DEVELOPMENT OF THE NONSTATIONARY MODEL FOR BEAM DYNAMIC SIMULATION IN MULTISECTIONAL ACCELERATORS

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Abstract

For simulation of transient beam dynamics in the accelerating systems we developed a code that is based on simultaneous solution of a wave equation and particle motion equations. If the processes under consideration are slow, there is a great simplification in the procedure of calculating of current integral which describes the current loading. For applying this approach to multisectional accelerators we developed a procedure of calculating of such current integrals for each accelerating section with consecutive injection of particles from the previous section into the following one.

1 INTRODUCTION

The acceleration of intense bunches in slow-wave structures has some features that have to be take into account under studying the dynamics of electron beams. One of the main effects is a beam loading. It includes changing of the characteristics of the accelerating wave under presence of bunched electron beam. Another problem is an excitation of higher order passbands[1], in particular one that lead to BBU (see, for example, [1-3]). Beam loading effects, especially transient, play an important role in the high current accelerators, so many efforts were and are made for developing the methods that could describe these effects properly. The most frequently used method is based on the equivalent circuit approach. This method gives possibility to obtain many useful transient characteristics, but in many cases the results are very approximate. The more general method is based on the solving (numerically or analytically) a nonhomogeneous wave equation. There are many approaches that can be used for simplifying the procedure of obtaining the transient characteristics. One of them can be used in that case when the transient process may be considered as slow one as compare with the fast oscillation of electromagnetic field [1,2,4,5]. In this paper we present our results of developing a mathematical model that was used for simulation of transient processes in multisectional accelerator.

2 ORIGINAL EQUATIONS

Let's consider a mathematical model that can be used for describing of the transient beam loading effects. Electromagnetic field which is excited in slow-wave structure by electron beam and external RF-source can be founded from the general theory of waveguide excitation [6]:

$$\vec{E} = \int d\omega \exp(-i\omega t)\,\{\sum_{s}[C_s(\omega,z)\,\vec{E}_s(\omega,\vec{r}) + C_{-s}(\omega,z)\,\vec{E}_{-s}(\omega,\vec{r})] + \frac{4\pi}{i\omega}\vec{j}^{\,l}_{\omega}\} + k.c., \tag{1}$$

$$\frac{dC_{\pm s}}{dz} \mp ih_s(\omega)\,C_{\pm s} = \pm\frac{1}{N_s}\int_{S_t}\vec{j}_{\omega}\,\vec{E}_{\mp s}(\omega,\vec{r})\,d\vec{r}_{\perp}, \tag{2}$$

where $\vec{E}_s(\omega,\vec{r}) = \sum_k \vec{E}_{s,k}(\omega,\vec{r}_{\perp})\exp(i\,2\pi\,k\,z/D)$ - eigen solenoidal functions of waveguide, ($\vec{E}_{-s} = \vec{E}^*_s$), $h_s(\omega)$ - a wave number, $\vec{j}_{\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\vec{j}(t)\exp(i\omega t)\,dt$. If the RF-signal of external source and current have a narrow frequency spectrum, it is natural to suppose that the Fourier transform of the electromagnetic field in the structure will be nonzero near frequencies that are multiple the working frequency ω_0 ($\omega = n\,\omega_0$) and can be represented in the form:

$$F = \sum_{n=1}^{\infty} F_n(t)\exp(-in\,\omega_0 t) + k.c, \tag{3}$$

where $F_n(t) = \int_{n\omega_0-\Delta/2}^{n\omega_0+\Delta/2} d\omega\,F_{\omega}\exp[i(n\omega_0-\omega t)]$ - functions that have a slow variation, $\Delta << \omega_0$ - frequency interval in which F_{ω} is nonzero. If we take into account only zero expansion in a small parameter Δ/ω_0, except of the wave number $(h_s(\omega) = h_s(n\omega_0) + \Delta\omega dh_s/d\omega)$, and neglect a quasistatic fields, we obtain:

