# Beam Current Limitation due to Single-Beam Collective Effects in the Ion Storage Ring of RIKEN RI-Beam Factory Project

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## *Abstract*

Ion-electron colliding experiments are planed in DSR. The current of ion beams in the ion storage ring of DSR is limited due to singe-beam collective effects. As the effects, here are considered the incoherent betatron tune shift due to the direct space charge, the longitudinal microwave instability, the beam bunching, and the longitudinal coupled-bunch instability. The limitation is estimated for the today-designed lattice of the ring from the viewpoint of the luminosity. Here, one treats of a 150 MeV/u $U^{92+}_{238}$ beam and a 400 MeV/u $He^{2+}_4$ beam with the momentum spread $10^{-3}$.

## 1 INTRODUCTION

The RIKEN Accelerator Research Facility group has been proposing "RIKEN RI Beam Factory" as a next project [1]. Ion-electron colliding and ion-ion merging experiments are planed at Double Storage Rings(DSR) of Multi-USe-Experimental-Storage ring system(MUSES). These experiments require high luminosity with small momentum spread and emittances. For this purpose, in the upstream side of the DSR will be installed a booster synchrotron where the intensity of ion beams is increased with a cooling-stacking technique and the momentum spread and emittances are cooled down. The expected full momentum spread $\Delta p/p$ may become at most $10^{-3}$, and the rms transverse emittances $10^{-6}\pi$ mrad. The improvement of beam qualities does not always mean the increase of luminosity, because it makes single-beam collective effects strong and hence limits the beam currents in the ion storage ring of DSR. Table 1 shows the parameters of the ring [2].

The current limitation of the ion colliding beam comes from singe-beam collective effects, that is, the betatron tune shift due to the direct space charge, the beam bunching, and longitudinal instabilities. In this paper, the limitation is estimated for the today-designed lattice of the ion ring. The beam is assumed to have the full momentum spread (6 rms) $\Delta\ p/p\ =\ 1\ 10^{-3}$ and the rms transverse emittance $10^{-6}\pi$ mrad. One treats of the limitation just in the lowest injection energy of the primary beam from a pre-booster, or 150 MeV/u for the heaviest ion beam and 400 MeV/u for the lightest ion beam. IF the full bunch length of the beam is beyond the detectable length of the collision section 0.4 m, just ions staying within the center 0.4 m of the bunch are considered to practically contribute to the colliding experiments. The luminosity is evaluated for the limited beam currents.

Table 1: Parameters of the ion storage ring of DSR.

| | |
|---|---|
| Circumference | 258.73 m |
| Max. beam energy for Z/A=0.5 | 1.45 GeV/u |
| Momentum compaction factor | 0.03772 |
| Betatron tune($\nu_x$ /$\nu_y$ ) | 7.42/5.81 |
| Natural chromaticity($\xi_x$ /$\xi_y$ ) | -11.2/-8.3 |
| Beta function at the collision point($\beta^*_x/\beta^*_y$ ) | 0.6/0.6 m |
| Dispersion at the collision point($\eta_x$ /$\eta_y$ ) | 0/0 |
| RF harmonics, or bunch number | 86 |

## 2 ELECTRON BEAM CURRENT LIMITATION

At a collision section of DSR, an electron beam circulating in the one ring collides at the colliding angle of zero with an 86-bunched ion beam circulating in the other ring. The linear tune shift of the ion due to the beam-beam effect is described as follows on the assumption where the charge distribution of the electron round beam is Gaussian with the rms $\sigma_{re}$ in the radius direction [3];

$$\xi_i = \frac{(1+\beta_i\beta_e)\beta^*_\perp N_{Be} r_p Z/A}{4\pi\sigma^{*2}_{re}\beta^2_i\gamma_i}$$

