MUON DYNAMICS AND IONIZATION COOLING AT MUON COLLIDERS

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Abstract

Muon Dynamics and beam cooling methods for muon colliders are presented. Formulations and effects of Ionization cooling as the preferred method used to compress the phase space to reduce the emittance and to obtain high luminosity muon beams are also included.

1 INTRODUCTION

Muon colliders potential to provide a probe for fundamental particle physics is very interesting. To obtain the needed collider luminosity, the phase-space volume must be greatly reduced within the muon life time. The Ionization cooling is the preferred method used to compress the phase space and reduce the emittance to obtain high luminosity muon beams. We note that, the ionization losses results not only in damping, but also heating. In ionization method, muons passing through a material medium lose momentum and energy through ionization interactions in transverse and longitudinal directions. The normalized emittance is reduced due to transvers energy losses. Although Ionization cooling studies dates back to 1970, experimentally it is a new technique that has not yet been demonstrated. We discuss the formalism including equations for damping rates of particles, optimization criterion, etc. We show how to estimate the optimal damping, energy and length of the system and illustrate the use of our formulations, with a numerical example.

2 FORMALISM & METHODS

At the present time the most promising way to cool muons seems to be the use of the ionization cooling. Damping rates (decrements) of individual particles in the absence of wedges (natural damping rate) are defined by the following formula:

$$\begin{aligned} \lambda_\perp &= -\frac{dE}{dz}_{ion}\ \frac{1}{2\beta^2\gamma m\, c^2} \\ \lambda_\parallel &= -\frac{1}{z}\ \frac{d}{dp}\left[\left(\frac{dE}{dz}\right)_{\rm ion}\ \frac{1}{v}\right] \end{aligned} \quad (1)$$

Where $\lambda_\perp$ and $\lambda_\parallel$ are natural transverse and longitudinal damping respectively. Here $\left(\frac{dE}{dz}\right)_{\rm ion}$ is the ionization losses of energy, m is the muon mass, β,γ are relativistic parameters, p,v are momentum and longitudinal velocity of muons being cooled.

It was established, that the sum of all increments is invariant of the cooling system.

$$\Lambda = 2\lambda_\perp + \lambda_\parallel \quad (2)$$

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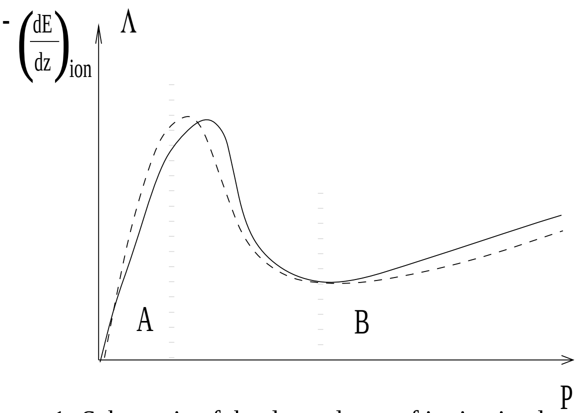

Figure 1: Schematic of the dependence of ionization losses on momenta.

This curve is also plotted in Fig. (1) (as the dotted line). Thus, we see that there are two natural regions for cooling: region A ("frictional cooling") and region B ("ionization cooling" for intermediate and high energies). Frictional Cooling (FC) is convenient only for cold (low energy) muons (e.g. Kinetic energy 10 to 150 KeV), and therfore it is difficult to use for high energy muon source, (in addition to big noises due to coulomb scattering etc.). Classical Ionization Cooling is useable for kinetic energy range of 30 to 100 MeV. Which due to abscence of "natural" longitudinal cooling it is necessary to use "wedges" for which R & D is needed (e.g. to choose a scheme including the optics, cooling, etc.). A propsal for such studies is being considered [6].

Other methods includes: electron cooling, stochastic cooling and channel cooling also have been studied: i) Electron cooling (EC), is suitable for the kinetic energy range of 5 to 30 MeV. To use EC, electron currents should be increased up to 10 mA and then a very good space charge and current compensation is necessary. ii) In stochastic cooling (Kinetic energy range of about GeV and more), it is necessary to diminish cooling time in few orders of magnitude (optical cooling?). In channel cooling (Kinetic energy range about GeV and more), it may be necessary to check principle of action in experiments with a number of crystals (not a single one) [3].

