Abstract
Longitudinal bunch distributions of subpicosecond and picosecond electron beams have been evaluated by the coherent transition radiation Michelson interferometer with the reconstruction procedure from interferograms. The results were compared to those given by a femtosecond streak camera which time resolution was 200 fs at FWHM. From the comparison, the validity of the method to evaluate the subpicosecond electron pulse width that would be close to the time resolution of the femtosecond streak camera has been discussed.

1 INTRODUCTION
We aim to product and measure the femtosecond electron beam which pulse length is shorter than the time resolution of the femtosecond streak camera (200 fs at FWHM) in near future. Now in the Nuclear Engineering Research Laboratory of University of Tokyo, the shortest bunch that can be generated is close to the time resolution. There are two promising methods to evaluate longitudinal pulse shapes of femtosecond electron bunches. The first one is the femtosecond streak camera [1]. The other is the coherent far-infrared transition radiation interferometry [2,3,4]. It is important to compare the results by the two methods in order to confirm the precision of both methods[5,6,7]. In this paper, we explains the construction of the Michelson coherent transition radiation interferometer and measure subpico- and picosecond electron pulses which are longer than the time resolution of the streak camera. Furthermore, the results were compared with each other and the reliability and improvement of the method was discussed.

2 MICHELSON INTERFEROMETER
It is known that the transition radiation with broad spectrum is emitted when an electron bunch passes through the boundary between two mediums which dielectric constants are different from each other. In case that the wavelength of the radiation is longer than the bunch length, the phase difference of the radiation emitted by each electron can be ignored so that the radiation becomes coherent. The Michelson interferometer for the measurement of the electron pulse width utilizes the coherent transition radiation. The longitudinal bunch shapes of the electron bunch can be deduced from the interferogram with the procedure of reconstruction. From the interferogram the power spectrum of the radiation is given by the Fourier transformation as follows,

\[ |E(\nu)|^2 = \frac{1}{4\pi|RT|^2} \int_{-\infty}^{\infty} S(\delta) e^{-i2\pi\nu\delta/c} d\delta, \quad (1) \]

where \( \nu \) is the wavenumber, \( S(\delta) \) is the light intensity of the recombined radiation at the detector which expressed in the time domain with an additional time delay \( \delta/c \) for the movable mirror minus the intensity at \( \delta \to \pm \infty \) and \( R,T \) are the coefficients of reflection and transmission at the beam splitter, respectively. And then the longitudinal bunch form factor can be obtained by,

\[ f_n(\nu) = \frac{\int_{-\infty}^{\infty} S(\delta) e^{-i2\pi\nu\delta/c} d\delta}{4\pi|RT|^2 I_e(\nu)}, \quad (2) \]

where \( N \) is the number of electrons in the bunches and \( I_e(\nu) \) is the radiation intensity emitted from a single electron. The longitudinal bunch distributions deduced with two methods of reconstruction. One is under assumption of symmetric bunch distribution and then only the inverse Fourier transformation is used. The other is under assumption of asymmetric bunch distribution and then the Kramers-Kronig relation is used with the inverse Fourier transformation as follows,

\[ h(z) = \int_{-\infty}^{\infty} f_1(\nu) \exp\left[i\phi_1(\nu) - 2\pi\nu t\right] d\nu, \quad (3) \]

\[ \phi_1(\nu) = -2\nu \int_{\nu}^{\infty} \frac{\ln[g(\nu') - g(\nu)]}{\nu'^2 - \nu^2} d\nu. \quad (4) \]

Furthermore, we must choose theoretical distribution functions of the electron bunch such as Gaussian distribution or exponential distribution. The results of these method are shown and the difference is discussed in the following chapter.
3 EXPERIMENT

3.1 Experimental setup
We performed this comparison at the S-band linac where the achromatic-arc-type magnetic pulse compressor is installed. In the experiment the longitudinal bunch distribution was controlled by tuning the energy modulation of the bunch in the accelerating tube for the magnetic pulse compression. We chose subpico- and picosecond (FWHM) pulse widths and performed the comparison between the femtosecond streak camera and the Michelson coherent transition radiation interferometry measurement as shown in Fig. 1. We measured the transition radiation in the far-infrared region emitted by the electron bunch at the Al-foil put in air after the 50-mm-thick Ti window at the end of the 35L linac. We used two liquid-He-cooled Si bolometers as a detector for the far-infrared radiation. The major beam parameters are as follows: the energy is 32 MeV, the pulse length 800 fs to 1.7 ps (FWHM) and the electron charge per single pulse 30 to 250 pC.

