SIMULATION OF THE TRANSVERSE MICROWAVE INSTABILITY OF AN INTENSE ION BEAM

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Abstract

The computer algorithm has been developed to study both longitudinal and transverse microwave instability problems of an intense ion beam for synchrotrons and storage rings. The simulation method is presented.

1 INTRODUCTION

The presence of a large number of particles in the beam leads to collective phenomena in circular accelerators and storage rings [1]. In consequence the dynamics of particle for intense beam differs from a single particle dynamics which is entirely determined by the guide fields. The important part of collective phenomena is the longitudinal and transverse instability problem of an intense beam.

Further it is supposed that the behavior of the beam is still mainly determined by the external guide fields and the self fields induced by a large number of charge particles moving in their surrounding present only a relatively small perturbation. Excluding the synchrotron radiation we restrict of phenomena which are important in the high intensity low energy ion circular accelerators and storage rings. In this case the characteristic times for longitudinal processes are $10^2 \div 10^3$ beam turns whereas for transverse phenomena only $10^{-1} \div 10^{-2}$ of beam turn. Due to this distinction the longitudinal motion has been settled as basic.

The macroparticle beam representation is used for the algorithm proposed. This method to simulate the longitudinal high intensity beam dynamics is well known [3] and realized in the practice computer codes [4]. On the base of the known macroparticle algorithm to simulate the longitudinal dynamics the simulation method to study the transverse particle motion based on the macroparticle technique and impedance beam-environment interaction has been developed.

2 MATHEMATICAL FORMALISM

For simplicity we ignore the possible coupling of different directions in the transverse plane and will use only the horizontal coordinate x.

2.1 Particle Dynamics

In betatron phase space $(x_\beta; \dot{x}_\beta)$ the particle motion is governed by differential equations [1,2]

$$\ddot{x}_\beta - \frac{\ddot{\varphi}}{\dot{\varphi}}\dot{x}_\beta + \dot{\varphi}^2 x_\beta = \frac{F_\perp(t,\theta)}{m_0\gamma} \quad (1)$$

$$\dot{\varphi} = \frac{d\varphi}{dt} = Q\omega \simeq \quad (2)$$

$$Q_0\omega_0(1-\dot{\tau}) + \omega_\xi\dot{\tau} + Q_0\omega_0\frac{\partial Q}{Q_0\partial\hat{x}^2}\hat{x}^2$$

here x_β and $\dot{x}_\beta = dx_\beta/dt$ - the transverse particle coordinate and velocity with respect to the equilibrium orbit; ω - particle angular velocity; Q - particle betatron tune; θ - particle azimuthal position in the machine; m_0, γ - particle rest mass and relativistic factor; $F_\perp(t,\theta)$ - transverse force due to the induced beam's electromagnetic field; $\dot{\varphi}$ - betatron phase advance per second; $\hat{x}$ - the amplitude of betatron oscillations.

The next definitions are used [1,2]

$$\xi = (dQ/Q_0)/(dp/p_0)$$

$$\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma_0^2}$$

$$\omega_\xi = Q_0\omega_0\xi/\eta$$

$$\dot{\tau} = \eta\left(\frac{dp}{p_0}\right) = -\frac{d\omega}{\omega_0}$$

The low mark 0 is referred to the nominal parameters of machine.

Induced electromagnetic force is defined by impedance beam-environment interaction model [1,2]

$$F_\perp(t,\theta) = \frac{i\beta_0}{2\pi R}\int\limits_{\omega=-\infty}^{\infty} Z_\perp(\omega)D_\perp(\omega,\theta)e^{-i\omega t}d\omega \quad (3)$$

where R - the average machine radius; $Z_\perp(\omega)$ - transverse coupling impedance; $D_\perp(\omega,\theta)$ - Fourier spectrum of the transverse dipole-moment current.

2.2 Transverse Impedance

In order to introduce the transverse coupling impedance the frequency description was used. There are several standard components of the total impedance $Z_\perp(\omega)$: space charge, resistive wall, broad band and parasitic resonator impedances. The mathematical formulas for these components are well known [2,6].