A milestone of the theory are equations for damping rates of an individual particle, discussed earlier. However, each section with ionization losses give not only damping, but a heating as well: transverse heating appears due to multiple Coulomb scattering and longitudinal one is due to so named "straggling" of the ionization losses (we note that, this straggling is produced by fast "knock-on" ionization electrons). Fig. 2 shows a conceptual schematic of ionization cooling. The beam is passed through some

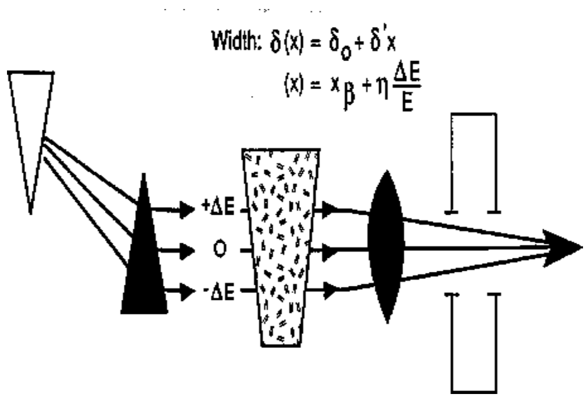

Figure 2: A schematic of cooling ionization through a wedge.

material in which the muons lose both transverse- and longitudinal-momentum by ionization loss (dE/dx). The longitudinal muon momentum is then restored by coherent re-acceleration, leaving a net loss of transverse momentum (transverse cooling).To achieve a large cooling factor the process is repeated many times. The transverse cooling can be expressed as

$$\frac{d\epsilon_n}{ds} = -\frac{1}{\beta^2}\frac{dE_\mu}{ds}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp(0.014)^2}{2\,E_\mu m_\mu\,L_R}, \qquad (3)$$

where β = v/c, ϵ_n is the normalized emittance, $\beta_\perp$ is the betatron function at the absorber, dE_μ/ds is the energy loss, and L_R is the radiation length of the material. The first term in this equation is the cooling term, and the second is the heating term due to multiple scattering. To minimize the heating term, a strong-focusing (small $\beta_\perp$) and a low-Z absorber (large L_R) is needed.

The energy straggling (heating) term is given by

$$\frac{d(\Delta E_\mu)^2_{\rm straggling}}{ds} = 4\pi\,(r_e m_e c^2)^2\,N_o\,\frac{Z}{A}\,\rho\gamma^2\left(1-\frac{\beta^2}{2}\right), \qquad (4)$$

where N_o is Avogadro's number, Z is the charge, ρ is the density, and the energy spread is given by:

$$\frac{d(\Delta E)^2}{ds} = \frac{d(\Delta E_\mu)^2_{\rm straggling}}{ds} - 2\,\frac{d\left(\frac{dE_\mu}{ds}\right)}{dE_\mu} < (\Delta E_\mu)^2 > \qquad (5)$$

where the first term is the heating due to straggling, and the second term is the cooling (or heating) due to energy loss, As illustrated in in Fig. 2, by introducing a transverse variation in the wedge (absorber) density or thickness, the energy spread, and the longitudinal emittance can be reduced.

3 ANALYSIS METHODS

From theoretical point of view, a situation with ionization cooling completely corresponds to a situation with radiation cooling whose theory is well developed. There is some standard "hierarchy" of methods for analyzing such systems: i) Moments methods, ii) Focker-Planck equation, iii) Multi-particles codes. i) Method of moments (MM) Using equation of motion for separate particles or kinetic equation, it is easy to derive equations for the second order moments ($\langle x^2\rangle, \langle xx'\rangle, \langle x'^2\rangle$ and etc.). (see, for example, [4]). Analysis of these equations permits us to find equilibrium emittances and to investigate evolution of emittances with time for linear external fields. It is necessary to be very careful when using this method (MM) e.g., 1) In transverse plane a strong focusing usually is used, and therefore "smoothing" of the motion (change $w_\perp^2(z) \to w_0^2$) must be done very accurately. 2) Often some authors have solved only the equations for $\langle x'^2\rangle$ (which, really, is proportional to beam temperature) *without* solving the equations for the last moments. However, to calculate evolution on time accurately we must solve the system of coupled equations for the moments [2, 4]. ii) Focker-Planck (FP) equation To find a form of a distribution function and to estimate beam losses due to diffusion it is possible to use the FP equation. The one-dimensional FP equation has the following form:

$$D\frac{\partial}{\partial I}\left(I\frac{\partial f}{\partial I}\right) + \alpha\frac{\partial}{\partial I}(fI) = \frac{\partial f}{\partial z} \qquad (6)$$

