High Brightness Low Frequency SR From a $2/\gamma$-Deviation ID

F. Méot
CEA/Saclay, DSM/DAPNIA/SEA, 91191 Gif sur Yvette, France

Abstract

It is shown that a $(2/\gamma)$-deviation insertion device produces highest SR brightness in the low frequency spectral range with a gain of four w.r.t. edge SR and up to several orders of magnitude w.r.t. body SR from a long dipole.

1 INTRODUCTION

Low frequency synchrotron radiation (SR) has always been a concern in many respects, e.g., as to beam diagnostics with visible and higher frequency SR. It is nowadays again a hot topic with the sake for quality long-wavelength and in particular infrared radiation production. We here show how low frequency SR properties cause a $(2/\gamma)$-deviation dipole to shift the SR peak energy density to the low frequency spectral range - liable to reach up to x-rays energy ranges in sufficiently high energy lepton machines, entailing a gain of four w.r.t. edge SR and up to several orders of magnitude w.r.t. body SR from a long dipole, and confinement of the radiation within a $\sqrt{2}/\gamma$ rms aperture cone.

2 LOW FREQUENCY SR

SR spectral angular energy density is given by [1]

$$\frac{\partial^2 W}{\partial\omega\partial\Omega} = 2\epsilon_0 c r^2 |\tilde{\vec{E}}(\omega)|^2 \tag{1}$$

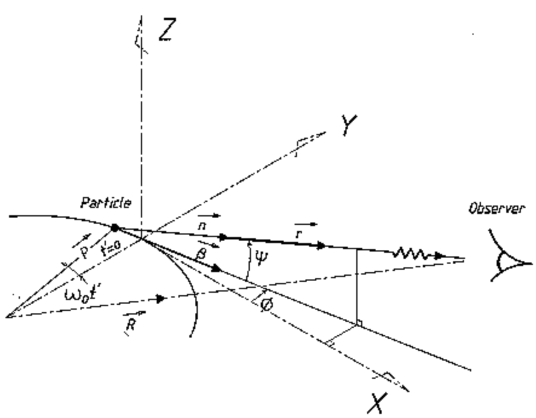

Figure 1: Reference frame and notations used in the text.

where r is the distance (assumed constant) from the particle trajectory region to the observer, $\tilde{\vec{E}}(\omega)$ is the Fourier transform of the electric field $\vec{E}(t)$ of the radiated wave, $\omega/2\pi$ is the spectral frequency and Ω is the solid angle. In the far-field approximation ($r \ll r^2$) the electric field of the radiated electromagnetic wave writes

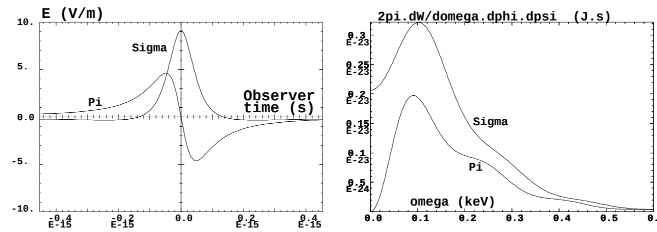

Figure 2: Typical shapes of the $E_{\sigma,\pi}(t)$ (left), and related spectral angular densities (right).

$$\vec{E}(t) = \frac{q}{4\pi\epsilon_0 c}\frac{\vec{n}(t') \times [(\vec{n}(t') - \vec{\beta}(t')) \times \dot{\vec{\beta}}(t')]}{r(t')(1 - \vec{n}(t') \cdot \vec{\beta}(t'))^3} \tag{2}$$

with, following conventional notations [1] (Fig. 1), $t' = t - r(t')/c$ and t = observer time, t' = particle time, $\vec{n}(t') = \vec{r}(t')/r(t')$ = observer direction, $c\vec{\beta}(t')$ = particle velocity, $\dot{\vec{\beta}}(t') = d\vec{\beta}/dt'$ and q = particle charge. $\vec{E}$ is normal to $\vec{n}$ and can be split into the two polarisation components $\vec{E_\sigma}$ parallel to the bend plane and $\vec{E_\pi}$ normal to $\vec{E_\sigma}$ and $\vec{n}$.