where $\beta_i$ and $\beta_e$ are velocities of the ion and the electron beams normalized by the light velocity $c$, respectively, $\beta^*_\perp$ the beta function at the collision section, $N_{Be}$ the number of electrons per bunch, $r_p$ the classical proton radius, $Z/A$ the charge to mass ratio of the ion, $\sigma^*_{re}$ the rms transverse electron beam size at the collision section, and $\gamma_i$ the ion mass normalized by the rest mass. $\beta^*_{\perp e}$ of the electron ring at the collision section is designed to be equal to $\beta^*_\perp$ of the ion ring. The colliding experiments are planed in the electron energy 0.3 GeV through 1 GeV. The rms emittance of the electron beam at 0.3 GeV and 1 GeV is $0.05\ 10^{-6}\pi$ mrad and $0.5\ 10^{-6}\pi$ mrad on the large-emittance mode of DSR, respectively. The stability of the lattice of the ion ring requires

$$|\cos\mu - 2\pi\xi_i\sin\mu| \leq 1$$

where $\mu$ is half the phase advance of the ring. One has the requirement $\xi_i\ \leq\ 0.04$, reserving little safety margin. Then, the threshold number of the electrons per bunch is evaluated as shown in Table 2.

On the other hand, the stored current of the electron beam is expected to be at least 300 mA, when the 0.3 GeV beam is injected in the ring. The electrons' number per bunch for 300 mA is shown in Table 2, the bunch number of the electron beam being 44 or 62 in colliding with 150 MeV/u ion beam or 400 MeV/u one, respectively. On the 1 GeV-energy side, it is seen to be smaller than the threshold number due to the beam-beam effect.

Table 2: The threshold of electrons' number per bunch due to the beam-beam effect and the number per bunch for the 300 mA electron beam.

| Ion × electron | 0.3 GeV $e^-$ | 1 GeV $e^-$ | 300 mA $e^-$ |
|---|---|---|---|
| 400 MeV/u $\mathrm{He}_4^{2+}$ | $1.2\ 10^{+10}$ | $1.3\ 10^{+11}$ | $2.5\ 10^{+10}$ |
| 150 MeV/u $\mathrm{U}_{238}^{92+}$ | $7.3\ 10^{+9}$ | $8.1\ 10^{+10}$ | $3.5\ 10^{+10}$ |

## 3 ION BEAM CURRENT LIMITATION

### *3.1 Incoherent betatron tune shift due to direct space charge*

For a round beam whose charge distribution is assumed to be Gaussian with the rms $\sigma_{ri}$ in the radius direction and Gaussian with the rms bunch length $\sigma_{Bi}$ in the longitudinal direction, the incoherent betatron tune shift of ions with small betatron amplitude due to direct space charge is described by [4]

$$\Delta\nu = -\frac{\beta_\perp N_{Bi} r_p Z^2/A}{4\pi\sigma_{ri}^2 \beta_i^2 \gamma_i^3 B_f}$$

where $\beta_\perp$ is the average beta function of the ion ring, $N_{Bi}$ the ions' number per bunch, and $B_f = \sqrt{2\pi}\sigma_{Bi}/2\pi R$ the bunching factor, $R$ being the average radius of the ring. When resonance compensation is carried out well, the shift of $|\Delta\nu| \leq 0.2$ is tolerable. Then, the ions' number per bunch $N_{Bi}$ for the full bunch length 0.4 m is shown in Table 3.

### *3.2 Impedance budget of the ion storage ring*

The lattice of DSR has been being optimized nowadays. The structure of the vacuum chamber and numbers of the elements have not yet determined. Here, the beam coupling impedances with the chamber are estimated on rough assumption of them. A candidate of the RF cavity for the ion ring is a $\lambda$/4 coaxial cavity with a higher order damper [5]. The longitudinal impedances except the resistive wall one are shown for the energies of 150 MeV/u and 400 MeV/u in Table 4.

### *3.3 Beam bunch*

In the energy region of the ring, the space charge force is dominant over the forces induced through the impedances. It acts on a beam to lengthen the bunch below the transition energy, when the beam intensity is high. The simulation of the beam bunching with electron cooling and RF voltage applying [5] shows the bunch-length dependence on the beam intensity, as shown in Table 5, with the electron current density 2.5 kA/m$^2$ at the cooling section. Ions staying within the bunch center of length 0.4 m are considered to practically contribute colliding experiments. For an $\mathrm{U}_{238}^{92+}$ beam, the longer the bunch length is, the more the number of the ions is. The $\mathrm{He}_4^{2+}$ beam is seen not to be bunched in such intensity as 4.2 $10^{+7}$ ions per bunch, as the 6-rms bunch length is larger than the RF wavelength of 3 m.

