CODE TO STUDY THE HIGH INTENSITY BEAM DYNAMICS IN THE ION LINEAR ACCELERATORS

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Abstract

To design of the high intensity ion linac the study of the effect of space charge on the beam quality can be done preliminary by simulation. CIC code was developed to research the dynamics of the bright beam in the periodic accelerating structures. The details are discussed.

1 INTRODUCTION

In the last time the interest to produce energetic intense ion beams is obvious. To design of a high current ion linac it is necessary to keep particle loss below an acceptable level and to have appropriate beam quality for injection into the following accelerator. For nonrelativictic energies the effects of nonlinear forces of the beam space charge and accelerating field determine mainly the beam quality. On the final stage of designing not only simplified theoretical beam dynamics but linac simulation with more or less real technical parameters is required.

The code LINCIC was written to calculate the particle traces of a bunched beam including space charge forces through a defined transport system of linac. The Monte Carlo simulation with a large number of macroparticles ($\sim 10^5$) incorporated with a fast Poisson solver is used.

2 CODE PROPERTIES

The LINCIC code is assigned for studies of space charge effects using a direct solution of 3D Poisson equation in the cylindrical pipe. It takes into account the macroparticle beam representation and grid description for external and self space charge fields.

2.1 Initial Distribution

The initial macroparticle distribution is an external data for code. The beam is represented by macroparticles defining the distribution in 6-dimensional phase space. It can be done either as result of the previous simulation or as artificial distribution. For last one to generate an arbitrary distribution the superposition of particle densities with elliptical symmetry is supposed. The basic particle distribution is

$$n(x_1, x_2, x_3, x_4, x_5, x_6) = f(R)$$

$$R^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2$$

where $x_1, x_2, x_3, x_4, x_5, x_6$ are canonical variables less then 1. The function $f(R)$ is limited and has an arbitrary dependence. The generation is fulfilled by reverse function method. After that the transformation to real parameters is required. Using the several model distributions with different weight centre, intensity and orientation it is possible to reproduce either the beam characteristics measured experimentally or the halo-core beam model.

It is evident that the initial distribution has an influence on the self fields and as result on the space charge dependent phenomena. Therefore it has to be reproduced carefully in accordance with the real assumptions.

2.2 Particle Motion

The general dynamics equations for a particle moving in the z-direction is expressed as

$$\frac{d\vec{p}}{dt} = \frac{q}{m_0 c}\left(\vec{E} + \frac{c}{\gamma}\vec{p} \times \vec{B}\right)$$

$$\frac{d\vec{r}}{dt} = \frac{c}{\gamma}\vec{p}$$

Here $\vec{p} = \gamma\vec{\beta}$ - modified particle momentum; $\vec{\beta} = \vec{v}/c$; $\vec{r}, \vec{v}$ - particle coordinate and velocity; c - light speed; q, m_0 - charge and rest mass of ion accelerated; $\gamma^2 = 1 + \vec{p} \cdot \vec{p}$ - relativistic factor; $\vec{E}, \vec{B}$ - electric and magnetic fields.

To integrate the motion equations the rectangular coordinates and time as independent variable are used. The integration is performed numerically by the Runge-Kutta method of forth order with a sufficiently small time step.

The fields $\vec{E}$ and $\vec{B}$ are divided into two parts: the external guide fields $(\vec{E}_{ex}, \vec{B}_{ex})$ and the beam self fields $(\vec{E}_s, \vec{B}_s)$. Both field type is determined on the cylindrical grid but calculated and entered by different ways.

2.3 External Fields

The linac transport system may consists of the rf elements, drifts, magnetic elements and others. The types of rf structure and the focusing methods have to be determined by the preliminary studies. The data of magnetic and rf linac structures are described in two external files and entered into the code. The first file contains in sequential order the information about each unit rf cell. To generate the geometry of the accelerating cell the electromagnetic code [1] is used for calculation the gap-to-period ratio as well as T-factor. Moreover the accelerating resonators may be optimized for maximal effective shunt impedance for the given aperture and the type of rf structure, the accelerating

gradients are calculated for given average rf power losses per unit length. Further an accurate electromagnetic field distribution on the cylindrical grid for each rf cell can be introduced into the code. However this is complicated process and it can be recommended on the final detailed linac studies.