In the case of low frequency SR the Fourier transform verifies $\sqrt{2\pi}\tilde{\vec{E}}(\omega) = \int_{\Delta t}\vec{E}(t)e^{-i\omega t}dt \overset{\omega\Delta t \ll 1}{\approx} \int_{\Delta t}\vec{E}(t)dt$ independent of the frequency ω under observation, with the integration interval Δt being the total duration of the observed impulse . This integral can be calculated in particle time variable t' under the form $\tilde{\vec{E}}(\omega) \approx \frac{1}{\sqrt{2\pi}}\int \vec{E}(t')\frac{dt}{dt'}dt'$ with $E(t') \equiv E(t(t'))$ while the $t(t')$ dependence can be obtained from $t' = t - r(t')/c$ which also provides, by differentiation expanded to order $(1/\gamma^2)$, $dt/dt' = 1 - \vec{n}(t') \cdot \vec{\beta}(t') \simeq [1 + \gamma^2\psi^2 + \gamma^2(\omega_0 t' - \phi)^2]/2\gamma^2$ (more details can be found in Ref. [2]).

Frequency validity domain

The above Fourier transform low frequency approximation is valid as long as $\omega\Delta t \ll 1$, where Δt can be obtained by integration of the dt/dt' equation, from $t' = 0$ to $t' = L/2c$, where L is the magnetic field extent. In the forward direction ($\phi = \psi = 0$) this gives $\Delta t = L(1 + \gamma^2\alpha^2/12)\,/\,2\gamma^2 c$ with α = total particle deviation. One therefore gets the validity condition

$$\omega \ll 2\gamma^2 c\,/\,L(1 + \gamma^2\alpha^2/12) \tag{3}$$

Electric field in particle time

We take (Fig. 1) $\vec{r}(t') = \vec{R} - \vec{p}(t') \approx$ constant with $\vec{R}$ (resp^{ly} $\vec{p}(t')$) = position of the observer (particle) in the laboratory frame, and ϕ (resp^{ly} ψ) = horizontal (vertical) component of the observation angle (ϕ is zero in the direction coinciding with $\omega_0 t' = 0$), $\omega_0 = c/\rho$ = rotation frequency with ρ = constant bending radius. In the (x,y,z) frame this leads to $\vec{n} = (\cos\psi\cos\phi, \cos\psi\sin\phi, \sin\psi)$ and $\vec{\beta} = \beta(\cos\omega_0 t', \sin\omega_0 t', 0)$. Substituting into Eq. 2 one gets

$$\begin{aligned} E_\sigma(t') &= \frac{q\omega_0\gamma^4}{\pi\epsilon_0 cr}\frac{(1+\gamma^2\psi^2)-\gamma^2(\omega_0 t'-\phi)^2}{(1+\gamma^2\psi^2+\gamma^2(\omega_0 t'-\phi)^2)^3}\mathrm{rect}\frac{t'}{2T'} \quad (4)\\ E_\pi(t') &= \frac{q\omega_0\gamma^4}{\pi\epsilon_0 cr}\frac{-2\gamma\psi\gamma(\omega_0 t'-\phi)}{(1+\gamma^2\psi^2+\gamma^2(\omega_0 t'-\phi)^2)^3}\mathrm{rect}\frac{t'}{2T'} \end{aligned}$$

where $\mathrm{rect}(x) = 1$ if $-\frac{1}{2} < x < \frac{1}{2}$ and zero elsewhere, allowing for the finite extent ($2cT'$) of the magnetic field.

The related spectra (as obtained by numerical calculation of Eqs. 1, 2) are shown in Fig. 2-right ; as expected the σ component does not go to zero with ω, due to the truncation of $E_\sigma(t)$ (Fig. 2-left) whereas $\int E_\pi(t)dt$ is zero because $E_\pi(t)$ is odd. The low frequency limit value as a function of the aiming direction (ϕ,ψ) can be obtained by analytical calculation of Eq. 1 from the Fourier transforms $\tilde{\vec{E}}(\omega) \approx \frac{1}{\sqrt{2\pi}}\int \vec{E}(t')\frac{dt}{dt'}dt'$ of Eqs. 4. The parameter $\gamma\alpha \equiv \gamma\omega_0 T'$ in Eqs. 4 can be understood as the ratio $2cT'/(2\rho/\gamma)$ of the magnetic field extent to the critical trajectory arc length $2\rho/\gamma$ that corresponds to the central positive arch in $E_\sigma(t)$: the larger that ratio, the smaller the low frequency spectral angular density.

