SINGLE BUNCH COLLECTIVE EFFECTS IN MUON COLLIDERS *

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Abstract

Theoretical examination is made of single bunch collective effects in the collider ring of a 2 Tev × 2 Tev Mu-Mu Collider complex. The situation involves an intense bunch, a short bunch, a small momentum compaction, a rather large impedance for the rf system, and luminosity life time limited by muon decay to of the order a thousand turns. The use in rings of techniques such as BNS damping, developed for linear colliders, is discussed. Qualitative descriptions and numerical simulation results are presented.

1 INTRODUCTION

The design of a high luminosity (2.5×10^{30}cm^{-2} per collision) muon collider ring, from the perspective of the physics of collective effects, has some unique features which need to be examined. (1) The bunch has a large charge: $N = 2 \times 10^{12}$. (2) The bunch is short: $\sigma_z = 3$ mm. (3) The momentum compaction α is very small: $\alpha \leq 10^{-6}$. (4) Muons have a very short life time: $\tau_\mu \simeq 41.6$ ms at 2 Tev, corresponding to a thousand "effective" turns in a ring with the circumference of 7 kilometers.

These features lead us to some unusual aspects of the ring operation: the intense bunch required for the high luminosity makes instabilities likely and very small α requires careful estimations of nonlinear corrections to the particle orbit and to the collective dynamics.

The longitudinal equation of motion of a particle traveling in a circular machine can be written as

$$z' = -\eta\delta, \quad \delta' = K(z), \tag{1}$$

where z is the oscillation amplitude with respect to the bunch center, $' = d/ds$, s measures distance around the ring, $\delta = dp/p$, $\eta = \alpha - 1/\gamma^2$, $\alpha = pdC/Cdp$. The force $K(z)$ that a particle experiences can be modeled as having two parts, one is due to the radio frequency (rf) cavities, and the other is from the wake fields generated by the interaction between beam and cavities or other components of its electromagnetic environment,

$$K(z) = K_{rf}(z) + K_{wake}(z), \tag{2}$$

where

$$K_{rf}(z) = eV_{rf}(z)/CE, \tag{3}$$

and

$$K_{wake}(z) = -(r_0/\gamma C)\int_z^\infty dz'\rho(z')W_0'(z-z'). \tag{4}$$

In Eq. (4), $T_0 = 2\pi/\omega_0 = C/c$, $E = \gamma m_\mu c^2$, $r_0 = e^2/m_\mu c^2$, $N = \int dz'\rho(z')$, and $C = 2\pi R$ is the circumference of the collider ring.

In Eq. (3), when the amplitude of synchrotron motion is small compared with the rf wavelength such that the rf voltage is linearized as $V_{rf}(z) = \hat{V}\sin(\omega_{rf}z/c) \approx \hat{V}\omega_{rf}z/c$ and the momemtum compaction is expanded as $\eta = \eta_1 + \eta_2\delta + \eta_3\delta^2$ (with the contributions from η_2 and η_3 negligible), the synchrotron oscillation frequency is $\omega_{s0} = (e\eta_1\, c\,\omega_{rf}\hat{V}/CE)^{1/2}$.

The transverse equation of motion is

$$y''(z,s) + (\omega_\beta^2/c^2)\, y(z,s) = -(r_0/\gamma C)\int_z^\infty dz'\rho(z')W_1(z-z')y(z',s). \tag{5}$$

In this paper, we discuss several single bunch collective effects, and illustrate the impacts and the cures of collective instabilities for the muon collider. Some important simulation results are presented.

2 STATIC EFFECTS

As a starting point we consider a TESLA-like rf system, and a quasi-isochronous lattice which has $\eta_1 = 10^{-6}$, with the contributions from η_2 and η_3 negligible. With a bunch length $\sigma_z = 3$ mm, $\sigma_\delta = 1.5 \times 10^{-3}$ rms energy spread and an 130MV of L-band rf, the muon bunch is matched to the rf and a muon would undergo $\sim .56$ synchrotron oscillations in one thousand turns.

2.1 Parastic Loss

The beam loses energy when it experiences the impedance of the rf cavities. We model the rf impedance by using Perry Wilson's scaling formula for the longitudinal wake function [1]. Explicitly, the wake function for the TESLA's rf frequency, $W_0'(z < 0) = 226(f_{TESLA}/f_{SLAC})^2 \times \exp[-(-zf_{TESLA}/0.1839f_{SLAC})^{0.605}]$, where

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$f_{TESLA} = 1.3$GHz and $f_{SLAC} = 2.856$GHz. Casuality requires: $W_0'(z) = 0$ if $z > 0$. Our simulation code computes the wake voltage $V_{wake}(z) = \int_z^\infty dz' \rho(z') W_0'(z - z')$, and the energy loss $\triangle\mathcal{E} = -\int_{-\infty}^\infty dz \int_{-\infty}^\infty dz' \rho(z') W_0'(z - z')$.