Where f = the distribution function ($f = f(I,z)$); I = action; z = longitudinal variable; D = the diffusion coefficient; and α = the damping coefficient. The FP equation must be solved using the following boundary condition:

$$f_m(I_m, z) = 0 \qquad (7)$$

where I_m is the aperture in I-variable. If $I_m = \infty$, Eq. (2) has a stationary solution:

$$f_0 = \frac{1}{I_o}\exp\left(-\frac{I}{I_0}\right) \qquad (8)$$

Where $I_0 = \frac{D}{\alpha}$.
Let us underline, that if $I_m >> I_0$, we can estimate particle losses using a simple formula:

$$\frac{1}{N}\frac{dN}{dz} \simeq D\frac{I_m\partial f_0}{\partial I}(I=I_m) = \frac{DI_m}{I_0^2}\exp\left(-\frac{I_m}{I_0}\right) = \alpha\frac{I_m}{I_0}\exp\left(-\frac{I_m}{I_0}\right) \qquad (9)$$

If $\frac{I_m}{I_0} \simeq$ constant,

$$\frac{\Delta N}{N} \sim \alpha L\frac{I_m}{I_0}\exp\left(-\frac{I_m}{I_0}\right) \qquad (10)$$

For example, if $\frac{I_m}{I_0} = 9$ and $L = \frac{5}{\alpha}$ (that corresponds to damping of an initial emittance in 150 times), three-dimensional diffusion losses $\frac{\Delta N}{N} \sim 0.017$

If condition $I_m >> I_0$ is not performed, we can find a beam evolution by numerical solution of FP equation. iii) Multi-particle codes. We see that method of moments

(MM) and Focker-Planck (FP) equation gives only limited information. A whole information about the beam, including large-angle scattering, interactions with residual gas, increase of emittances due to nonlinearity of the external field and so on can be found only by multi-particle codes. Map methods are not convenient for our problem due to presence of large noises. However, we note, that it is a correct procedure that can be used to validate accuracy of the final results of numerical calculations.

4 OPTIMIZATION OF PARAMETERS

Problem of choice of optimal parameters for the muon cooling system is open, and its solution is far from trivial due to infinite numbers of possible options. The process of optimization can usually be divided into five stages: 1) choice of optimization criterion; 2) calculation of "primary" parameters by use of simple analytical models; 3) choice of design of focusing and accelerating systems; 4) search of optimum for chosen focusing and accelerating systems by use of more sophisticated algorithms; 5) validation of chosen scheme by use of multi-particles codes, etc. Here we discuss the first two points (others are not of scope of this paper).

• Optimization: Luminosity of collider L is defined by the following expression:

$$L \sim \frac{N^2 f}{g_x g_y} = \frac{N^2 f}{\epsilon_\perp^f \cdot \beta_\perp^f} \tag{11}$$

Where $N=$ a number of muons per bunch, $f=$ mean repetition frequency of collisions, $\epsilon_\perp^f=$ emittance at collision point and $\beta_\perp^f=\beta$-function at collision point. Usually $\beta_\perp^f$ is limited by condition:

$$\beta_\perp^f \geq \sigma_z^f \tag{12}$$

where σ_z^f is a longitudinal bunch size. Let us assume, that: 1) $\frac{\Delta p_f}{p}$ is known (monochromatic experiments); 2) we can redistribute emittances inside a given six-dimensional phase volume. Then, taking into account losses in the cooling system, we can rewrite Eq. (11) in the following form:

$$L \sim \frac{N_0^2 \exp\left(-\frac{2}{cT_0}\int_0^z \frac{dz}{\gamma(z)}\right) D^2}{\sqrt{V_6^N \cdot \epsilon_\parallel^f}} \cdot \left(\frac{\Delta p}{p}\right)_\parallel^f \tag{13}$$

Here "N_0" is a number of particles at an entrance of the cooling system, "exp" describes muon decay, "D" describes muon losses in cooling section, and "V_6^N" is an invariant six-dimensional phase volume of muon beam.

