# TRACKING STUDIES IN THE LONGITUDINAL PHASE SPACE FOR THE TESLA DAMPING RING DESIGN

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*Abstract*

The TESLA linear collider will require bunch lengths in the range of 0.3 to 0.7 mm at the interaction point (IP) for the different parameter sets presently under discussion.

In order to achieve this the bunches leaving the TESLA damping ring will pass a single stage bunch length compressor. To keep the demands of the latter moderate, the TESLA damping ring needs to fulfill limitations in respect to the longitudinal phase space (in the range of 4 mm to a maximum of 1 cm for the bunch length and 0.1 % for the energy spread). The possibility of bunch lengthening effects and other instabilities due to wake fields need to be and have been studied. The assumed wake fields of various components are presented and incorporated in a tracking method using a quasi-Green's function approach to simulate their effect on the longitudinal dynamics of the beam. The results are compared to analytical estimations.

## 1 INTRODUCTION

The TESLA linear collider design [1] foresees damping rings in order to reduce the beam phase space volumes down to the design emittance for collider operation.

The normalised emittance of the positron beam injected into the ring which is assumed to $\gamma\epsilon_i$ = 0.01 m leads to a necessary vertical emittance reduction factor of $2 \times 10^{-5}$. This in combination with a storage time equal to the cycle time $T_c$ of 200 ms determines the vertical damping time and hereby the necessary energy loss per turn.

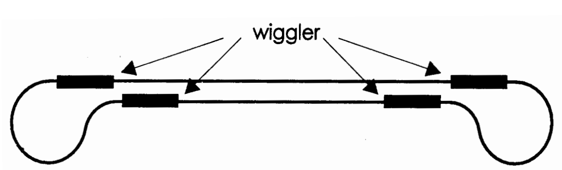


Figure 1: General layout of the "dog-bone"-shaped damping ring (ref. [1]).

In order to avoid the need for a costly additional ring tunnel, the TESLA damping ring is designed so that most of the lattice can be installed in the linac tunnel. Additional tunnels are only required for the two "loops" at the ends of the straight sections (see Figure 1).

The main complication which arises for the TESLA damping ring is due to the pulse structure of the linac. The train of the 1130 bunches per pulse has a length of 0.8 ms, or 240 km respectively. The bunch train will be therefore stored in a compressed mode, with a bunch spacing much smaller than in the linac.

### 1.1 Parameters

The parameters (for a one cm bunch) have been previously studied in detail in [2] and are to be found in Table 1.

As the radiation comes mainly from the wigglers with the piecewise constant magnetic field $B_0 = 1.5\,\mathrm{T}$, the energy spread in the TESLA damping ring (DR) is circumference independent.

| Parameter | | | units |
|---|---|---|---|
| Circumference | C | 20 | km |
| Rev. frequency | $f_0$ | ca. 15 | kHz |
| DC/bunch | $I_b$ | 0.09 | mA |
| Energy | E | 3.3 | GeV |
| Loss per turn | $U_0$ | 12 | MeV |
| Energy spread | $\sigma_{0\varepsilon}$ | 3.44 | MeV |
| RF Voltage | $\widehat{V}$ | 24 | MV |
| Overvoltage factor | q=e$\widehat{V}/U_0$ | 2.0 | |
| for bunch rms length $\sigma_s = 1\,\mathrm{cm}$: | | | |
| Synchr. tune | $\nu_s$ | 0.087 | |
| Mom. compaction | $\alpha$ | $2.7 \times 10^{-4}$ | |

Table 1: TESLA dog-bone damping ring parameters relevant for this study

In [2] it was assumed that the rms bunch length $\sigma_s$ was fixed at approx. 1 cm. For sake of definiteness the overvoltage factor $q$ was chosen to be two whereby the necessary values of the momentum compaction factor $\alpha$ and the synchrotron tune $\nu$ were determined so as to obtain the desired bunch length.

Here $q$ has been left at two, but the momentum compaction has been looked upon as variable to obtain different rms bunch lengths $\sigma_s$ and to study the wake field effects. The bunch length is proportional to the square root of $\alpha$.

## 2 SIMULATION TECHNIQUE

The simulation is based upon the usage of macroparticles and the incorporation of wake field effects into the equations of motion with the help of quasi-Green's functions (see [3]).

