Transverse RF Focussing in Jefferson Lab Superconducting Cavities

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Abstract

We have investigated the RF transverse focussing effect in a five-cell CEBAF-type superconducting accelerating cavity on the electron beam produced by the photoemission gun of the Jefferson Lab free-electron laser. We compared different analytical models with numerical simulations using the PARMELA "particle pushing" code that incorporates a MAFIA model of the CEBAF-type cavities. Some preliminary measurement performed in the Jefferson Lab free-electron laser are also presented.

1 INTRODUCTION

With the need of increasing the brightness of electron beams to drive sources such as free-electron lasers or synchrotron sources, achieving a very short bunch length is of primary concern. For such purposes one commonly uses magnetic compression by means of wigglers or chicanes. Also it is common, especially in beam formation processes, to provide bunching using the accelerating sections. For such purposes, the bunches are injected in the accelerating section with a significant offset phase with respect to the maximum accelerating electric field. One generally changes the slope of the longitudinal phase space and provides compression at the expense of energy spread. A potential inconvenience from this technique arises from the radial electric field $\mathbf{E}_r$ [1] which is generated by the accelerating field $\mathbf{E}_z$ to fulfill the Maxwell-Ampère relation div$\mathbf{E} = 0$. Because of this radial field, one can conceive that it might affect the transverse beam dynamics. In this paper, we compare analytical models with results from numerical simulations for the case of a high-gradient CEBAF-type 5-cells superconducting cavity. It is found that the cavity couplers have considerable effects on the transverse dynamics that cannot be associated for any analytical model based on cylindrically symmetric cavities. In a last section we present preliminary measurements being performed at the Jefferson Lab FEL injector [2].

2 ANALYTICAL MODELS

2.1 The RF transverse equation of motion

The accelerating RF field in a cylindrically symmetric RF structure $\mathbf{E}_z$ induces a radial field which is given by $\mathbf{E}_r(r,z) = -\frac{r}{2}\frac{d}{dz}\mathbf{E}_z$. From the equation of motion $\frac{dp}{dt} = e\mathbf{E}_r$, noting that $\frac{dp}{dt} = mc\frac{d}{dt}(\gamma r') = mc^2(\gamma' r' + \gamma r'')$ (where we have defined $r' \doteq dr/dz$ and assumed the electrons are relativistic i.e. $\beta \simeq 1$) and that $e\mathbf{E}_r = -mc^2\frac{r(z)}{2}\gamma''$, one can derive a transverse equation of motion in an RF structure:

$$r''(z) + \frac{\gamma'(z)}{\gamma(z)}r'(z) + \frac{\gamma''(z)}{2\gamma(z)}r(z) = 0 \tag{1}$$

where $\gamma'(z)$ is the normalized energy gradient $\gamma'(z) = \frac{eE_z(z)}{mc^2}\cos(\Delta\phi)$, $\Delta\phi$ is the phase of injection with respect to the maximum energy gain phase, and mc^2 is the rest mass of an electron. The previous second order equation can be numerically integrated, and the solution associated with the initial conditions ($x = 1$, $x' = 0$) and ($x = 0$, $x' = 1$) will respectively give (m_{12}, m_{22}) and (m_{11}, m_{21}), the elements of the RF structure transverse transfer matrix. An approximate analytic solution of Eq.(1) has been derived by Chambers [3].

2.2 Rosenzweig-Serafini & Chambers Models

The Rosenzweig-Serafini generalized model directly gives the transfer matrix of an RF structure of arbitrary mode including harmonic content of the RF field [4]. If one assumes the CEBAF-type cavity to be a pure π-mode cavity, this model simplifies to the Chambers model, and the transfer matrix reduces to (see Eq.(13), p 1601 of Ref. [4] with $\eta(\phi) = 1$):

$$\begin{cases} m_{11} = \cos(\alpha) - \sqrt{2}\cos(\Delta\phi)\sin(\alpha) \\ m_{12} = \sqrt{8}\frac{\gamma_i}{\gamma'}\cos(\Delta\phi)\sin(\alpha) \\ m_{21} = -\frac{\gamma'}{\gamma_i}\left(\frac{\cos(\Delta\phi)}{\sqrt{2}} + \frac{1}{\sqrt{8}\cos(\Delta\phi)}\right)\sin(\alpha) \\ m_{22} = \frac{\gamma_i}{\gamma_f}\left(\cos(\alpha) + \sqrt{2}\cos(\Delta\phi)\sin\alpha\right) \end{cases} \tag{2}$$

where $\gamma_{i,f}$ are the initial and final reduced Lorentz factors, the angle α is defined as $\alpha = \frac{1}{\sqrt{8}\cos(\Delta\phi)}\ln(\gamma_f/\gamma_i)$ and $\Delta\phi$ denotes the phase of the injection of the particle with respect to the on-crest phase. γ' is the averaged (over the RF structure) energy gradient: $\gamma' = \frac{eV_{RF}}{mc^2}\cos(\Delta\phi)$.

