BEAM EMITTANCE IN THE ELECTRON STORAGE RING WITH A STRONG BETATRON AND SYNCHROTRON OSCILLATIONS COUPLING

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Abstract

The report describes the calculational technique of beam emittance in the electron storage ring with due account of betatron and synchrotron oscillations. The formulas for estimating the electron distribution function in the 4-D phase space are derived. The beam emittance is calculated for the case when frequencies of betatron and synchrotron oscillations are of the same order. It is shown that the emittance calculated by the present formulas differs from that given by Sands.

1 INTRODUCTION

The tendency of evolution in electron storage rings to RF systems with a higher frequency and amplitude of accelerating voltage is now observed. As a result, the synchrotron oscillation frequency increases. Thus, there is a problem on the boundaries of applicability of approximations, which are supposed for calculation of equilibrium sizes of an electron beam in storage rings. A main approximation, which is used in theory, is the submission of movement in a horizontal plane as a superposition fast (betatron) of oscillations which were not connected to a modification of energy of particles, and slow (synchrotron) of oscillations. This approximation is based on the supposition, that the frequency of synchrotron oscillations has significantly less frequency of betatron oscillations. It is possible to assume, that for higher frequency of synchrotron oscillations this approximation can be of no use.

2 MATHEMATICAL ALGORITHM

The present work is devoted to development of mathematical algorithm for calculation of stored beam sizes in a horizontal plane taking into account of betatron and synchrotron of oscillations coupling.

The equation of electron motion in the storage ring can be written as [1]

$$\frac{dx_l}{d\theta} - \sum_m^4 A_{lm} x_m = Y_l \ , \qquad (1)$$

here $x_1 = \dfrac{dx_2}{d\theta}$;

x_2 - deviation of an electron from equilibrium orbit in a horizontal plane;

x_3- relative deviation of energy from equilibrium values;

x_4 - deviation from equilibrium value of a phase,

θ - azimuth coordinate.

The matrix **A** describes an operation of electromagnetic fields of the storage ring in a linear approximation taking into account the synchrotron radiation. The matrix ***A*** is periodic, $\mathbf{A}(\theta+2\pi) = \mathbf{A}(\theta)$. Y_l - - component of a force vector describing an operation of radiation quantum fluctuations. Let's take into account in (1) only the most essential component of fluctuations, namely fluctuation of energy Y_3, by putting $Y1=Y2=Y4=0$.

Using the results of work [2], it is possible to write a stationary distribution function of particles in space of components of the vector $\boldsymbol{x}(x_1,x_2,x_3,x_4)$.

$$D(x) = [(2\pi)^4 Det\|G_{lm}(\theta)\|]^{-1/2} \exp\left\{-\frac{1}{2}\sum_{l,m}^4 G_{lm}^{-1} x_l x_m\right\},$$

(2)

here G_{lm}^{-1} - inverse matrix of G_{lm}. The matrix G_{lm} are periodic solutions of the matrix equation:

$$\frac{dG_{lm}}{d\theta} = \left[\mathbf{A}(\theta), \mathbf{G}(\theta,\theta) + \mathbf{G}(\theta,\theta)\mathbf{A}^{+}(\theta)\right]_{lm} + \delta_{lm}\sigma_m^2(\theta) \ ,(3)$$

The sign "+" - means a transposition, δ_{lm} - Kronecker symbol, $\sigma_1=\sigma_2=\sigma_4=0$. The value σ_3 - intensity of a noise is calculated in a number of articles [1, 3, 4]; $\sigma_3(\theta+2\pi)=\sigma(\theta)$.

The performances of a stationary cumulative distribution function, moment of the 2-nd order $\langle x_l x_m \rangle$, are determined by the formulas [2]

$$\langle x_l x_m \rangle = \lim_{\theta\to\infty} G_{lm} \qquad (4)$$

The existence of limits in (4) assumes availability of an asymptotic stability of solutions of the equation (1) for $Y_l=0$.

