Beam-Loading Issues and Requirements for the KEKB Crab RF System

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Abstract

A crab cavity is being developed for KEKB. This paper discusses beam-loading on the crab cavity, a coupled-bunch instability caused by the crabbing mode, and the tolerance for possible errors.

1 INTRODUCTION

A finite-angle crossing scheme eliminates parasitic collisions, and simplifies the interaction region design in high-luminosity colliders. On the other hand, the luminosity and/or lifetime can be degraded due to beam-beam effects. The crab-crossing scheme [1], [2] is considered to solve the potential problems encountered with the finite angle crossing. The crab crossing employs four crab cavities: two cavities in each ring. The crab cavity is required to provide a high transverse deflecting voltage. In addition, it must be a "damped cavity", in which all parasitic modes are heavily damped in order to avoid coupled-bunch instabilities. A superconducting "squashed crab cavity" was developed under the KEK-Cornell collaboration [3]. Its damping scheme was verified and the high-field performance was tested with a one-third scale cavity [4].

At KEK we have decided to develop the crab RF system as a viable fall-back solution for the finite-angle crossing of KEKB [5]. The ring parameters and required kick voltage are shown in Table 1. The superconducting squashed crab cavity has been adopted as the basic design. The R&D effort is being conducted aiming at fabricating full-scale niobium cavities [6].

In this paper we first discuss the beam-loading on the crab cavity. Since the operating mode is a dipole mode (TM110), the beam-loading is different from that on accelerating cavities. We also discuss coupled-bunch instabilities caused by the crabbing mode, as well as the tolerance for possible errors.

2 BEAM LOADING ON THE CRAB CAVITY

2.1 Beam-induced voltage

In this paper the transverse shunt impedance ($\bar{R}_\perp$) is defined as

$$\bar{R}_\perp \equiv \frac{V_{\perp c}^2}{P_c}, \tag{1}$$

Table 1: Parameters for the Crab Cavity in KEKB.

Ring		LER	HER	
Beam energy	E	3.5	8.0	GeV
RF frequency	$\omega_{RF}/2\pi$	508.887		MHz
Crossing angle	φ	±11		mrad
Beta function at IP	β_x^*	0.33	0.33	m
Beta function at crab	β_{crab}	20	100	m
Required kick	$V_{\perp c}$	1.41	1.44	MV

where $V_{\perp c}$ is the kick voltage and P_c is the dissipation power on the cavity surface. (Another notation, $R_\perp$, which is usually used in instability calculations, is related as $R_\perp \equiv \frac{k}{2}\bar{R}_\perp$, where k is the wave number.)

We now consider the case in which the kick voltage is in the horizontal (x) plane (horizontal crossing). We assume that bunches pass parallel to the cavity axis, displaced by Δx. The single-bunch beam loading ($V_{\perp b0}$) is given by

$$V_{\perp b0} = -j\frac{\omega_a}{2}\left(\frac{\bar{R}_\perp}{Q_0}\right) q\,(k\Delta x), \tag{2}$$

where q is the charge, ω_a the resonant angular frequency, and Q_0 the intrinsic Q-value.

If the bunch spacing is much smaller than the filling time of the cavity (as in KEKB), the superposition of the successive single-bunch beam loadings gives a continuous beam-indeced voltage ($V_{\perp b}$),

$$V_{\perp b} = V_{\perp br}\cos\psi e^{j\psi}, \tag{3}$$

where

$$V_{\perp br} = -j\frac{I_b\bar{R}_\perp}{1+\beta}k\Delta x, \tag{4}$$

$$\tan\psi = 2Q_L\frac{\omega_a - \omega_{RF}}{\omega_a}. \tag{5}$$

Here, I_b is the total beam current, β the input coupling, ψ the tuning angle, and Q_L the loaded-Q value.

2.2 Vector relations

In the following, a positron beam ($q > 0$, $I_b > 0$) is considered. (An electron beam can be treated similarly.) Figure 1 shows a vector relation for the crabbing mode. In this figure,

$V_{\perp g}$ is the generator voltage, $V_{\perp gr}$ the generator voltage on resonance, α_L the angle between $V_{\perp gr}$ and $V_{\perp c}$, and ϕ_c the angle of $V_{\perp c}$ with respect to the beam. It is similar to that for the accelerating mode, except for the beam-induced voltage, $V_{\perp br}$ and $V_{\perp b}$. The phase of $V_{\perp br}$ with respect to the bunch phase is 90 or 270 degrees, according to $\Delta x < 0$ or $\Delta x > 0$ (180 degrees for the accelerating mode). Furthermore, the amplitude of $V_{\perp br}$ is dependent on Δx.