A bunch is represented by a variable number $N_{\mathrm{par}}$ of macroparticles $(\tau_k, \varepsilon_k)$ where $\tau_k$ is the particle position in time with origin at zero of the main RF and $\varepsilon_k$ the particle energy deviation from nominal energy E.

The tracking equations in the program are such that it is possible to define different sections within the machine and track during one turn from one sector to another. The equations of motion are described by practically the same equations as given here for a full revolution for simplicity where the phase space coordinates on turn $n+1$ are related to those on turn $n$ by

$$\tau_k^{n+1} = \tau_k^n + \frac{\alpha \mathrm{T}_0}{\mathrm{E}}\varepsilon_k^n - \frac{\alpha \mathrm{T}_0 \mathrm{U}_0}{2\mathrm{E}} \quad (1)$$

$$\varepsilon_k^{n+1} = \varepsilon_k^n + \mathrm{e}\sum_{i=1}^{m} \widehat{\mathrm{V}}_i \sin(\omega_{\mathrm{rf,i}}\tau_k^n + \phi_i) - \mathrm{U}_0 - \frac{2\mathrm{T}_0}{\mathrm{T}_\varepsilon}\varepsilon_k^n + 2\sqrt{\frac{\mathrm{T}_0}{\mathrm{T}_\varepsilon}}\sigma_{0\varepsilon}R + \mathrm{e}W(\tau_k^{n+1}) \quad (2)$$

with $\mathrm{T}_0$ the beam revolution period, $\mathrm{e}\widehat{\mathrm{V}}_i$ the peak energy gain from the RF $i$, $\omega_{\mathrm{rf,i}}$ the angular frequency of the RF $i$, $\phi_i$ the phase off-set from main RF (with index $i = 1$, $\phi_1 = 0$), $\mathrm{T}_\varepsilon$ the radiation damping time for energy oscillations, $R$ a Gaussian distributed random number with mean = 0 and rms = 1, and $W$ the wake field.

The second to last term on the right hand side of the second equation describes the energy variation due to quantum excitation by photon emission. The third term on the right hand side of the first equation reflects the continuous radiation loss, here between two successive turns.

Instead of using Green's functions (delta-wakes) to calculate the wake fields quasi-Green's functions are used, the wakes of Gaussian bunches with a smaller rms value than the bunch studied; in the following abbreviated by QGF.

To determine the wake field seen by a macroparticle a binning technique is used. The bunch represented by the $N_{\mathrm{par}}$ macroparticles is divided into bins with a pre-defined constant width. Then $\tau_{\mathrm{c}}(i)$, the centre of mass for every bin $i$, and $n_{\mathrm{b}}(i)$, the number of particles in bin $i$, are determined. The wake field effect on each bin is calculated as:

$$W_{\mathrm{b}}(i) = \frac{Q}{N_{\mathrm{par}}}\sum_{j=1}^{N_{\mathrm{bin}}} n_{\mathrm{b}}(j) \cdot \mathrm{QGF}(\tau_{\mathrm{c}}(i) - \tau_{\mathrm{c}}(j)). \quad (3)$$

The wake field $W(\tau_k)$ seen by a macroparticle $(\tau_k, \varepsilon_k)$ is then calculated by linear interpolation. Therefore there is no artificial jump of the wake fields seen by the macroparticles due to the binning, i.e. the transition between two bins is continuous. Note, that the numerical approximation has an acasual part to it as $\mathrm{QGF}(z) \not\equiv 0$ for $z < 0$.

The maximum relative error (relative to the maximum wake) for the below stated wakes of the impedance model and stated QGFs varied in the cases of a $5\,\mathrm{mm}$ and $1\,\mathrm{cm}$ Gaussian bunch between one and five percent (further details can be found in [4]).

## 3 IMPEDANCE MODEL

The impedance model used here is based on the one established by Shiltsev in [2], originally done for a $1\,\mathrm{cm}$ bunch.

There it was concluded that the bellows, beam position monitors (BPMs), vacuum ports, tapers, etc. all contributed to an inductive impedance $Z/n$ totaling $17\,\mathrm{m}\Omega$, corresponding to an inductance $L$ of $1.8 \times 10^{-7}$ H. For the simulation advantage is taken of the fact that an inductive impedance $Z = -i\omega L$ corresponds to a wake field which is inversely proportional to the derivative of the charge distribution: $W(s) = -L \cdot c^2\lambda'(s)$. This is computed for a Gaussian bunch of an appropriate length and used as a QGF.