2.3 Krafft Model

This model [5], which has been derived under the approximation of relativistic beam ($\beta \simeq 1$), gives the transverse transfer matrix of an RF structure to be:

$$\begin{cases} m_{11} = 1 - \frac{\cos(\Delta\phi)\int_{-\infty}^{+\infty}E_z(z)\cos(\omega z/c)dz}{2\gamma_i mc^2} \\ m_{12} = L/\gamma_i \\ m_{21} = -\Big(\cos^2(\Delta\phi)\int_{-\infty}^{+\infty}E_z^2(z)\cos^2(\omega z/c)dz \\ \quad +\sin^2(\Delta\phi)\int_{-\infty}^{+\infty}E_z^2(z)\sin^2(\omega z/c)dz\Big)\frac{1}{4\gamma_i} \\ m_{22} = 1 + \frac{\cos(\Delta\phi)\int_{-\infty}^{+\infty}E_z(z)\cos(\omega z/c)dz}{2\gamma_i mc^2} \end{cases} \tag{3}$$

where γ_i is the reduced energy of the particle at injection. The $E_z(z)$ quantity represents the energy gradient at the location z inside the cavity.

2.4 Comparison

We present in Figure 1 the matrix elements computed with the three different models described in the previous section for the case of CEBAF-type cavity. The injection energy was choosen to be 5 MeV and the cavity gradient was set to 11 MV/m; both of these parameters are close to the experimental conditions of the measurement reported later in this paper. The Chambers model gives the same results as the numerical integration of the equation of motion (the results are indistinguishable on the figure except for the m_{21}). The Krafft model approximately gives comparable results except for the m_{12}. This is believed to be due to an approximation made in the model that was generally applied for higher energy beams.

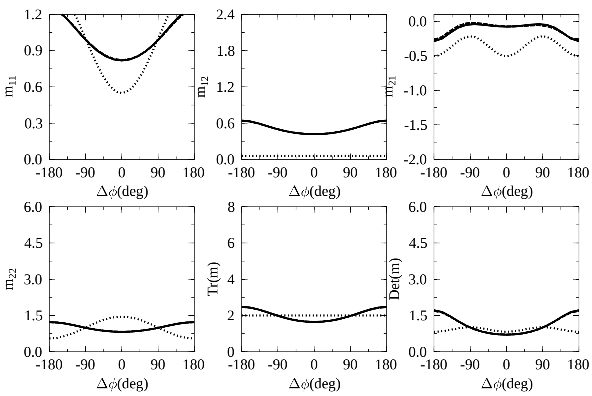

Figure 1: Transfer matrix elements, determinant, and trace for the different models: numerical integration of the equation of motion (solid line), Chambers-Rosenzweig-Serafini model (dashed line) and Krafft model (dotted line).

3 NUMERICAL SIMULATION

The numerical simulations are performed with the "particle pushing" code PARMELA. The electrons are tracked by integrating the equation of motion using the RF field map as external field. In order to perform the numerical simulation we generated a gaussian distribution with various first-order moments ($\langle x\rangle$ and $\langle x'\rangle$ set to 0 or 1) and computed the beam first moments after a CEBAF-type cavity. To estimate what is the effect of the higher-order-mode (HOM) and input couplers, we use two different cavity models: a MAFIA model [7] that incorporates the couplers and a SUPERFISH model that assumes the cavities to be cylindrically symmetric. The computed matrix elements, determinant and trace are gathered in Figure 2. The numerical model based on the MAFIA field map predicts the focal length (defined as $-1/m_{21}$) to be more sensitive with respect to the injection phase than the cylindrically symmetric SUPERFISH model. In particular, the 3D MAFIA model gives a much smaller focal length than the cylindrical model for large off-crest phase. On the other hand the SUPERFISH model is in better agreement with the analytical models presented in Figure 1.

