Design Study of Quasi-periodic Undulator

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Abstract

A new type of undulator, the quasi-periodic undulator (QPU) is considered, which generates irrational harmonics in the radiation spectrum. This undulator consists of an array of magnet blocks aligned in a quasi-periodic way, and consequently leads to a quasi-periodic motion of electron passing through QPU. The radiation spectrum by the electron of oscillating quasi-periodically contains no rational harmonics but irrational ones. A combination of the QPU and a conventional crystal/grating monochromator can provide pure monochromatic photon beam for synchrotron radiation experiments because the irrational harmonics are not diffracted by a monochromator in the same direction.

1 INTRODUCTION

Conventional undulators which have ever been built have a periodic magnetic structure so that the relativistic electrons have sinusoidal orbit in the device to emit the radiation with sharply peaked spectrum. The motion of electrons in the undulator leads to a generation of rational harmonics of radiation such as the third and fifth harmonics in addition to the fundamental radiation unless the reflection parameter K is very small.

For many user's experiments, monochromatic radiation from undulator is very useful, but a mixing of the higher harmonics is not welcome because it causes an increment of noise. The higher harmonics of radiation are usually removed by using a total reflection mirror which absorbs the radiation above the critical energy at a given reflection angle. The other way to remove the higher harmonics is to detune a double crystal monochromator so as to utilize the wider Darwin width of the fundamental reflection. However, these conventional techniques are somewhat difficult to be adopted in the hard x-ray region because of a small critical angle of total reflection or a very narrow Darwin width even for the fundamental radiation.

Recently, an undulator which suppresses higher harmonics by introducing intentional phase error in periodic structure or by adding a horizontal magnetic field has been proposed in order to avoid the higher harmonic contamination [1, 2, 3, 4]. From a different viewpoint, we propose a new type of undulator which never generates the rational higher harmonics [5, 6].

The idea came up from an analogy between an equation of x-ray scattering by matter and that of synchrotron radiation spectrum. The intensity of x-rays diffracted by a one-dimensional scatterer is [7]

$$I\left(q\right)=\left|\int_{-\infty}^{\infty}\rho\left(r\right)e^{-2\pi iqr}dr\right|^{2}, \tag{1}$$

where $\rho\left(r\right)$ is the electron density of scatterer. On the other hand, the spectral angular intensity distribution of synchrotron radiation is [8]

$$\frac{d^2I(f)}{dfd\Omega}=\left|\int_{-\infty}^{\infty}F\left(t\right)e^{2\pi ift}dt\right|^{2}, \tag{2}$$

$$F\left(t\right)=\frac{\vec{n}\times\left[\left\{\vec{n}-\dot{\vec{r}}\left(t\right)\right\}\times\ddot{\vec{r}}\left(t\right)\right]}{\left[1-\dot{r}_n(t)\right]^2}e^{-2\pi ifr_n(t)/c}, \tag{3}$$

$$r_n\left(t\right)=\vec{n}\cdot\vec{r}(t)\ . \tag{4}$$

Here, we neglect the coefficient for simplicity.

As we see above, Eqs. (1) and (2) have the same form. Therefore, we realize that we can treat the radiation from a quasi-periodic undulator in the same manner as the diffraction from a one-dimensional quasi-crystal.

2 CREATION OF QUASI-PERIODICITY

One of the ways for creating a one-dimensional quasi-periodic lattice is to project the two-dimensional periodic lattice points in a window onto a line inclined with an irrational gradient against the two-dimensional lattice axis. Figure 1 shows an example of the two-dimensional square lattice with the nearest neighbor distance of a, in which open and full circles refer to the scattering centers of positive and negative contributions for x-ray or electron, respectively.

In order to produce the quasi-periodic array of scattering centers, we next draw a window AA'BB' inclined with a slope of $\tan\alpha$ as shown in Fig. 1. In this example, we selected an irrational inclination of $\tan\alpha=1/\sqrt{5}$. Then, we project the lattice points contained in the window onto the inclined axis AA' (hereafter we refer to this axis as $R^{\|}$). From Fig. 1, we find intuitively that the points projected onto $R^{\|}$ have coordinates of $\left(R_j^{\|},0\right)$'s with two kinds of inter-site distances, $d=a\sin\alpha$ and $d'=a\cos\alpha$ having a ratio of $d'/d=1/\tan\alpha$. The points are aligned in an aperiodic fashion defined by [9]

$$R_j^{\|}=ja\cos\alpha+a\left(\sin\alpha-\cos\alpha\right)\left[\frac{j\tan\alpha}{1+\tan\alpha}+1\right], \tag{5}$$

where the symbol [ ] represents the greatest integer operator.