2.3 *Transverse Dipole-Moment Current*

The transverse dipole moment current is defined as [5]

$$D_{\perp}(t,\theta)=\sum_{j=1}^{N} d_j(t,\theta)=\sum_{j=1}^{N} x_j(t,\theta) I_j(t,\theta) \quad (4)$$

$$I_j(t,\theta)=q\omega_j \sum_{m=-\infty}^{+\infty} \delta\left[\theta_j(t)-\theta-2\pi m\right] \quad (5)$$

here N - total particle number; x_j, θ_j - transverse and azimuthal coordinates of particle numbered j; $I_j(t,\theta)$ - j-particle current on azimuth θ.

For coasting beam the unperturbed particle motion is governed by the following equations

$$\theta_j(t)=\omega_j t+\theta_j^0$$

$$I_j(t,\theta)=q\frac{\omega_j}{2\pi}\sum_{m=-\infty}^{+\infty} e^{-im[\theta_j(t)-\theta]} \quad (6)$$

$$x_j(t,\theta)=\hat{x}_{\beta j}\cos(Q_j\omega_j t+\varphi_j^0)+x_0^j(\theta)$$

here the upper index 0 denotes initial parameter value; $x_0^j(\theta)$ - particle displacement different from the betatron motion on the azimuth θ.

Finally

$$D_{\perp}(t,\theta)=\sum_{j=1}^{N} q\frac{\omega_j}{2\pi}\hat{x}_{\beta j}\cdot \quad (7)$$

$$\sum_{m=-\infty}^{+\infty}\cos\left[(m+Q_j)\omega_j t+\Delta\varphi_m^j+\varphi_j^0\right]+$$

$$\sum_{j=1}^{N} q\frac{\omega_j}{2\pi}x_0^j(\theta)+\sum_{j=1}^{N} q\frac{\omega_j}{2\pi}x_0^j(\theta)\sum_{m=1}^{+\infty}\cos(m\omega_j t+\Delta\varphi_m^j)$$

$$\Delta\varphi_m^j=m(\theta_j^0-\theta)$$

In frequency domain the first sum is two spectral bands near $(m-Q_0)\omega_0$ and $(m+Q_0)\omega_0$ frequencies ($m=1,2,\ldots,\infty$), while the second and third sums are the spectral band near $m\omega_0$ frequency. The shape of the band spectrum depends from the particle momentum distribution, ξ and η parameters and the number of azimuthal harmonic m.

For bunched beam the conception of band formation is not changed, but the band spectrum will consist from the synchrotron satellite bands [5].

3 COMPUTER REALIZATION

As it was mentioned above the longitudinal simulation is carried out by standard macroparticle algorithm [3]. Then we suppose that parameters of the longitudinal macroparticle dynamics are known.

Applying the macroparticle beam representation and the binning technique all spectrum amplitudes of $D_{\perp}(t,\theta)$ can be determined. Here in eqs.(4,5) the sum is carried out over the ensemble of macroparticles. On the uniform mesh used for longitudinal calculations the transverse dipole moment current is defined in the grid points by standard weight method

$$D_{\perp}(t,\theta_k)=\sum_{j=1}^{M} d_j(t,\theta)F(\theta,\theta_k) \qquad k=(0\div N_b) \quad (8)$$

here $F(\theta,\theta_k)$ - the weight function, M - the number of macroparticles.

According to the spectrum description considered in the previous section the next main assumption is supposed: the orbital wave with a positive harmonic number $n=|m|$ consists from the superposition of three waves which have the same orbital number n but different phase velocities

- "fast" wave $(m>0)$ with phase velocity slightly differ from
$$\Omega_n^+=\left(1+\frac{Q_0}{n}\right)\omega_0 \quad (9)$$
- "slow" wave $(m<0)$ with phase velocity near
$$\Omega_n^-=\left(1-\frac{Q_0}{n}\right)\omega_0 \quad (10)$$
- "synchronous" wave with phase velocity near
$$\Omega_n^0=\omega_0 \quad (11)$$

The first two waves are referred to "pure" betatron motion while the third wave is determined of every kind deviations of particle motion from the betatron one (for example - the closed orbit distortions).