3 SR FROM A $(2/\gamma)$-DEVIATION ID

From what precedes it appears that the low frequency energy density is maximised upon truncating the E_σ impulse (at $\pm\tau_c = \pm 2(1+\gamma^2\psi^2)^{3/2}/\omega_c$, with $\omega_c = 3\gamma^3 c/2\rho$ = critical frequency), in order to preserve the sole central positive arch of the impulse. Doing so maximises the integral $\int E_\sigma(t)dt$ and hence the intensity in the σ component, in the low frequency spectral range. This is achieved by observing the centre of a $(2/\gamma)$-deviation dipole ; as shown below the gain on the spectral angular energy density so obtained w.r.t. body SR from a long dipole reaches orders of magnitude in the low frequency range, this last being determined by[1] (Eq. 3 with $\gamma\alpha = 2$), $\omega \ll 3\gamma^2 c/2L = 3\gamma^3 c/4\rho = \omega_c/2$.

The deviation in the dipole verifies $BL/B\rho = 2/\gamma$. Taking $B\rho = p/q \simeq E/c = \gamma E_0/c$ (with E = particle energy and E_0 = rest mass) this fixes the magnetic field integral, namely (B in Tesla, L in meter)

$$BL = 2E_0/c \approx 1/300 \;\; (T.m) \qquad \text{(for electrons)} \quad (5)$$

Typically $L \approx$ centimeters and $B \approx$ kGauss.

[1] Note that, the smaller L, the wider the low frequency approximation range. For instance, one gets, $\omega \ll 10^{16}/L$ (rad/s) ($\lambda \gg 0.2\,10^{-6}L$ (m)) at 2.5 GeV, $\omega \ll 4\,10^{18}/L$ (rad/s) ($\lambda \gg 5\,10^{-10}L$ (m)) at 45 GeV.

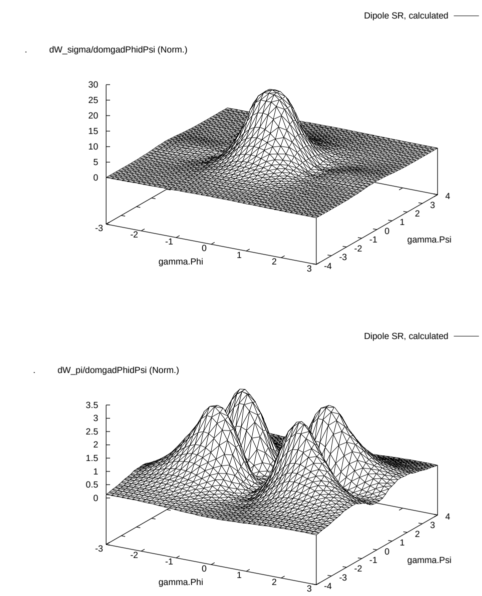

Figure 3: Spectral angular energy density from a $(2/\gamma)$-deviation dipole (after Eq. 6). *Upper plot* : σ component ; *lower plot* : π component.

Radiation properties

By substitution into Eq. 1, and given the total deviation $\alpha = 2/\gamma$ the low frequency Fourier transforms $\tilde{\vec{E}}(\omega) \approx \frac{1}{\sqrt{2\pi}}\int \vec{E}(t')\frac{dt}{dt'}dt'$ of Eqs. 4 provide the spectral angular energy density in the low frequency approximation ($\omega \ll \omega_c/2$)

$$\begin{aligned} \frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega} &= \frac{q^2\gamma^2}{\pi^3\epsilon_0 c}\left(\frac{2-\gamma^2\phi^2+\gamma^2\psi^2}{4(1+\gamma^2\psi^2)+(\gamma^2\phi^2+\gamma^2\psi^2)^2}\right)^2 \quad (6)\\ \frac{\partial^2 W_\pi}{\partial\omega\partial\Omega} &= \frac{q^2\gamma^2}{\pi^3\epsilon_0 c}\left(\frac{2\,\gamma\phi\,\gamma\psi}{4(1+\gamma^2\psi^2)+(\gamma^2\phi^2+\gamma^2\psi^2)^2}\right)^2 \end{aligned}$$