The peak wake voltage is further scaled from Wilson's formula to give 17V/pC/m at $1\sigma_z$ for a Gaussian beam. This choice of wake amplitude makes it consistent with the TESLA rf cavity studies [2]. The beam-loading factor, defined as $\triangle\mathcal{E}$/particle/V_{rf}, is 10% when only the cavity losses are included. The energy loss due to the resistive-wall are estimated to roughly equal the rf losses [3], but have not been explicitly included in our calculation. Other losses have not been calculated, and may lead to an increase in the rf voltage. These losses will need to be replenished even if the momentum compaction is reduced to $\eta_1 = 10^{-7}$, as may be required because of microwave instabilities.

2.2 Potential Well Distortion

The effects of radiation and diffusion of muons are small in a muon lifetime (radiation damping time $\sim 10^6$ turns), so that, unlike in electron rings, equilibrium is not achieved by radiation damping. The intense muon bunch generates significant wakes, and these wakes in turn cause significant changes in the rf potential. This potential-well distortion causes oscillation of the bunch center, bunch size, and distribution function in the rf bucket. Fig. (1) shows the oscillations of the rms bunch size and bunch energy spread. The bunch centroid tends to move forward to a higher rf voltage, so that the energy loss can be compensated. As a result, it makes a counter-clockwise rotation in $\delta - z$ phase space, as shown in Fig. (2). The parasitic losses and the bunch centroid shift are compensated for by injecting the beam with an rf phase offset of 0.082 radians with respect to the bunch center, as shown by the dashed line in Fig. (2).

3 COHERENT EFFECTS

3.1 Microwave Instability

The longitudinal microwave instability is presently considered the most serious challenge to maintaining a short bunch. Presently studies are underway to examine the limits this instability places upon Z/n. The ring parameters obviously will not satisfy the Keil-Schnell criterion for stability, but rather we hope to reduce the growth rate to an acceptable amount during the 1000 turns of beam storage. Specifically, the growth rate and the damping rate of the longitudinal microwave instability of a short bunch is [4]

$$\tau_{growth}^{-1} = n\omega_0 \sqrt{\frac{\eta I_p Z_0^{||}/n}{2\pi E/e}}, \quad \tau_{damping}^{-1} = n\omega_0 \eta \sigma_\delta, \quad (6)$$

where $n = \omega/\omega_0$. If $\eta = 10^{-6}$ and $Z/n \simeq 1\Omega$, then for the ring parameters, $\tau_{growth}^{-1} \simeq 8.7n \times 10^{-3}\text{s}^{-1}$ and $\tau_{damping}^{-1} \simeq 0.4n \times 10^{-3}\text{s}^{-1}$. The instability growth rate is much faster than the damping rate.

The microwave instability growth rate is weaker at smaller η values, and the instability may require the lattice to operate at $\eta = 10^{-7}$. At this value of η the particles barely move longitudinally, and the possibility and consequences of compensating for the wake potential with rf are being considered. In the absence of significant longitudinal motion the main problem is to maintain an energy spread within the longitudinal acceptance of the ring.

3.2 Beam-Break-Up

For times much shorter than the synchrotron oscillation period, particles are almost frozen longitudinally in the bunch, and the transverse wakefield dynamics has many similarities with that in a linac [5]. In a linac, the transverse wake field generated by the head of the bunch drives the tail, causing Beam-Break-Up (BBU). A dimensionless parameter that characterizes the BBU strength is [6]

$$\Upsilon(z) = N r_0 |\langle W_1(z) \rangle| c / 4\omega_\beta \gamma, \quad (7)$$

where ω_β is the betatron angular frequency, and $\langle W_1(z) \rangle = \int_z^\infty dz' W_1(z - z')\rho(z')$ is the convoluted total transverse wake function. The tail of an off-axis bunch doubles its offset in a number of turns $n \simeq 1/\Upsilon$, so long as $n \ll 1/2\nu_s$, i.e., when the particle's synchrotron motion can be ignored. Here ν_s is the synchrotron tune.

Simulation results for the BBU-like instability using a resonator model are shown in Fig. (3). The main point is that while the motion is unstable, it is easily cured with only a small amount of BNS damping, as discussed below.

3.3 Head-Tail Instabilities

When the transverse oscillation frequency is modulated by the energy oscillation, the chromaticity, which is the slope of the frequency to the energy, builds up a head-tail phase that bootstraps from the first half synchrotron period to the next, and drives the system into instability without threshold. This head-tail instabilities occurs in both transverse and longitudinal motions [7]. The effect of transverse head-tail (THT) instability is small when $\eta_1 \leq 10^{-6}$.