Then for simplicity we will take into account the "fast" and "slow" waves only. In the coordinate system moving with the beam these waves may be presented as travelling waves

$$D_m^S(t,\phi)= \quad (12)$$
$$C_{1m}\cos(m\phi+Q_0\omega_0 t)+D_{1m}\sin(m\phi+Q_0\omega_0 t)=$$
$$\cos m\phi(C_{1m}\cos Q_0\omega_0 t+D_{1m}\sin Q_0\omega_0 t)+$$
$$\sin m\phi(D_{1m}\cos Q_0\omega_0 t-C_{1m}\sin Q_0\omega_0 t)$$

$$D_m^F(t,\phi)= \quad (13)$$
$$C_{2m}\cos(m\phi-Q_0\omega_0 t)+D_{2m}\sin(m\phi-Q_0\omega_0 t)=$$
$$\cos m\phi(C_{2m}\cos Q_0\omega_0 t-D_{2m}\sin Q_0\omega_0 t)+$$
$$\sin m\phi(D_{2m}\cos Q_0\omega_0 t+C_{2m}\sin Q_0\omega_0 t)$$

here ϕ - azimuthal coordinate in the beam system.

It should be pointed that in common case the coefficients $C_{1m}, C_{2m}, D_{1m}, D_{2m}$ are time dependent, but the dependence is weak. Then the expressions (12,13) are valid for small time interval.

To calculate four unknown coefficients for each harmonic m the Fourier analysis of transverse dipole moment current for two time points is necessary. It can be done by the following steps [6]: Fourier analysis at fixed time, then to conserve the spectrum information we turn off the particle interaction and carry out the macroparticle tracking for time step Δt ($Q_0\omega_0\Delta t = \pi/2$ is recommended), then the second Fourier analysis is done.

Applying the wave definitions (12,13) and some mathematical manipulations the induced electromagnetic force (3) is expressed as [6]

$$F_{\perp}(t,\phi) = F_0(\phi)+ \tag{14}$$

$$[QC(\phi)\cos Q_0\omega_0 t + QS(\phi)\cos Q_0\omega_0 t]$$

$$QC(\phi) = \sum_{m=0}^{\infty}\left[\left(M_1^S + M_1^F\right)\cos m\phi + \left(M_2^S + M_2^F\right)\sin m\phi\right]$$

$$QS(\phi) = \sum_{m=0}^{\infty}\left[\left(M_2^S - M_2^F\right)\cos m\phi - \left(M_1^S - M_1^F\right)\sin m\phi\right]$$

$$\begin{aligned} M_1^S &= RZ_m^S C_{1m} + IZ_m^S D_{1m} \\ M_2^S &= -IZ_m^S C_{1m} + RZ_m^S D_{1m} \\ M_1^F &= RZ_m^F C_{2m} + IZ_m^F D_{2m} \\ M_2^F &= -IZ_m^F C_{2m} + RZ_m^F D_{2m} \end{aligned} \tag{15}$$

$$RZ_m^{SF} = -\frac{\beta_0}{2\pi R} ImZ_{\perp}[(m \mp Q_0)\omega_0]$$

$$IZ_m^{SF} = \frac{\beta_0}{2\pi R} ReZ_{\perp}[(m \mp Q_0)\omega_0]$$

Using the form (14) the integration of particle dynamics equations (1,2) is not problem. Some simple examples are considered in [6].

Finally, the computer realization of proposed algorithm to simulate the transverse particle dynamics of an intense ion beam in circular accelerators including an induced electromagnetic fields consists of the next stages per each time step

- binning technique to define the transverse dipole-moment current;
- determination of the transverse dipole-moment current spectrum in the frequency domain;
- transverse induced force calculation by using the definition of transverse impedance and reverse Fourier transformation;
- integration of the particle motion equations.

In practice the total time to simulate the transverse dynamics is a few times more then it takes for longitudinal simulation. However there is possibility to estimate the dangerous processes leading to particle losses and beam quality degradation.

4 CONCLUSIONS

The computer code using the algorithm discussed above was developed to study both longitudinal and transverse collective phenomena for high intensity proton storage ring of the Moscow Meson Factory. Throw off the different factors acting on transverse motion it is possible to carry out the more detailed studies, for example:

- the influence of longitudinal dynamics on transverse one (excitation of the betatron oscillations during the longitudinal microwave instability);
- transverse dynamics versus the closed orbit distortions due to the injection law or imperfections;
- transverse dynamics versus the deviation of the average beam momentum from the designed value.

5 REFERENCES

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