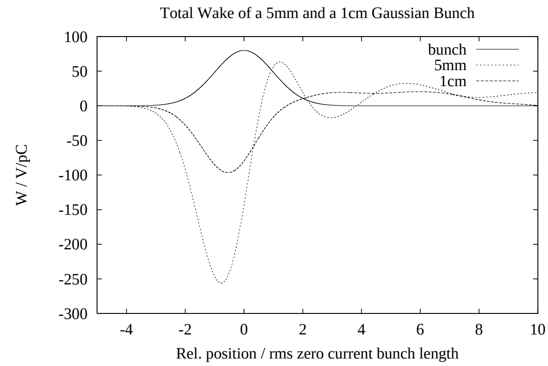


Figure 2: The wake field (all components) of a 5 mm and a 1 cm Gaussian Bunch.

The resistive wall effect has been accounted for, too. The longitudinal monopol resistive wall impedance of a round pipe in the long-range approximation is well-known and the corresponding formula for the wake field of a Gaussian charge distribution has been used to calculate QGFs. An aluminum vacuum chamber with a conductivity $\sigma_0 = 3.5 \times 10^7\Omega^{-1}m^{-1}$ is assumed. In the straight sections the pipe will have a radius of $b = 5\,\mathrm{cm}$, in the wiggler section ($2\,\%$ of the circumference) a radius of $b = 1\,\mathrm{cm}$ and in the arcs ($\sim 6\,\%$ of circumference) $b = 3\,\mathrm{cm}$.

The exact geometry of the RF cavities for the TESLA DR is not known yet. For this study eight 4-cell superconducting cavities, such as installed at HERA but scaled in frequency from $500\,\mathrm{MHz}$ to $433\,\mathrm{MHz}$ have been assumed. The longitudinal wake fields of Gaussian bunches with $\sigma_s$ equal to 2 and $3\,\mathrm{mm}$ have been calculated with the MAFIA T2 program and have been used as QGFs. For a $0.5\,\mathrm{mm}$ bunch only one cell was used.

Furthermore the installation of 40 very fast kickers has been assumed (see [5]). The wakes have been calculated using the MAFIA T3 module. The maximum absolute wake created by a Gaussian bunch passing a fast kicker varies approximately as $|W_{\max}^{\mathrm{FK}}| \sim 1/\sigma_s^{3/4}$.

## 4 SIMULATION RESULTS

For the simulation different numbers of macroparticles, number of sections, and different QGFs varying in bunch length for the various wakes were used. The rms values for the bunch length and the energy spread per run were calculated as the average values over the last 500 of the 3000 turns for which the simulation was run (this being approximately equivalent to eleven damping times and the cycle time).

The individual components have been studied in detail, the results can be found in [4].

| $I$ mA | $\alpha = 5 \times 10^{-5}$ Tracking $\sigma_s$ cm | $\sigma_\epsilon$ MeV | H $\sigma_s$ cm | $\alpha = 7 \times 10^{-5}$ Tracking $\sigma_s$ cm | $\sigma_\epsilon$ MeV | H $\sigma_s$ cm | $\alpha = 2.7 \times 10^{-4}$ Tracking $\sigma_s$ cm | $\sigma_\epsilon$ MeV | H $\sigma_s$ cm |
|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.44 | 3.47 | 0.44 | 0.52 | 3.48 | 0.51 | 1.06 | 3.59 | 1.02 |
| 0.09 | 0.56 | 3.47 | 0.56 | 0.62 | 3.48 | 0.62 | 1.09 | 3.59 | 1.05 |
| 0.18 | 0.64 | 3.48 | 0.64 | 0.69 | 3.49 | 0.69 | 1.13 | 3.59 | 1.09 |

Table 2: Results for the total wake and the three studied momentum compaction factors. Average tracking results (one section; $N_{\rm par} = 100\,000$ and $200\,000$) compared with those obtained by the Haissinski equation, abbreviated by "H". The design current is $0.09\,\rm mA$.