For the longitudinal motion, the longitudinal chromaticity involves the lowest non-linear part of slip factor: η_2. The bucket height and the growth time of the longitudinal head-tail (LHT) instability are both proportional to η_1/η_2. Different designs of the lattice lead to very different results for the geometry of bucket and the collective effects. It is assumed here that the contributions of η_2 and η_3 to the dynamics are sufficiently small, even if $\eta_1 = 10^{-7}$, that they can be neglected [8]. Simulations indicate that the longitudinal head-tail instability can be controlled by not allowing η_2/η_1 to become too large. Detailed studies of the acceptable parameter ranges are underway.

4 DAMPING MECHANISMS

Since the synchrotron radiation damping is negligible and the ring is quasi-isochronous (so that the effect of Landau damping is very small), neither of these are likely to damp collective instabilities. “External” mechanisms, such as BNS damping, may be needed to cure the instabilities. The BNS damping can be achieved by a radio frequency quadrupole (RFQ), which introduces a betatron tune spread across the bunch such that the bunch tail experiences a larger betatron focusing than the bunch head [6]. Fig. (3) shows that the BBU-like instability is stabilized when a small BNS tune spread is applied to the beam. One should note that, the BNS damping works for the ring only when the potential-well distortion is compensated by rf phase offset, such that the bunch shape remains approximately stationary. This is because the amount of BNS tune spread obtained from the prescribed formula $\triangle\nu_\beta(z)/\nu_\beta = \Upsilon(z)/\pi\nu_\beta$, involves the bunch's density profile. To maintain the correct BNS detuning condition, the bunch shape should not seriously deviate from its initial state; otherwise, one needs to adjust the BNS tune spread accordingly. Investigations are underway to determine if such an rf quadrupole is feasible. In addition, the transverse chromaticity, which causes betatron tune spread, may provide some Landau damping of the instability.

5 CONCLUSIONS

Various single bunch collective effects have been examined. The longitudinal microwave instability is, at present, seen to be the greatest threat to maintaining the bunch length. Operation at $\eta_1 = 10^{-7}$ is being considered, along with ideas for compensating the energy variation induced by the longitudinal wake. The transverse strong head-tail instability with the small η is seen to be BBU-like and can be stabilized by BNS damping. Other instabilities are not believed to be severe over the short storage times.

Acknowledgment

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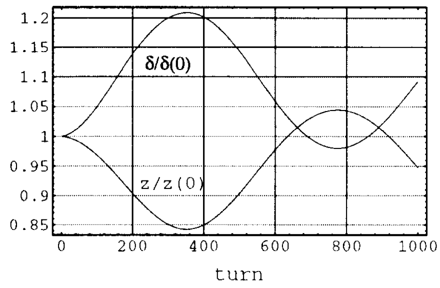

Figure 1: Scaled rms bunch size and rms energy-spread vs. turn, where at injection: $z(0) = 3$mm, $\delta(0) = 0.15\%$. Note that, $\eta_1 = 10^{-6}$ and $\eta_2 = \eta_3 = 0$.

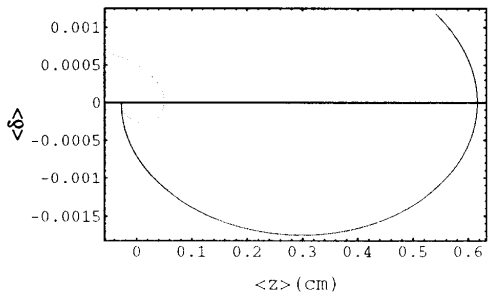

Figure 2: Center of energy-spread vs. center of bunch size. The rf phase offsets are, $\phi = 0, 0.082$ radian, for the solid line and dashed line, respectively. Note that, $\eta_1 = 10^{-6}$ and $\eta_2 = \eta_3 = 0$.

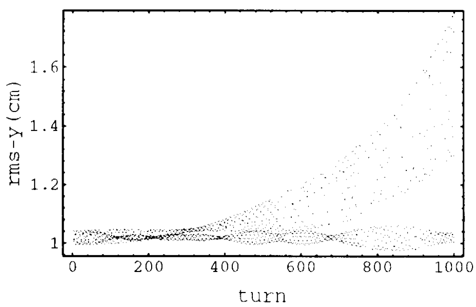

Figure 3: Blows up of the rms beam size, due to the BBU-like effect, where $1/\nu_s = 1784$. Note that $R_s/Q = 18225(\Omega)$, $b_{cavity} = 1.3$cm for the resonator model, injection error $\Delta y = 0.2$cm, and $\Upsilon(1\sigma_z) \simeq 0.017$. After BNS damping is applied, $\triangle\nu_\beta(1\sigma_z)/\nu_\beta \simeq 6 \times 10^{-5}$, the beam size fluctuates only slightly around 1 cm, a nominal injection beam size.