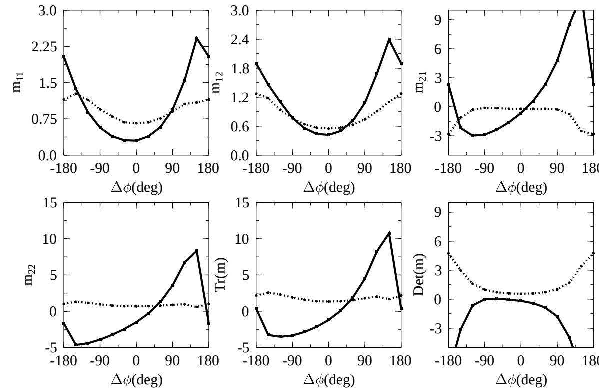

Figure 2: Transfer matrix elements, determinant, and trace computed with the PARMELA code. The solid lines are results using the 3D map field generated by MAFIA while the dotted lines have been obtained using a cynlindrically symmetric SUPERFISH model.

4 EXPERIMENTAL RESULTS

4.1 Experimental Method

The experimental setup to measure the transfer matrix element of a CEBAF-type cavity utilizes a 350 keV electron beam produced out of a photoemisson gun at the Jefferson Lab IRFEL. The emitted beam is imaged by means of two solenoidal lenses at the entrance of a CEBAF-cryounit which contains two superconducting radiofrequency (SRF) accelerating cavities. The first cavity is nominally operated with maximum acceleration (i.e. $\Delta\phi = 0$ deg) whereas the second cavity is operating off-crest ($\Delta\phi = -19.7$ deg) to provide the necessary longitudinal-phase-space slope for matching the momentum compaction of a downstream chicane. The beam energy after the first cavity is approximately 5.3 MeV. The purpose of the experiment is to measure the transfer matrix of the second cavity for different phases $\Delta\phi$. After the second cavity, the beamline contains a quadrupole and a multislit mask used to determine the beam parameters [8].

4.2 Results

We have recently completed a first test to see whether we were able to measure the beam parameters over a large RF phase span. As shown in Figure 3, we were able to scan in phase from -200 to -50 deg (the on-crest phase of the cavity being -120 deg), with the quadrupole turned off. In the future experiment we intend to use the quadrupole to readjust the beam envelope at each point, to be able to measure the beam parameters over the whole 360 deg of RF phase. At present the only limitation appears to be that the

OTR power might be too low for phases close to $\Delta\phi \simeq$-180 deg. At the moment we are unable to measure the beam parameters at the entrance of the cavity, and we know that comparison of the measurement presented in Figure 3 with PARMELA predictions are difficults because the injector is not yet fully optimized. Nevertheless, the preliminary data shown in Figure 3 qualitatively support the RF focussing: the α-Twiss parameter is varying very rapidly versus the injection phase, while the betatron function is slightly increasing with positive off-crest phase, meaning that the focal length is decreasing which is in accordance with simulations. The emittance is approximately constant over a wide range which is in agreement with the constancy of the matrix determinant for phase values within $\pm$90 of the on-crest phase.

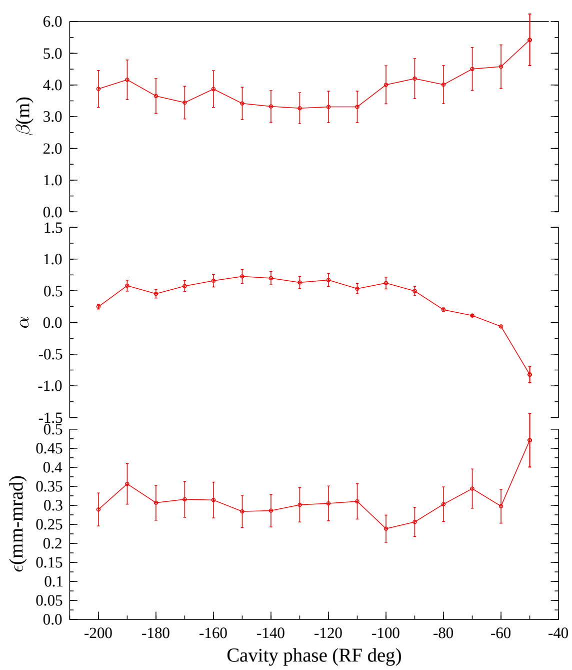

Figure 3: Experimental measurement of the beam parameters at CEBAF cavity exit for different injection phase. The on-crest phase is -120 deg.

5 CONCLUSION

Analytical models based on cylindrical symmetry do not necessarily give results in agreement with more precise calculation including coupler effects. Numerically, these coupler effects lead to non trivial changes in the beam dynamics. A preliminary measurement performed in the Jefferson Lab IRFEL injector demonstrates the major effect of RF focussing on the beam dynamics and the possibility of performing a detailed parametric experiment varying the phase over the whole 360 deg. Such an experiment is planned for next autumn in the Jefferson Lab FEL once we have finished optimizing and understanding the beam dynamics in this machine.

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