A PARTICLE CORE MODEL FOR SPACE CHARGE DYNAMICS IN RINGS*

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Abstract

High power circular accelerators and storage rings have both stringent beam loss requirements and significant space charge forces. It is therefore important to study the space charge dynamics and halo formation in rings. For linear accelerators, particle core models (PCM) have proven to be computationally fast, tractable to analytic methods, and to provide insight into the physics of halo formation. However, due to complications arising from the interaction of space charge and dispersion, until now PCM have not been developed for circular accelerators. We present a self-consistent PCM for the calculation of transverse space charge dynamics in rings. The model includes the effect of dispersion in representing the space charge force of the beam as a superposition of effective envelope equations for pure betatron motion. Also, space charge correction terms appear in the betatron focusing functions and in the dispersion function. The overall dynamics involves the interactions of the envelope with the lattice, the tracked particles with the envelope, and the tracked particles with the lattice. Particle tracking calculations are presented comparing the effects of space charge in similar doublet, FODO, and uniform focusing lattice configurations.

1 INTRODUCTION

The analysis and understanding of space charge effects for particle beams in linear accelerators has been greatly facilitated by the use of particle core models (1). PCMs represent the collective dynamics of the beam by envelope equations that contain the effects of the lattice focusing forces, the beam emittances, and the space charge. PCMs are also employed to provide space charge forces for particle-tracking calculations to study both collective and individual particle dynamics. The alternatives to PCMs are Particle in Cell (PIC) models (2), which are totally self-consistent but less amenable to direct theoretical analysis and computationally more intensive than are PCMs. One advantage of PCMs is the ability to track specified particles individually or in small groups. PIC calculations require the entire particle distribution to evaluate the space charge forces for tracking.

Until now, PCMs have been applied primarily to straight channels with strong space charge forces. However, with the advent of several applications involving rings with high beam intensities and small beam loss requirements (3), the application of PCMs to rings is necessary. However, the representation of the beam via the envelope equations must be generalized to include the effects of dispersion. One treatment of this dispersion problem has recently been carried out at University of Maryland (4), and we present a somewhat different approach here.

2 PARTICLE CORE MODEL FOR RINGS

Our purpose is to extend the particle core model to rings by including dispersion. In this section we present a brief description of the derivation of a PCM for rings together with the principal features of the simplest version of this model. Detailed derivations of the model can be found in Refs. 5 and 6.

The model is derived within the framework of an accelerator ordering scheme, valid for high intensity rings. Using this ordering and canonical transformations, a Hamiltonian that decouples the betatron motion from the longitudinal motion and from the dispersion is derived. Noteworthy results of this process include transverse canonical coordinates and momenta that represent pure betatron oscillations with dispersion removed and an equation for the dispersion function with space charge correction. Next, a statistical approach, based on the kinetic distribution function in the canonical phase space of the separated Hamiltonian, is applied to perform a moments analysis of the beam at each energy. Effective emittances, conserved to leading order, are defined for the transverse canonical coordinates, and rms envelope equations are derived for the azimuthal variation of the standard deviations of the transverse canonical coordinates. These emittances and envelope equations are decoupled from dispersion and pertain to pure betatron motion only. The effect of dispersion appears only in determining the spatial charge distribution for the evaluation of the space charge forces. This is carried out by integrating the charge distribution implied by the envelope equations over the beam energy distribution. The integration can be done to varying levels of sophistication, but in any case, closure requires the resulting space charge distribution to be consistent with that appearing in the dispersion and envelope equations. To ensure this, an iterative process may be required. We now present a very simple version of a

PCM with dispersion for rings based on the assumption of a K-V distribution.

Letting q be one of the transverse coordinates and p its conjugate momentum, the effective emittance is given by

$$\varepsilon_q = 4[\sigma_p{}^2\sigma_q{}^2 - \langle \Delta p \Delta q \rangle^2]^{1/2}\,, \tag{1}$$

where $\langle\ \rangle$ denotes an average over the beam distribution function taken at fixed energy. The quantities σ_p and σ_q are the rms standard deviations of p and q, respectively, also at fixed energy. The emittances, conserved to leading order, are regarded as constants in the model. The rms envelope equations for x and y can be written:

$$\begin{aligned} a'' - \frac{\varepsilon_x{}^2}{a^3} + \left[K_x(s) - \frac{2K}{A(A+B)}\right]a = 0 \\ b'' - \frac{\varepsilon_y{}^2}{b^3} + \left[K_y(s) - \frac{2K}{A(A+B)}\right]b = 0 \end{aligned} \tag{2}$$

where $a = 2\sigma_x$; $b = 2\sigma_y$; primes denote derivatives with respect to s, the azimuthal distance along the lattice; $K_x(s)$ and $K_y(s)$ are the focusing forces of the lattice magnets; and K is the generalized perveance of the beam (7). These envelope equations describe pure betatron oscillations about the closed orbit at each energy in the beam. A similar form is obtained for the dispersion equation with space charge:

$$D_x{}'' + \left[K_x(s) - \frac{2K}{A(A+B)}\right]D_x - \frac{1}{\rho} = 0\,, \tag{3}$$

where ρ is the local bending radius. The quantities A and B are the elliptical axes of the K-V distribution, which satisfy

$$A = \left[a^2 + 4D_x{}^2\left\langle\delta^2\right\rangle_{tot}\right]^{1/2}, \qquad B = b\,, \tag{4}$$

where $\left\langle\delta^2\right\rangle_{tot}$ is the standard deviation of the momentum spread $\frac{\Delta p}{p_0}$, so that A contains separate contributions from betatron oscillations and from dispersion.

Because Eqs. 2-4 are interdependent, it is necessary to solve them as a system. For particle tracking, the space charge forces are calculated in this simple model by assuming a K-V distribution with axes A and B. We now present results of calculations with this PCM for doublet, FODO, and uniform focusing rings with parameters similar to those of the SNS accumulator ring (3).

3 CALCULATED RESULTS

Particle core model calculations with tracking were carried out for matched K-V beams in each of three lattices: doublet, FODO, and uniform focusing. These lattices are fourfold symmetric, 221 m in circumference, and have bare tune values of ν_x=5.85 and ν_y=5.70. The beam parameters for these calculations are also similar to the SNS baseline design. Specifically, an H^- beam of energy 1 Gev and maximum energy spread ± 9.4 Mev is assumed. The number of beam particles, 3.08*10^{14} for a nonbunched circulating beam, corresponds to 2.0*10^{14} particles for a bunched beam with bunching factor equal to 0.65. We use a coasting beam and neglect azimuthal variations in charge density. The rms beam emittances are chosen to be 100 mm-mrad in both the $x - x'$ and $y - y'$ planes. The tracking calculations are carried out for a total of 1250 turns.

In all three lattices the tracking results show a mild spreading of the beam core and no halo generation. The rms emittances remain almost constant and the average tune shifts are in the range 0.08-0.12, bracketing the calculated Laslett tune shift of 0.105 for the K-V distribution in the uniform focusing case. The tune spreads are small, ~0.04, as expected for a K-V distribution. For the doublet lattice, these results differ from those obtained using a PIC model, which gives significant halo generation in the $y - y'$ plane. The difference between the calculated PIC and PCM behaviors here may be due to a space charge instability accessible to the PIC model but not to the envelope equations, or to a numerical instability in the PIC code. Numerical and beam diagnostic studies are being carried out to clarify these behaviors.

Calculations for mismatched K-V beams have been carried out for all three lattices with the mismatch set by the PCM and the macroparticle distributions made consistent with the envelopes. Again, for all three lattices and up to 25% initial radial core mismatch, the PCM results in little beam growth and no halo generation. This result contrasts with PIC calculations in which both beam core growth and substantial halo occur.

In the case study of the three lattices, the tunes were selected to avoid resonances. The dynamic behavior changes significantly in the vicinity of a core resonance. Figure 1 shows the ratios of the envelope radii for a slightly perturbed envelope to those of the matched periodic solutions for the FODO lattice with the bare tunes changed to ν_x=6.035 and ν_y=5.885. For this case there is a core structure resonance with ν_x^{env}=12.00, and its effects can be seen in the amplitudes $\frac{a}{a_{matched}}$, plotted continuously over 400 turns in Fig. 1. It is also seen that $\frac{b}{b_{matched}}$ varies little. Figure 2 shows, for the particles in the $x-x'$ phase plane, the extent of the effect of this resonance on the beam.