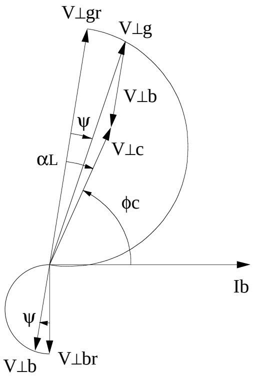

Figure 1: Vector relation for the crabbing mode ($\Delta x > 0$).

From the vector relation we obtain

$$\tan \alpha_L = \frac{\tan \psi - Y \cos \phi_c}{1 + Y \sin \phi_c}, \quad (6)$$

$$\mid V_{\perp gr} \mid \cos \alpha_L = \mid V_{\perp c} \mid (1 + Y \sin \phi_c), \quad (7)$$

where $Y \equiv \pm \mid V_{\perp br}/V_{\perp c} \mid$ (positive sign for $\Delta x > 0$ and negative sign for $\Delta x < 0$).

Since $\mid V_{\perp gr} \mid$ is related to the input power (P_g) as

$$\mid V_{\perp gr} \mid = \frac{2\sqrt{\beta}}{1+\beta}\sqrt{\bar{R}_\perp P_g}, \quad (8)$$

the required power to maintain the crabbing voltage is obtained from Eqs. 4, 7 and 8 as

$$P_g = \frac{(1+\beta)^2}{4\beta \bar{R}_\perp} \times \left\{ \frac{1}{\cos \alpha_L} \left(\mid V_{\perp c} \mid + \frac{I_b \bar{R}_\perp}{1+\beta} k \Delta x \sin \phi_c \right) \right\}^2 . \quad (9)$$

2.3 *Loaded-Q value and required power*

As long as the beam orbit is kept just on-axis, the required power can be minimized, if we set $\beta = 1$. In this case Q_L is about $\sim 10^9$ for a superconducting crab cavity. However, a high Q_L is undesirable, since the system becomes extremely sensitive to an orbit change. A displacement with $\Delta x > 0$ increases P_g, as shown in Eq. 9. Figure 2 shows P_g as a function of Q_L for displacements of Δx=0, 0.5 and 1.0 mm. The best choice for KEKB is $Q_L \sim 10^6$. Then, P_g is relatively small and is not very sensitive to orbit change. A generator power of 50 kW is required to accomodate to an orbit change of 1 mm. Another reason for avoiding a high Q_L is that the resonant frequency becomes extremely sensitive to any mechanical vibration or other effects. The above choice seems to also be good in this respect; superconducting cavities in TRISTAN have been successfully operated with $Q_L \sim 10^6$.

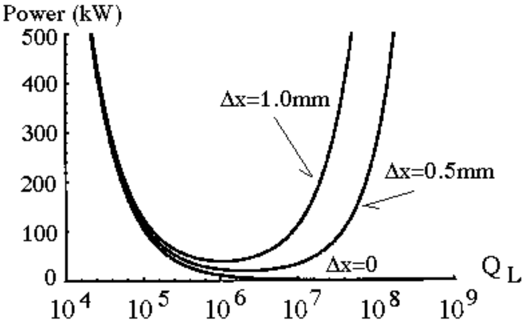

Figure 2: Required power for an off-axis beam.

3 COUPLED-BUNCH INSTABILITY DUE TO THE CRABBING MODE

While parasitic modes, including the TM010 mode, are heavily damped [3], the crabbing mode, itself, can cause a coupled-bunch instability. It is similar to an instability caused by the accelerating mode of accelerating cavities.

3.1 *Transverse*

The growth rate is given by [7]

$$\tau^{-1} = -\frac{e\omega_0 I_b}{4\pi E} \times \beta_{crab} \sum_p \left\{ Re Z_T(\omega_p^{(\mu)+}) - Re Z_T(\omega_p^{(\mu)-}) \right\}, \quad (10)$$

where

$$\omega_p^{(\mu)+} = \{(p-1)M + \mu + \nu_x\} \omega_0, \quad (11)$$

$$\omega_p^{(\mu)-} = \{pM - \mu - \nu_x\} \omega_0, \quad (12)$$

$$Z_T(\omega) = \frac{\frac{1}{2}\left(\frac{R_\perp}{Q_0}\right) Q_L}{1 - jQ_L \left(\frac{\omega_a}{\omega_{RF}} - \frac{\omega_{RF}}{\omega_a}\right)} \times \frac{\omega_a}{\omega_{RF}}. \quad (13)$$

Here, ω_0 is the revolution angular frequency, M the number of bunches, μ the coupled-bunch mode number, and ν_x the betatron tune. If ω_a is equal to ω_{RF}, the growth of all coupled-bunch modes (μ) are cancelled out by its damping; the impedances at the growth and damping frequencies are symmetric with respect to ω_{RF}, as shown from Eqs. 11 — 13. An instability can occur when $\omega_a \neq \omega$ and the growth rate depends on ν_x. Figure 3 shows the growth time (τ) for the KEKB HER with ν_x = 45.52, as a function of Δf ($= \omega_a - \omega_{RF}$). With two crab cavities installed in each ring, the growth time is longer than the radiation damping time (40 ms), when -15< Δf <+12 kHz.