$$\vec{E} = \sum_s\sum_n \exp(-in\omega_0 t)\,\{C_{s,n}(t,z)\exp[ih_s(n\omega_0)z]\,\vec{E}_s(n\omega_0,\vec{r}_{\perp}) + C_{-s,n}(t,z)\exp[-ih_s(n\omega_0)z_0]\,\vec{E}^*_s(n\omega_0,\vec{r}_{\perp})\} + c.c., \tag{4}$$

$$\frac{\partial C_{\pm s,n}}{\partial z} \pm \frac{1}{v_{g,s,n}}\frac{\partial C_{\pm s,n}}{\partial t} = \pm\frac{\exp[\pm ih_s(n\omega_0)]}{N_s(n\omega_0)}\int_{S_t}\vec{j}_n\,\vec{E}_{\mp s}(n\omega_0,\vec{r}_{\perp})\,d\vec{r}_{\perp}, \tag{5}$$

where $v_{g,s,n} = 1/(dh_s/d\omega)_{\omega=n\omega_o}$ - a group velocity of the s-th eigen wave. Summation in (4) is taken on over all types of waves that propagate at frequencies $\omega = n\omega_0$. Field attenuation can be taken into account by adding terms $\pm\alpha_{s,n}C_{\pm s,n}$ to the left-hand side of Eq.(5).

For obtaining the system of equations that describe an acceleration process with a current influence we ought to

[1] At present, these effects are often called as Wake Field Effects

connect $\vec{j}_n$ and $C_{\pm s,n}$. In general case this connection can be found from kinetic equation

$$\frac{\partial f}{\partial t}+\vec{v}\,\frac{\partial f}{\partial \vec{r}}+e\left(\vec{E}+\frac{1}{c}[\vec{v}\,\vec{H}]\right)\frac{\partial f}{\partial \vec{v}}=0\,, \quad (6)$$

Solving Eq.(6) by the characteristic method, we find the expression for a current:

$$\vec{j}=\int_{-\infty}^{t}dt_0\int d\vec{r}_{\perp 0}\int d\vec{p}_0\, f_0(\vec{r}_{\perp 0},t_0,\vec{p}_0)\,\vec{v}_l\,v_{z0}\,/\,v_{zl}\times$$
$$\times\delta[t-t_l(z_0,\vec{r}_{\perp 0},\vec{p}_0)]\,\delta[\vec{r}_\perp-\vec{r}_{\perp l}]\,\delta[\vec{p}-\vec{p}_l]\,, \quad (7)$$

where $t_l,\vec{r}_{\perp l},\vec{p}_l$ are the solutions of the motion equations.

Determination of a slow-varying amplitude $\vec{j}_n$ by applying Fourier transformation to Eq.(7) is a difficult procedure, so it is often used some averaging operators that give possibility to find this value with definite accuracy. One of them is:

$$\overline{M}_n\vec{j}=\frac{1}{T_0}\int_{t-T_0/2}^{t+T_0/2}dt'\exp(in\omega_0 t')\,\vec{j}(t') \quad ,T_0=2\pi/\omega_0\,. \quad (8)$$

Substituting into (8) the Fourier expansion of the current, we obtain

$$\overline{M}_n\vec{j}=\int_{-\infty}^{\infty}d\omega\frac{\sin[\pi(n\omega_0-\omega)/\omega_0]}{\pi(n\omega_0-\omega)/\omega_0}\,\vec{j}_\omega\exp[i(n\omega_0-\omega)\,t]=$$
$$=\vec{j}_n+\sum_{k\neq n}\frac{(-1)^{n-k}}{(n-k)\,i\omega_0}\frac{d\,\vec{j}_k}{d\,t}\exp[i\,(n-k)\omega_0\,t]+\ldots \quad (9)$$