Table 3: Threshold ions' number per bunch due to the incoherent tune shift $|\Delta\nu| \leq 0.2$ for the full bunch length 0.4 m.

| | |
|---|---|
| 150 MeV/u $\mathrm{U}_{238}^{92+}$ | $1.6\ 10^7$ |
| 400 MeV/u $\mathrm{He}_4^{2+}$ | $2.1\ 10^9$ |

Table 4: The imaginary parts of the longitudinal impedances $\Im[Z_\parallel/n]$ $[\Omega]$ of the ion ring of DSR at the low frequency.

| Beam energy | 150 MeV/u | 400 MeV/u |
|---|---|---|
| Space charge | -1600$\jmath$ | -730$\jmath$ |
| Bellows | 1.4$\jmath$ | 2.0$\jmath$ |
| Flanges | 0.05$\jmath$ | 0.07$\jmath$ |
| Transition sections | 0.20$\jmath$ | 0.28$\jmath$ |
| Slits of vacuum ports | 0.005$\jmath$ | 0.005$\jmath$ |
| Clearing electrodes | 0.13$\jmath$ | 0.19 $\jmath$ |
| Beam monitors | 0.04$\jmath$ | 0.06 $\jmath$ |
| RF equivalent BB | 0.04$\jmath$ | 0.02$\jmath$ |

### *3.4 Longitudinal instability*

The microwave instability is undesirable, because it induces bunch lengthening when the ions' number per bunch goes beyond a threshold, and the lengthening decreases the luminosity. The threshold peak current $I_{pi}$ is given with Keil Schnell criterion under the Gaussian momentum distribution [6];

$$\frac{I_{pi} e Z/A}{2\pi m_0 c^2 \beta_i^2 \gamma_i |\eta| (\Delta p/p)_{rms}^2} \left| \frac{Z_\parallel}{n} \right| \leq 1$$

where $e$ is electronic charge, $\eta$ the slippage factor, and $(\Delta p/p)_{rms}$ the rms momentum spread. The equation shows the threshold ions' number per bunch for the full bunch length 0.4 m as shown in Table 6, which is smaller than that due to the tune shift. The tune shift is seen to be 0.08 and 0.05 for the microwave threshold of 150 MeV/u $\mathrm{U}_{238}^{92+}$ and 400 MeV/u $\mathrm{He}_4^{2+}$, respectively. When the bunch length is longer than 0.4 m, the threshold number increase proportionally to the length. The $\mathrm{U}_{238}^{92+}$ bunched beams and $\mathrm{He}_4^{2+}$ ones shown in Table 5 are seen not to meet the microwave instability. For the coasting beams, the threshold ions' numbers per 0.4 m are about 2.4 times as much as those for the bunch length 0.4 m. If the momentum spread is smaller than $10^{-3}$, the threshold number becomes smaller, resulting in the smaller luminosity.

When a bunched beam circulates with the angular frequency $\omega_0$ in a ring, it can have longitudinal frequency components $\omega_p = \omega_0(pB + m + n\nu_s)$ without perturbation, $B$ being the number of bunches, $p$=0, $\pm1$, $\pm2$, $\cdots$

Table 5: Bunch length (6 $\sigma_{Bi}$). Two numbers in the parentheses tell the percentage of ions staying within the bunch center 0.4 m and the final RF voltage in the bunching, respectively.