The matrix of solutions of the equation (3) is symmetrical $G_{lm} = G_{ml}$. From 16 equations (3) for G_{lm} 10 equations are independent for G_{11} , G_{22}, G_{33}, G_{44}, G_{12}, G_{13}, G_{14}, G_{11} ,, G_{24}, G_{34}. For convenience we shall introduce labels: $G_{11} = y_1$, $G_{22} = y_2$, $G_{33} = y_3$, $G_{44} = y_4$, $G_{12} = y_5$, $G_{13} = y_6$, $G_{14}= y_7$, $G_{23} = y_8$, $G_{24} = y_9$, $G_{34} = y10$.

$$\frac{dy_n}{d\theta} = \sum_{j=1}^{10} B_{nj} y_j + \sigma_n^2 \delta_{nj} \qquad (5)$$

According to the restrictions introduced for components $\boldsymbol{Y_l}$ and introduced new labels, distinct from zero will be only magnitude σ_3.

The solution of the equation (5) can be presented as

$$y_1 = c_1 z_{11} + c_2 z_{12} + ... + c_{10} z_{1,10} + \sum_{k=1}^{10} z_{1k} \int_0^\theta \varphi_k(\tau) d\tau$$

$$y_2 = c_1 z_{12} + c_2 z_{22} + ... + c_{10} z_{2,10} + \sum_{k=1}^{10} z_{2k} \int_0^\theta \varphi_k(\tau) d\tau$$

....

$$y_{10} = c_1 z_{10,1} + c_2 z_{10,2} + ... + c_{10} z_{10,10} + \sum_{k=1}^{10} z_{10,k} \int_0^\theta \varphi_k(\tau) d\tau, \tag{6}$$

here the functions Z_{ik} are elements of matriciant $\mathbf{Z}$ equation, composed from solutions, (5) for $\sigma_n = 0(n = 1,2,...,10)$, and $\varphi_k = \sigma_3(\theta) {D_{3k}}/{Det\|\mathbf{Z}\|}$, $\sigma_3(\theta) \propto \kappa^2(\theta)$, κ - curvature of equilibrium orbit. D_{3k} - cofactor of k - element in 3 line of matriciant $\mathbf{Z}$. Matriciant of solutions $\mathbf{Z}$ for the equation (5) it is possible to calculate as follows. According to [5] general solutions of the equation (5) for σ_m=0 looks like

$$\mathbf{Z} = \mathbf{MCM}^+, \tag{7}$$

here $\mathbf{M}$ - matriciant of solutions of the equation (1) for Y_l = 0, $\mathbf{C}$ - symmetrical constant matrix $C_{lm} = C_{ml}$. From (7) it is possible to choose necessary elements and to make matriciant $\mathbf{Z}$ for the equations (5). By virtue of an asymptotic stability of solutions of the equation (1) for Y_l = 0

$$\langle x_l x_m \rangle = \lim_{\theta\to\infty} y_i = \lim_{\theta\to\infty} \sum_{k=1}^{10} z_{ik} \int_0^\theta \varphi_k(\tau) d\tau \tag{8}$$

For determination of limits in (8) it is necessary to calculate elements of matriciant $\mathbf{M}$ using the entry of solutions in the Floquet-form. It will allow to integrate in (8) with the help of expansions of periodic Floquet-functions, included in Z_{ik} and φ_k in Fourier series.

Let's formulate main stages of calculations:

1. With the help of programs "DECA" [6] are calculated matriciant $\mathbf{M}$ and Floquet-parameters ($\mu_{1,2} = I_1 \pm i\omega_1, \mu_{3,4} = I_2 \pm i\omega_2$).

2. There are sixteen Floquet-functions must be found through elements of matriciant $\mathbf{M}$. The inverse problem is decided the matriciant elements are expressed using Floquet-functions.

3. Elements of matriciant $\mathbf{Z}$ are calculated with the help of expressions (7) the cofactor of k-th element from the third line of a matriciant $\mathbf{Z}$ are determined.

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5. The periodic functions which are included in Z_{ik} and φ_k are decomposed in Fourier series.

6. The integration $\int_0^\theta \varphi_k$ is made.

7. The calculation of the sums $\sum_{k=1}^{10} z_{ik} \int_0^\theta \varphi_k(\tau) d\tau$ and passage to the limit for $\theta \to \infty$ are made.

3 CONCLUSION

The mathematical algorithm for calculation of stored beam sizes in a horizontal plane is developed taking into account of betatron and synchrotron of oscillations coupling. It will be used for the modelling of beam dynamics in new storage rings which have large synchrotron oscillation frequency with DeCA code.

4 REFERENCES

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