As to frequency validity domain one for instance gets, with $L = 0.1$ m (hence $B = 1/300L = 333$ Gauss) and with 2.5 GeV electrons, $\omega \ll \omega_c/2 = 3\gamma^2 c/2L \approx 10^{17}\mathrm{rad.s}^{-1}$ ($\lambda \gg 0.02\,\mu$m) ; more detailed study [2] shows that Eqs 6 are excellent model up to $\omega \approx \omega_c/10$.

The spectral energy density is obtained by integration and amounts to $\frac{dW_\sigma}{d\omega} = \frac{q^2}{2\sqrt{2}\pi^2\epsilon_0 c}\mathrm{Asinh}(1)$ ($3.053\,10^{-37}$ $(J.s)$ for q = elementary charge) and $\frac{dW_\pi}{d\omega} = \frac{q^2}{2\pi^2\epsilon_0 c}\left(\sqrt{2}\,\mathrm{Asinh}(1) - 1\right)$ ($1.207\,10^{-37}$ $(J.s)$ for q = elementary charge), leading to the partition $\frac{dW_\sigma/d\omega}{dW_\pi/d\omega} = 2.53$. The rms openings and π-peak directions can be obtained from Eqs 6) [2], giving (Fig. 3) $\phi_{\sigma-rms} = \psi_{\sigma-rms} = \phi_{\pi-rms} = \psi_{\pi-rms} = \sqrt{2}/\gamma$ and $\phi_{\pi-peak} = \pm 2/\gamma\sqrt{3}$, $\psi_{\pi-peak} = \pm\sqrt{2}/\gamma\sqrt{3}$.

These properties stay unchanged at constant γ, i.e., at constant deviation $2/\gamma$ or equivalently at constant field integral BL (Fig. 4). The spectral angular energy density in the low frequency range decreases when the particle deviation departs from $2/\gamma$, and its ϕ and ψ projections as well,

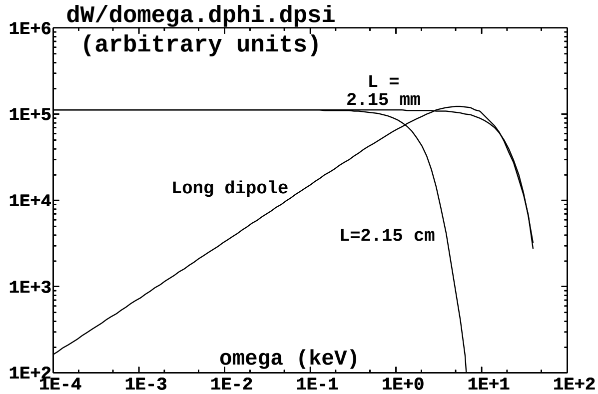

Figure 4: Energy density in the forward direction ($\phi = \psi = 0$), as obtained from numerical calculation of Eqs. 1,2 (constant over the frequency range as stated in Eq. 3). The graph compares the $K^2_{2/3}$-like spectrum from a 2.5 GeV electron in field B=1.55 T ($\rho = 5.4$ m), and the σ-spectrum from a $(2/\gamma)$-dipole, in two cases : the dipole is either a $(2\rho/\gamma)$-long piece of the former, with length $L = 1/300B = 2.15$ mm (Eq. 5) or, the dipole is ten times longer with ten times weaker field.

whether α decreases (thus causing the positive arch of the electric field impulse received by the observer to be truncated within $\pm\tau_c$) or whether α increases (causing part of the negative tails to be introduced in the electric field impulse as observed during a time $t > 2\tau_c$, which is detrimental to $\int E_\sigma dt$ and hence to low frequency radiation) ; the latter effect is accompanied by an increase of the radiation opening with an overall effect of about constant integrated power per unit of magnet length.