Fig. 1 Experimental setup

3.2 Procedure of analysis
On the basis of the procedure of analysis as mentioned in chapter 2, we analyzed these pulses from the interferograms which we got by the Michelson interferometer. Because of nonuniform transparency of the 100-mm-thick Mylar beam splitter and diffraction loss of long-wavelength components, the bunch form factor was obtained within rather limited range. Therefore we have to use theoretical bunch form factors assuming the Gaussian or exponential distribution out of the range.

4 RESULTS AND DISCUSSION
The interferogram of the subpicosecond electron pulse is shown in Fig. 2.

Fig. 2 Interferogram of the subpico-second electron pulse.

The experimental result of the bunch form factor is shown by the solid curve and that of theoretical by dashed curve in Fig. 3. In the figure, we choose the Gaussian distribution as the theoretical curve since the exponential function has unphysical long tails in both sides and the simultaneous observation of the bunch shapes by the streak camera indicates that the Gaussian is closer to the real bunch distribution.

Fig. 3 Bunch form factor

The dashed curves in Fig. 3 represent those of three bunch length (400, 500 and 600 fs at FWHM). We used the measured bunch form factor in the range of 9.5 - 18 cm\(^{-1}\) and the theoretical bunch form factor out of the range for the analysis. In this case, we adopted the
bunch form factor of the Gaussian of 500 fs (FWHM) and extrapolated this to the range under 9.5 cm$^{-1}$ and over 18.0 cm$^{-1}$.

Finally, we got the bunch distribution derived by the interferometry as shown by the solid curve in Fig. 4. The dashed curve in the figure is one of the pulse shape taken by the streak camera. For example, the result by the interferometry under the assumption of asymmetric Gaussian distribution gives 550 fs bunch length at FWHM while that by the streak camera becomes 650 fs. The calibration of the camera was also performed by using a Ti:Sapphire laser. Then the error at FWHM was found out to be 370 fs (FWHM) assuming the law of error propagation. After the above error is subtracted, the net pulse length becomes 550 fs.

Differences of the pulse width with two theoretical distributions are shown in Table 1 and 2. From these tables, Gaussian fitting gives good agreement with the results of the streak camera. We have known that the distribution of the real electron bunch shows approximately Gaussian from images taken by the streak camera obtained almost simultaneously. Next asymmetric and symmetric results of the Gaussian fitting indicate almost same, otherwise in the case of exponential fitting the symmetric results are much shorter. Consequently the pulse width by the interferometry and those by the streak camera agree with each other with appropriate choice of the Gaussian distribution function.

![Graph](image)

Fig. 4 Bunch distribution by the interferometry (solid) and that by the streak camera (dashed)

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<thead>
<tr>
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<th>Gaussian Fitting</th>
<th>Exponential Fitting</th>
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<tbody>
<tr>
<td>Symmetric</td>
<td>520 fs</td>
<td>420 fs</td>
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<tr>
<td>Asymmetric</td>
<td>550 fs</td>
<td>570 fs</td>
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Table 2. Reconstructed pulse width (FWHM) of picosecond electron pulses by the interferometer (The results of the streak camera was 1.8 fs).

5 CONCLUSION

From the comparison of the diagnostics by the interferometry with the streak camera, the reliability of the interferometric method to evaluate subpico- and picosecond electron bunches was confirmed with appropriate assumption of the longitudinal Gaussian distribution function of electron in the bunches.

6 REFERENCES