4 CONCLUSIONS

A self-consistent particle core model for transverse beam dynamics in rings, including the effects of space charge and dispersion, has been derived. The model includes rms envelope equations for betatron oscillations, a space-charge-corrected dispersion function, and a prescription for the evaluation of the space charge potential, all coupled together self-consistently. In addition to describing the collective dynamics of the beam, this model provides space charge forces for particle tracking calculations. Computational results using the PCM show tune shifts in good agreement with PIC code and analytic values, but the dynamics accessible to the PCM exclude many higher order space charge instabilities that are accessible to the PIC model.

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REFERENCES

1. O'Connell, J.S., Wangler, T.P., Mills, R.S., and Crandall, K.R., "Beam Halo Formation from Space-Charge Dominated Beams in Uniform Focusing Channels," in *Proc. of the 1993 Particle Accelerator Conf.*, Washington, D.C., May 17-20, 1993, pp. 3657-9; Riabko, A., Ellison, M., Kang, X., Lee, S.Y., Liu, J.Y., Li, D., Pei, A., and Wang, L., "Hamiltonian Formalism for Space Charge Dominated Beams in a Uniform Focusing Channel," in *Proc. of the 1995 Particle Accelerator Conf. and International Conf. on High-Energy Accelerators*, Dallas, TX, May 1-5, 1995, pp. 3182-4..

2. Hockney, R.W. and Eastwood, J.W., *Computer Simulation Using Particles*, New York: Adam Hilger, 1988.

3. National Spallation Neutron Source Conceptual Design Report, Volumes 1 and 2, NSNS/CDR-2/V1,2, (May, 1997).

4. Venturini, M. and Reiser, M., *Phys. Rev. E* **57**, 4725-32 (1998).

5. Lee, S.Y. and Okamoto, H., *Phys. Rev. Lett.* **80**, accepted for publication (1998).

6 Holmes, J.A., Galambos, J.D., Olsen, D.K., and Lee, S.Y., "An RMS Particle Core Model for Rings," in *Proc. of the Workshop on Space Charge Physics in High Intensity Hadron Rings*, Shelter Island, NY, May 4-7, 1998, to be published by American Institute of Physics.

7. Wangler, T.P., "Emittance Concept and Growth Mechanisms," in Space Charge Dominated Beams and Applications of High Brightness Beams, AIP Press, 1996, pp. 3-22.

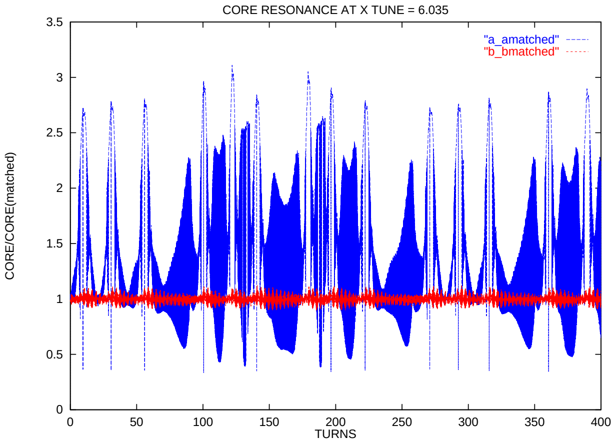

Figure 1: Ratios of resonant to matched envelope radii for quadrupole resonance in horizontal plane.

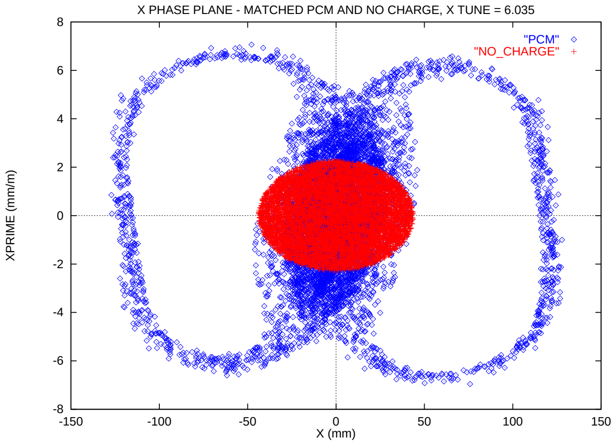

Figure 2: Particles in horizontal phase plane at 1250 turns for PCM and no space charge models and quadrupole resonance.