Comparison with classical SR model

The properties of low frequency SR in a finite extent magnetic field as described above sensibly depart from what the classical theory of low frequency SR from an infinitely long dipole provides. As to the σ component in the forward direction one has (the π component is zero)

$$\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega} = \frac{3q^2\gamma^2}{8\pi^3\epsilon_0 c}\frac{\Gamma(\frac{2}{3})^2}{2^{\frac{1}{3}}}\left(\frac{\omega}{\omega_c}\right)^{\frac{2}{3}} \qquad (7)$$

whereas for the $(2/\gamma)$-dipole (Eq. 6 with $\phi = \psi = 0$)

$$\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega} = \frac{q^2\gamma^2}{4\pi^3\epsilon_0 c} \qquad (8)$$

which leads to the ratio

$$\left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{Classical} : \left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{\left(\frac{2}{\gamma}\right)-dipole} = \frac{3\Gamma(\frac{2}{3})^2}{2^{\frac{4}{3}}}\left(\frac{\omega}{\omega_c}\right)^{\frac{2}{3}} : 1 \qquad (9)$$

Taking $\omega = 1.88\,10^{15}\,\mathrm{rad{\cdot}s^{-1}}$ ($\lambda = 1\,\mu$m) and $\rho = 5\,$m in Eq. 7, one gets $\omega/\omega_c \approx 1.8\,10^{-4}$ for E = 2.5 GeV, and the ensuing ratio, highly in favour of the $(2/\gamma)$-dipole, $\left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{Classical} : \left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{\left(\frac{2}{\gamma}\right)-dipole} \approx 1 : 140$

As to the openings, the $(2/\gamma)$-dipole SR is confined in a cone with $\sqrt{2}/\gamma$ rms aperture independent of ω, while that of the long dipole radiation aperture is much wider, namely [1], $\phi_R \approx \alpha/2 \approx$ mrad in the bend plane and $\psi_R \approx 10/\gamma$ (for $\omega \approx 2\,10^{15}\,\mathrm{rad{\cdot}s^{-1}}$) proportional to $\omega^{1/3}$ in the vertical plane.

Fig. 4 more generally compares the spectral angular energy density in the forward direction ($\phi = \psi = 0$), over the effective spectral extent (obtained by numerical computation). It can be checked that the $(2/\gamma)$-dipole energy density in the low frequency range does not depend on its length contrary to what occurs at high frequency (the shorter the $(2/\gamma)$-dipole, the wider the spectrum).

Comparison with edge SR

The gain in the σ-component energy density from a $(2/\gamma)$-dipole w.r.t edge radiation can be estimated from the shape of the observed electric field impulse, as follows. In the case of the $(2/\gamma)$-dipole the full positive arch of the σ impulse is received by the observer. In the case of the peak edge radiation (i.e., in the direction $1/\gamma$ from the magnet end [2]) the observer receives in addition a part of the negative tail of that impulse. Following the low frequency Fourier transform approximation the energy density ratio [Edge:$(2/\gamma)$-dipole] is the square of the ratio of the electric field integrals, that is to say, 1:4 if the edge radiation is issued from an infinitely long dipole, a bit more if this last is not very long (since in such case, the negative tail of the edge electric field impulse is truncated, and hence the area under $E_\sigma(t)$ is a little more than half the area under the sole positive arch).

Analytical calculations provide [2]

$$\left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{Edge} : \left.\frac{\partial^2 W_\sigma}{\partial\omega\partial\Omega}\right|_{(\frac{2}{\gamma})-dipole} = \left(1 + \frac{2}{\gamma\alpha}\right)^2 : 4 \qquad (10)$$

which tends to $1 : 4$ if $\gamma\alpha \gg 1$.

4 CONCLUSION

It is shown that a $(2/\gamma)$-deviation device causes the low frequency SR spectral angular energy density to be maximised, confined within a $\sqrt{2}/\gamma$ rms aperture cone, entailing a gain of up to four w.r.t. edge SR and up to several orders of magnitude w.r.t. body SR from a long dipole, and making it a highly recommendable candidate as a versatile, long-wavelength radiation source.

5 REFERENCES

[1] A. Hofmann, Theory of synchrotron radiation, SSRL ACD-Note 38, Sept. 1986.

[2] F. Méot, A theory of low frequency far-field synchrotron radiation, report DSM/DAPNIA/SEA-98-05, CEA/Saclay, 1998.