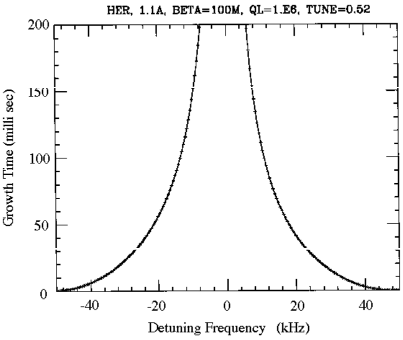

Figure 3: Growth time of the transverse coupled-bunch instability caused by the crabbing mode.

3.2 *Longitudinal*

In addition to the transverse instability, a longitudinal coupled-bunch instability can be excited when the beam passes off-axis. If both a leading bunch and a following test particle are off-axis by Δx, the longitudinal force acting on the test particle is given by

$$F_z = -eqW_1'(s)(\Delta x)^2, \qquad (14)$$

where

$$W_1'(s) = \frac{\omega_a}{2}k\left(\frac{\bar{R}_\perp}{Q_0}\right)\frac{\omega_a}{c} \times \cos\left(\frac{\omega_a}{c}s\right) exp\left(-\frac{\omega_a}{2cQ_L}s\right). \qquad (15)$$

Eq. 14 and 15 show that $\bar{R}_\perp(k\Delta x)^2$ corresponds to the shunt impedance for the longitudinal instability.

With the KEKB (LER) parameters, τ = 84 ms for Δx = 1 mm and τ = 9 ms for Δx = 3 mm, when the resonant frequency hits the upper synchrotron side band. Since it is the $\mu = 0$ coupled-bunch mode, damping is provided by the accelerating cavities (4.4 ms). Nevertheless, it is still desirable to tune the cavity at a slightly lower side of ω_{RF}.

4 TOLERANCE FOR ERRORS

4.1 *Phase errors*

Phase error between $V_{\perp c}$ and the beam This error shifts the bunch center horizontally at the IP. If we require that the displacement should be much smaller than the bunch size, we obtain

$$|\delta\phi| << \frac{\omega_{RF}}{c\tan\varphi}\sqrt{\varepsilon_x\beta_x^*}. \qquad (16)$$

$\mid \delta\phi \mid << 4.3$ degrees with the KEKB parameters.

This error comes from:

1. an RF reference phase error between HER and LER;
2. a change in the synchronous phase (ϕ_s) according to the bunch current; and
3. a different ϕ_s between bunches due to a bunch gap.

In the LER, ϕ_s changes from 72 degrees at zero current to 69.6 degrees at full current, while there is no significant change in the HER. This causes a relative phase error of 2.4 degrees, which is not acceptable. Thus, the change in ϕ_s should be compensated. One possible method is to introduce a phase offset which is programmed according to the bunch current.

The bunch phase modulation due to a 10% bunch gap in the HER is 2.7 or 4.9 degrees, depending on the cavity choice [5]. The relative displacement between the HER and LER can be cancelled out by introducing a compensation gap in the LER.

Phase error between two crab cavities in the same ring This error causes a closed-orbit distortion (COD) in the whole ring. The r. m. s. COD due to this error is given by

$$\Delta x_{rms} = \frac{c\tan\varphi}{2\omega_{RF}\sqrt{\beta_x^*}} \times \frac{1}{\sin\pi\nu_x}\sqrt{\beta_{ave}} \times \delta\phi, \qquad (17)$$

where β_{ave} is an average of β_x in the ring. If we require $\Delta x_{rms} < 50\mu$m, the tolerance for KEKB is $\mid \delta\phi \mid < 1.6$ degrees.

4.2 *Amplitude error*

The amplitude error of $V_{\perp c}$ causes an error in the tilt angle. It is probably acceptable if $\Delta\varphi << \sigma_x^* / \sigma_z$. Then, we obtain

$$\mid \frac{\Delta V_{\perp c}}{V_{\perp c}} \mid \cong \frac{\mid \Delta\varphi \mid}{\varphi} << \frac{\sigma_x^*}{\varphi\sigma_z}. \qquad (18)$$

With the KEKB parameters, $|\Delta V_{\perp c}/V_{\perp c}| << 1.75$.

4.3 *COD in the crab cavity*

As mentioned before, an orbit displacement of ± 1 mm is acceptable in view of the available power. It is still desirable to control the orbit within $\sim \pm 0.1$ mm for stable operation. A possible method to do that is to control the steering magnets according to the input power to the crab cavity, since it is a good indicator of the orbit displacement in the cavity.

5 REFERENCES

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