q_i}} &= \frac{1}{J}\sum_{i=1}^{n} J_i\sqrt{\frac{m_i}{q_i}} \\ &= \frac{1}{J}\sum_{i=1}^{n} \rho_i v_i\sqrt{\frac{m_i}{q_i}} \\ &= \frac{1}{J}\sum_{i=1}^{n} \rho_i v_n\sqrt{\frac{m_n}{q_n}}\end{aligned} \tag{22}$$

where the last equality sign results from the square root of Eq. (21).

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Using Eq. (22) and the definition of ρ and J as the total charge and current density, one obtains

$$\begin{aligned}
\left[\frac{m}{q}\right]_{\text{eff}} v_{\text{eff}}^2 &= \left[\left(\sum_{i=1}^{n} c_i \sqrt{\frac{m_i}{q_i}}\right) v_{\text{eff}}\right]^2 \\
&= \left[\left(\sum_{i=1}^{n} c_i \sqrt{\frac{m_i}{q_i}}\right) \frac{1}{\rho} \left(\sum_{j=1}^{n} \rho_j v_j\right)\right]^2 \\
&= \left[\frac{1}{J} \left(\sum_{i=1}^{n} \rho_i v_n \sqrt{\frac{m_n}{q_n}}\right) \frac{1}{\rho} \left(\sum_{j=1}^{n} \rho_j v_j\right)\right]^2 \\
&= \left[\frac{1}{J} \left(\sum_{i=1}^{n} \rho_i\right) \frac{1}{\rho} \left(\sum_{j=1}^{n} \rho_j v_j\right)\right]^2 \frac{m_n}{q_n} v_n^2 \\
&= \frac{m_n}{q_n} v_n^2
\end{aligned} \tag{23}$$

which explicitly proves that Eq. (20) holds for $i = n$.

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