It is follows from (9) that in the case of slow processes ($d\,j/dt \ll \omega_0\,j$) applying of the averaging operator to function gives the value of its slow-varying amplitude. Taking this into account, we can write the right side of Eq.(5) in the such form:

$$\int_{S_t} d\,\vec{r}_\perp\,\vec{j}_n\,\vec{E}_{\mp s}=$$
$$=\int_{S_t} d\,\vec{r}_\perp\vec{E}_{\mp s}\int_{t-T_0/2}^{t+T_0/2}d\,t'\,\frac{\exp(i\,n\,\omega_0 t')}{T_0}\,\vec{j}(t',\vec{r})= \quad (10)$$
$$=\int_{G(t,z)} d\,t_0\int d\,\vec{r}_{\perp 0}\int d\,\vec{p}_0\;f(t_0,\vec{r}_{\perp 0},\vec{p}_0)\exp(in\omega_0 t_l)\times$$
$$\times\vec{E}_{\mp s}(n\omega_0,\vec{r}_{\perp l})\,\vec{v}_l v_{z0}\,/\,v_{zl}\;\;,$$

where the integration region $G(t,z)$ over the fly-in time determines from the condition

$$t-T_0/2<t_l(z,t_0,\vec{r}_{\perp l},\vec{p}_0)<t+T_0/2\;. \quad (11)$$

Eq.(5) with (10) and with the motion equations represent the system of equations that describe nonstationary dynamics of electron beam acceleration in slow-wave structure. But this system is still difficult for simulation, so we have made some additional assumptions. The the most important one is such - amplitude of accelerating wave has a small variation during the particle fight time through the interaction region.

This assumption gives possibility to simplify the procedure of calculating the integral (10), but it lay out restrictions on the time characteristics of the processes under consideration - they must be slow not only in comparison with the oscillations on the working frequency, but also with the relaxation time that is determined by the particle flight time through the interaction region. Detail analyze shows that this assumption restrict the frequency width of signals - it must be less than 50 MHz. Similar restrictions (even more rigid owing to the presence of couplers) flow out from the dispersive properties of slow-wave structures because their first passband has width of the order 100-200 MHz.

Simplification of the procedure of calculating of the integral in the right-hand part of Eq.(5) connects with circumstance that the integration region *G(t,z)*, which is determined by Eq. (11), can be found in an explicit form. Indeed, under these conditions the particle fly-in time $t_l(t_0,z)$ into the point with coordinate *z* is approximately periodic function t_0, so many-connected region $G(t,z)$ can be replaced on the single-connected one:

$$t-T_0/2<t_0<t+T_0/2 \quad (12)$$

Under this, calculations of the motion equations of particles which flied into structure at moments that are determined by Eq.(12) can be conducted with a field amplitude taken at time *t*.

In multisectional accelerators we have several structures. Each section has its own RF-source. All sources are synchronized, but there may be staggering the timing of the klystron pulses with respect to the beginning of the current pulse. The described above method, in principle, can be applied for each accelerating section. For doing that we have to know the function $f_0(t_0,\vec{r}_{\perp 0},\vec{p}_0)$ at the entrance of the considered section, that is, we must know a solution of kinetic equation (6) for previous sections. This demand returns us in the initial state and this approach can not simplify the task.

For using the described above method in the case of multisectional accelerators, we note that in general case there is no necessity to understand under $f_0(t_0,\vec{r}_{\perp 0},\vec{p}_0)$ in Eq.(10) (and in all subsequent formulas) the distribution function at the entrance of considered structure. We can consider it as the initial distribution function in some initial cross-section (*z*= 0). In this case, the problem of correct description re-lay on the determination of the integration regions over the fly-in time in this cross-section $G_i(t,z)$ (*i* - section number) in current integrals of each section. In general case it do not simplify the task, but in the case of linacs such approach may be useful. Indeed, in multisectional linacs the main role plays particles that are “trapped” by accelerating wave and the beam consists of a series of short bunches, except the initial stage of acceleration. Each bunch moves in such way that it remains be “trapped” and be in synchronism with the accelerating wave despite a small particle movement inside the bunch. It is signified that the phase shift relative the accelerating wave do not exceed π. So, if we know the phase velocity of the accelerating structures, we can determine the fly-out time of each bunch from the *i*-th section with the accuracy of one period of RF-oscillation and, consequently, the fly-in time in the (*i*+1)-th section. Then the integration region for the *i*-th section under our assumptions will be:

$$t - T_0/2 - T_{d,i}^{\Sigma} < t_0 < t + T_0/2 - T_{d,i}^{\Sigma}$$

where the mean flight time $T_{d,i}^{\Sigma} = \sum_{s=1}^{i-1} T_{d,s}$ is defined by the phase velocity of the accelerating wave. As the value of the mean flight time $T_{d,i} = L_i/(\beta_{ph}^{i} c)$ of the *i*-th section is constant, we can make the time shift on the value of $T_{d,i}^{\Sigma}$, so that the integration region in each section will be defined by Eq.(12).

The described above method can be also used in the case of quasi-constant impedance or even constant amplitude sections.

3 SIMULATIONS

On the basis of this method we developed numerical code for simulation the transient beam loading processes in electron linacs. This code permit to simulate the transient beam dynamics consecutively in each section that is very suitable under study of the influence of a staggering the timing of klystron pulses on the energy spread.

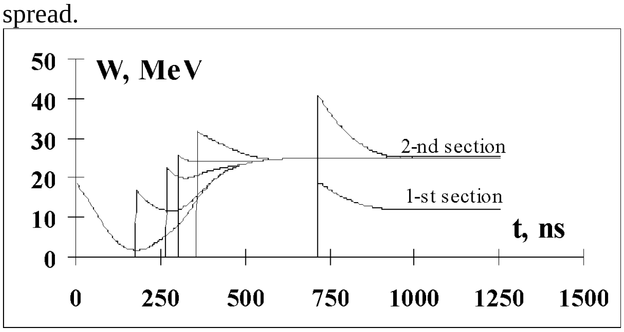

Fig.1. Mean bunch energy vs. time

Using this program we have made a simulation of the beam dynamics in the initial part of the Kharkov 2-GeV accelerator in the high current regime.

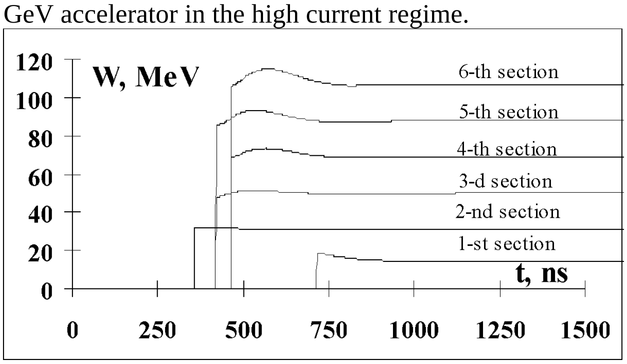

Fig.2. Mean bunch energy vs. time

The studied scheme included a prebuncher, an injection section (*L*=3 m) with variable phase velocity and a train of constant impedance acceleration sections with $\beta_{ph} = 1$ and *L*=4.4 m. The dependencies of the mean bunch energy (we inject 32 “particles” in each RF-period) on the time for the various moments of beginning the injection of current pulse into the second section are shown on Fig.1 (injection current I_{in}=0.8 A, current at the exit of the first section I_{out}=0.65 A). Under simulation we took into account the variations of amplitudes and phases of RF-signals at the section entrances. The dependencies of the mean bunch energy on the time for the first six sections under optimum time shifts between RF and beam pulses are shown on Fig.2 (injection current I_{in}=0.5 A, current at the exit of the first section I_{out}=0.4 A).

4 CONCLUSIONS

Developed code is happened to be useful for working out different problems. We are planning to include in it the radial movement of particles and Coulomb interactions for more strict description of the initial part of the acceleration process.

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