In the last code version the field distribution in the accelerating gaps is approximated by a wave with cos-like edges for a given value of E_0T (E_0 is the electric field average over the cell, T is the transit time factor) calculated for the realistic field distribution on the gap axis. This representation has a satisfactory conformity with reality and allows to increase the accuracy and reduce the time of numerical integration of the macroparticle motion.

The magnetic data file contains the information about the placement and strength of magnetic elements. The field shape is also presented with the smooth cos-like edges due to the reasons mentioned above.

2.4 Self Field Calculation

The self field calculations are carried out for charge bunch in the cylindrical conducting pipe supposing the periodical conditions in longitudinal z-direction. Therefore the problem is to solve Poisson equation

$$\Delta\varphi(r,\theta,z) = -\frac{\rho(r,\theta,z)}{\varepsilon_0} \tag{1}$$

with boundary conditions

$$\begin{aligned}
&\varphi(r,\theta,z) = \varphi(r,\theta+2\pi,z)\\
&\varphi(r,\theta,z) = \varphi(r,\theta,z+L)\\
&\varphi(r_0,\theta,z) = 0
\end{aligned}$$

Here $\theta \in [0,2\pi]$ - azimuthal angle; $r \in [0,r_0]$ - radial coordinate; $z \in [z_0, z_0+L]$ - longitudinal coordinate; L - longitudinal period length; r_0 - pipe radius; ε_0 - vacuum dielectric constant; Δ - Laplace operator; ρ - charge density; φ - electric potential.

The solution of eq.(1) is defined in the coordinate system moving with the bunch where we suppose that the currents and self magnetic field are neglected. Here the potential is expressed as

$$\varphi(r,\theta,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \tag{2}$$

$$\left\{\left[S^{(1)}_{nm}(r)\cos n\theta + S^{(2)}_{nm}(r)\sin n\theta\right]\cos\frac{2\pi}{L}mz + \left[S^{(3)}_{nm}(r)\cos n\theta + S^{(4)}_{nm}(r)\sin n\theta\right]\sin\frac{2\pi}{L}mz\right\}$$

$$S^{(i)}_{nm}(r) = \sum_{k=1}^{\infty}\frac{F^{(i)}_{knm}}{\left(\frac{\mu^{(n)}_k}{r_0}\right)^2 + \left(\frac{2\pi}{L}m\right)^2} J_n\left(\frac{\mu^{(n)}_k}{r_0}r\right) \tag{3}$$

where J_n and $\mu^{(n)}_k$ - standard Bessel function of order n and its zero of number k respectively; $F^{(i)}_{knm}$ - coefficients of the Bessel-double Fourier decomposition of the right side of eq.(1).

For computer realization the solution of the eq.(1) is determined on the cylindrical grid uniformed both in longitudinal,axial and radial directions. Using the finite series in eq.(2), cubic spline approximation for $S^{(i)}_{nm}(i = 1 \div 4)$ and some mathematical manipulations the fast Poisson solver has been worked out in the Fourier-spline form on the grid defined [2]. The tests have shown the higher accuracy of the proposed numerical algorithm due to the application of the radial cubic spline approximation. While the calculation time is slightly more then the time of standard finite difference method.

To determine the $\vec{E}_s$ and $\vec{B}_s$ the analytical differentiation of potential (2) and Lorentz transformations to the lab system are yielded. The dimensions of the coordinate mesh is chosen in dependence of the bunch parameters (sizes, charge density distribution, macroparticle beam representation) and computer resources.

3 CONCLUSIONS

The computer code discussed briefly may be used in any program complex to design the high intensity low energy ion linacs. The main prescription is to study the delicate space charge effects on the final stage of an accelerator design (such as the beam halo formation due to space charge, initial distribution, mismatching of different accelerator parts, tolerances). A typical calculation time for a $\sim$ 400-cell simulation ($\sim 10^5$ macroparticles, 200 time steps per cell, $16\times32\times64$ grid dimensions) is ~ 70 hours for ALPHA500 processor [3].

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5 REFERENCES

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