The main component contributing to the total wake seen by the particles in the damping ring for the momentum compaction factors $\alpha = 5 \times 10^{-5}$ and $\alpha = 7 \times 10^{-5}$ is the one due to the inductive impedance followed by that of the fast kickers, whereas for a $1\,\rm cm$ Gaussian bunch ($\alpha = 2.7 \times 10^{-4}$) it is vice versa. Note, that the absolute inductive wake maximum $|W^{\rm IN}_{\rm max}| \propto 1/\sigma_s^2$.

The bunch lengthening factors for the design current and for $I = 0.18\,\rm mA$ were equivalent to those for the purely inductive wake and match the results obtained by the Haissinski equation (see [6]). They therefore can be understood by the potential-well distortion theory. At higher currents, e.g. $I = 0.27\,\rm mA$ and $0.36\,\rm mA$, the results when using one section were such that the rms energy grew and the rms bunch lengths were noticeably larger than the predictions of the Haissinski equation. However for two sections the above stated differences disappeared (see [4]).

In the case of the two smaller momentum compaction factors a $1\,\rm mm$ inductive, $0.5\,\rm mm$ resistive wall, a $0.5\,\rm mm$ ($1\,\rm mm$) cavity, and a $2\,\rm mm$ fast kicker QGF were used. The bin width was $0.2\,\rm mm$. For $\alpha = 2.7 \times 10^{-4}$ a $1\,\rm mm$ inductive, resistive wall, and cavity wake, and a $3\,\rm mm$ fast kicker QGF were used; the width of the bins was $0.4\,\rm mm$.

Tracking simulations where all wake field effects except for the inductive wake were taken into account showed for the currents $I = 0.09\,\rm mA$ and $0.18\,\rm mA$ only a slight bunch lengthening and no signs of any instability.

A validity of the Boussard criterion [8] (microwave instabilty threshold) – which is not necessarily correct for short bunches – could not be established with the above mentioned QGFs. The only instability which was found was when studying the wake due to the fast kickers. The bunch became unstable at a current for which when solving the Haissinski equation a second potential-well minimum occured.

## 5 COMPARISON AND CONCLUSION

A comparison of wake field calculations with the MAFIA T3 module for a four button BPM monitor (in an elliptical pipe) has shown that for smaller bunches the wake can not be described by only an inductive part. Such a wake has been taken and multiplied by an appropriate factor so that its first absolute maximum corresponds to that of the assumed inductive wake attributed to the BPMs in the impedance model. The outcome was that total bunch lengthening was somewhat smaller. Further the assumed inductive wake to the BPMs in [2] seems to be unrealistically large ($L = 4.3 \times 10^{-8}\,\rm H$) so that the total bunch lengthening is momentarily presumably overestimated.

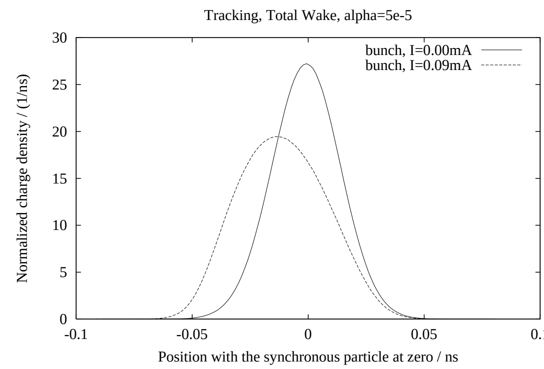


Figure 3: The average normalised charge density over the last 500 turns as seen by a tracking run with $N_{\rm par} = 100\,000$. Total wake for $\alpha = 5 \times 10^{-5}$, design current.

Further studies are planned. An improved impedance model is most certainly necessary, especially in respect to those components which contribute to the inductive wake component. In how far a vacuum chamber object appears inductive depends critically on the bunch length.

To date the prediction is that the shortest bunch studied would see at the design current a bunch lengthening factor of up to $30\,\%$ due to potential-well distortion.

Tracking simulations have been applied to the SLC damping rings (the old chamber but with bellow sleeves). The results (see [4]) agree well with those in [7]. Our thanks goes to Karl Bane for kindly supplying the wake field data without which the comparison would not have been possible.

## 6 REFERENCES


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