Thus we can introduce "merit factor" which describes a quality of muon cooling system. We obtain

$$R = \frac{D^2 \exp\left[-\frac{2}{cT_0}\int_0^z \frac{d_z}{\gamma(z)}\right]}{\sqrt{V_6^N}} \tag{14}$$

Note that, the dependence on V_6^N may be stronger. With account of all the circumstances, we can write

$$R \sim (V_6^N)^\alpha \tag{15}$$

with α in interval (0.5; 2/3).

• Calculation of primary parameters

Let us neglect muon losses ($D = 1$, $\exp\left[-\frac{2}{cT_0}\int_0^z \frac{dz}{\gamma(z)}\right] = 1$). Then to maximize R we must minimize V_6^N. Taking into account standard formula for equilibrium phase volumes, we have:

$$V_6^N = \frac{W_\perp^2 \cdot W_\parallel}{\alpha_\parallel\left(\alpha_\perp^0 - \frac{\alpha_\parallel}{2}\right)^2} \tag{16}$$

This equation has a solution relative to $\alpha_\parallel$:

$$\alpha_\perp^{\min} = \alpha_\parallel^{\min} = \frac{2\alpha_\perp^0}{3}; \tag{17}$$

$$(\sqrt{V_6^N})_{\min} = \frac{27}{8}\frac{W_\perp^2 \cdot W_\parallel}{(\alpha_\perp^0)^3} \tag{18}$$

However, a situation will change if we take into account the muon losses.

As an example, let us consider a simplest case: 1) $D = 1$; 2) $\lambda_\parallel << \Omega$; 3) $\gamma = \gamma_0$. In this case (for $\lambda_\perp = \lambda_\parallel$)

$$R \simeq \frac{\exp[-2L/cT_0\,\gamma_0]}{(V_6^N)_{\min}^{\frac{1}{2}}\left[1 + \frac{\epsilon_\perp^0 - \epsilon_{eq}}{\epsilon_{eq}}\exp(-\lambda L)\right]^{-3/2}} \tag{19}$$

It is easy to see that in this case there is an optimal system length L, which is defined by the following equation:

$$L_0 = \frac{1}{\lambda}\ell n\left(\frac{\frac{AB}{\lambda}}{\frac{3}{2}B - \frac{A}{\lambda}}\right)^{-1} \tag{20}$$

Where $A = \frac{2}{cT_0\,\gamma_0}$ and $B = \frac{\epsilon_\perp^0 - \epsilon_{eq}}{\epsilon_{eq}}$. Numerical Example: We consider a numerical example with $\gamma_0 = 1.5$; $\lambda = 5 \cdot 10^{-2} m^{-1}$; $B = 10$. Then we obtain:

$$\begin{aligned} \tau_0^{\max} &= \lambda L = 3.6 \\ R_{\max} &\simeq \frac{0.6}{\sqrt{(\sqrt{V_6^N})_{\min}}} \end{aligned} \tag{21}$$

5 REFERENCES

[1] A. Skrinsky, Morges,1971; V. Parchochuk and A. Skrinsky*Phys. of Elementary Part. and Atomic Nucl.***12**(1981).

[2] Z. Parsa, ed., *Beam Stability and Nonlinear Dynamics*, AIP-Press CP **405**, 1997.

[3] Z. Parsa and P. Zenkevich, D. Cline, ITP-Report (1996)

[4] Z. Parsa, Emittance and Moments in Accelerators and Beamlines Parts (I) and (II), Reports and BNL-44618, (10/1989);

[5] Z. Parsa, Ionization Cooling and Muon Dynamics, in *Physics Potential & Development of Muon-Muon Colliders*, AIP-Press, Woodbury, NY (in Press).

[6] Muon Collider Feasibility Study, BNL Report BNL-52503 (1996); Muon Collab; MUCOOL Collab (S. Geer) Proposal, April 15, 1998; AIP CP 352; AIP CP 372.

[7] Z. Parsa, ed., *Future High Energy Colliders*, AIP CP **397**, 1997.