| 150 MeV/u $\mathrm{U}_{238}^{92+}$ | | 400 MeV/u $\mathrm{He}_4^{2+}$ | |
|---|---|---|---|
| Ions/bunch | 6 $\sigma_{Bi}$ | Ions/bunch | 6 $\sigma_{Bi}$ |
| $0.9\ 10^{+6}$ | 0.8 m (75 %, 120 kV) | $4.2\ 10^{+7}$ | 4.1m (30 %, 8 kV) |
| $4.5\ 10^{+6}$ | 1.4 m (65 %, 50 kV) | $2.1\ 10^{+8}$ | 4.5 m (20 %, 6 kV) |
| $2.3\ 10^{+7}$ | 2.4 m (45 %, 27 kV) | - | - |

Table 6: Threshold ions' number per bunch due to the microwave instability for the full bunch length 0.4 m.

| | |
|---|---|
| 150 MeV/u $U_{238}^{92+}$ | $4.5\,10^6$ |
| 400 MeV/u $He_4^{2+}$ | $3.8\,10^8$ |

etc., $\nu_s$ the synchrotron tune, $m$ (=0, 1, 2, $\cdots$, B-1) the coupled-bunch mode, and $n$ (=0, $\pm$1, $\pm$2, $\cdots$ etc.) the synchrotron mode. With perturbation, the frequency is coherently shifted, in the Sacherer-Zotter formalism [7], by

$$\Delta\omega_{\parallel}^{m,n} = -j\frac{|n|}{|n|+1}\frac{eZ/AI_i\omega_0\eta}{6B_i^3\pi\beta_i^2E_i\nu_s}\frac{\sum_{p=-\infty}^{+\infty}\frac{Z_{\parallel}(\omega_p)}{\omega_p/\omega_0}h_n(\omega_p)}{\sum_{p=-\infty}^{+\infty}h_n(\omega_p)}$$

where $I_i$ is the beam current, $B_i = 6\sigma_i/2\pi R$ the bunching factor for one bunch, and $h_n(\omega_p)$ the spectral power density of the $n$-th synchrotron mode. The above equations are applicable to the single bunch, also. The shortest growth time of the coupled-bunch mode in the low synchrotron modes is shown in Table 7, which has been calculated with the program ZAP [8] that has been modified to be applicable to ion beams by the authors. The electron cooling force has not been taken into account in the program. The single-bunched $U_{238}^{92+}$ beam is seen to be stable. The natural synchrotron frequency spread comes from a nonlinear sinusoidal RF bucket. In the other cases, the growth modes of the instability are Landau-damped through the spread.

## 4 LUMINOSITY

The luminosity for the ion-electron collision in some cases shown in Table 8 is estimated with [9]

$$L = \frac{N_{Be}N_{Bi}f_{REVi}B}{2\pi(\sigma_{re}^{*2}+\sigma_{ri}^{*2})}$$

where $f_{REVi}$ is the ion's revolution frequency, and $\sigma_{ri}^*$ is $\sigma_{ri}$ at the collision section. The upper limit of the luminosity is decided by the coasting-beam threshold of the microwave instability. For the $U_{238}^{92+}$ beam of one-third of the threshold intensity, the limit can be nearly got by bunching the beam. The bunching is effective in getting near the upper limit just for very heavy-ion beam, though it itself is effective in increase of the luminosity for light-ion beams or heavy-ion beams when the intensity is weak.

Table 7: The shortest growth time [s] of the coupled-bunch $m$-th mode of the longitudinal coupled-bunch instability in the low $n$-th synchrotron mode.

Single-bunch instability

| | 150 MeV/u $U_{238}^{92+}$ | | 400 MeV/u $He_4^{2+}$ | |
|---|---|---|---|---|
| $n$ | $4.5\,10^{+6}$ | $2.3\,10^{+7}$ | $4.2\,10^{+7}$ | $2.1\,10^{+8}$ |
| 1 | -0.2 | -0.2 | $3\,10^{+4}$ | $9\,10^{+3}$ |
| 2 | -0.3 | -0.03 | $5\,10^{+3}$ | $2\,10^{+3}$ |
| 3 | -6 | -0.03 | $2\,10^{+3}$ | $8\,10^{+2}$ |
| 4 | -72 | -0.2 | $2\,10^{+3}$ | $6\,10^{+2}$ |

Multi-bunch instability

| | 150 MeV/u $U_{238}^{92+}$ | | | | 400 MeV/u $He_4^{2+}$ | | | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $m$ | $4.5\,10^{+6}$ | $m$ | $2.3\,10^{+7}$ | $m$ | $4.5\,10^{+6}$ | $m$ | $2.1\,10^{+8}$ |
| 1 | 85 | 48 | 19 | 18 | 85 | 16 | 85 | 6 |
| 2 | 51 | 7 | 85 | 12 | 85 | 5 | 85 | 2 |
| 3 | 51 | 2 | 85 | 9 | 85 | 4 | 85 | 1 |
| 4 | 51 | 1 | 85 | 13 | 85 | 4 | 85 | 1 |

Table 8: Luminosity[/cm$^2$s] for the ion-electron collision. The beam$^\dagger$ is a coasting beam of which the intensity is equal to the threshold for the microwave instability, and of which $6\sigma_{Bi}$ is 5.2 m for the flat line density along 3 m.

150 MeV/u $U_{238}^{92+}$

| $N_{Bi}$ ($6\sigma_{Bi}$) | 0.3 GeV electron | 1 GeV electron |
|---|---|---|
| $4.5\,10^6$ (1.4 m) | $2.7\,10^{25}$ | $8.6\,10^{25}$ |
| $2.3\,10^7$ (2.4 m) | $9.5\,10^{25}$ | $3.2\,10^{26}$ |
| $8.1\,10^7$ (5.2 m)$^\dagger$ | $9.9\,10^{25}$ | $3.3\,10^{26}$ |

400 MeV/u $He_4^{2+}$

| $N_{Bi}$ ($6\sigma_{Bi}$) | 0.3 GeV electron | 1 GeV electron |
|---|---|---|
| $2.1\,10^8$ (4.5 m) | $4.5\,10^{26}$ | $6.5\,10^{26}$ |
| $6.8\,10^9$ (5.2 m)$^\dagger$ | $9.7\,10^{27}$ | $1.4\,10^{28}$ |

## 5 CONCLUSION

The luminosity for the ion-electron collision in DSR is limited by following single-beam collective effects. As ion beams, one considers the primary 150 MeV/u $U_{238}^{92+}$ beam and the primary 400 MeV/u $He_4^{2+}$ beam with the full momentum spread $10^{-3}$. The electron current in the low energy side is limited by the beam-beam effects. The longitudinal microwave instability limits the ion-beam current more strongly than the incoherent betatron tune shift due to direct space charge. The beam bunching with the electron cooling and RF voltage applying can hardly bunch the ion beam as shortly as the microwave instability occurs. The longitudinal single-bunch instability of the bunched beam does not occur, or is Landau-damped. The longitudinal multi-bunch instability is Landau-damped. The maximum luminosity is realized to be 1$\sim$ $3\,10^{26}$ /cm$^2$s for the 150 MeV/u $U_{238}^{92+}$ beam, and about $10^{28}$ /cm$^2$s for the 400 MeV/u $He_4^{2+}$ coasting beam. The maximum luminosity for most other species of ion beams are seen to be realized just in the cases of coasting beams.

The beam bunching does not go forward in the high intensity beam in the today-designed storage ring of which the transition energy is beyond the operation energy. One should search for the lattice of which the transition energy is below the operation energy, where the beam bunching goes forward easily in the high intensity.

## 6 REFERENCES

[1] Y.Yano, A.Goto, T.Katayama, and RARF Accelerator Group, RIBF-A-1-1995 (1995).

[2] N.Inabe and T.Katayama, in these proceedings.

[3] L.R.Evans and J.Garete, CERN 87-03, 159 (1987).

[4] A.Hofmann, CERN 94-01, 329(1994).

[5] M.Takanaka and T.Katayama, in these proceedings.

[6] A.Hoffmann, CERN 77-13, 139(1977).

[7] J.L.Laclare, 11th International Conference on High-Energy Accelerators, 526(1980).

[8] M.S.Zisman, S.Chattopadhyay, and J.Bisognano, LBL-21270 (1986).

[9] K.Potter, CERN 85-19, 318(1985).