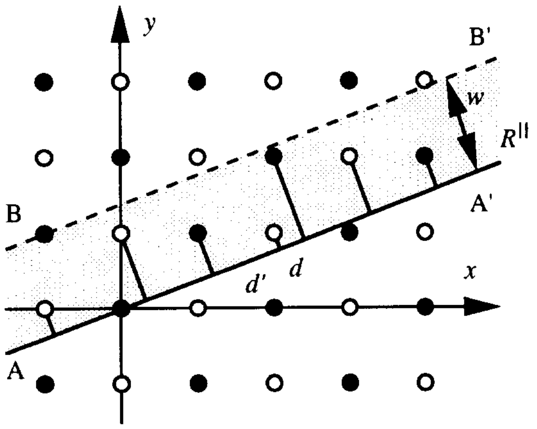

Figure 1: Creation of one-dimensional quasi-periodic lattice from a two-dimensional square lattice consists of positive and negative scattering centers. A quasi-periodic one-dimensional lattice is created on the AA' line.

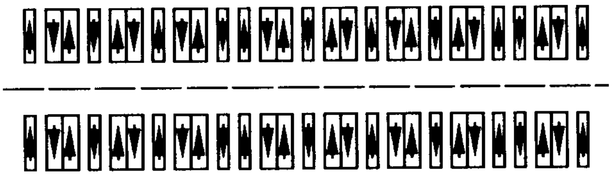

Figure 2: Magnetic structure of QPU. An arrow in each magnet block represents the direction of magnetization. The magnets in a row are aligned in a quasi-periodic order.

As we realize from Eqs. (1) and (2), the intensity in each equation is given by the square of the Fourier transform of the function $\rho\,(r)$ or $F\,(t)$. Therefore, the intensity distribution in q-space (reciprocal space) or in f-space (frequency space) is determined in a simple way by assuming the positions of $+/-$ scatterers described in Eq. (5).

3 REAL MAGNET CONFIGURATION OF QPU

Figure 2 shows the QPU magnet structure which we have designed as a prototype. Arrows in the magnet blocks represent the directions of magnetization. A magnet block isolated from its neighbors is thinner by a factor 0.7 to reduce the strength of the on-axis magnetic field to the same magnitude as the other non-isolated regular magnet blocks. The magnets at the left end and those at the right end are thinner by a factor of 0.35 for the purpose of the end correction. In this example, we chose $\sqrt{5}$ for an irrational number (and consequently $d'/d = \sqrt{5}$) in order to avoid including an accidental rational harmonic in the spectrum. The order of magnet arrangement was determined by Eq. (5).

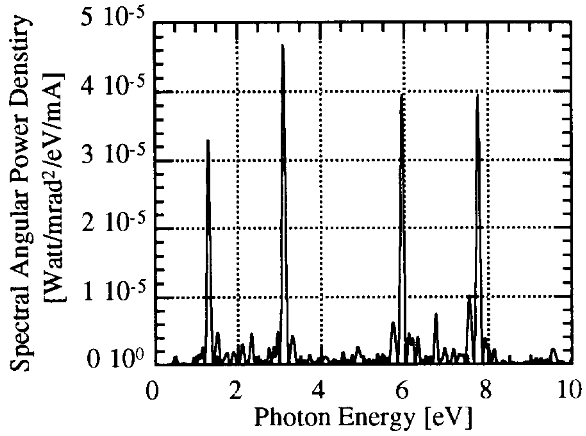

Figure 3: Spectral angular power density from QPU. N_{pole} = 27, gap = 36 mm, electron beam = 200 MeV, 1 mA.

4 SPECTRAL RADIATION FROM QPU

We calculated the spectral angular power density emitted from the QPU in Fig. 2 by using Eq. (2). The number of magnetic poles was set to be 27 including both end-magnets and the total length about 1.2 m. The inter-pole distances d and d' were 25.0 mm and 55.9 mm, respectively. Figure 3 shows the radiation spectrum at the gap 36 mm.

To prove the practical validity of the QPU, we installed the undulator in the electron storage ring NIJI-IV at ETL. The radiation spectrum from the QPU is observed and the experimental result is reported in another article [10].

5 DISCUSSIONS

We proposed a very different idea from the conventional undulator to suppress the rational harmonics. By analogy with diffraction from a quasi-crystal, the quasi-periodic array of magnets in an undulator were supposed to be capable of generating irrational higher harmonics, *i.e.* suppressing the rational harmonics.

We calculated the radiation spectra from QPU by using Eq. (2) in case of $d'/d = \sqrt{5}$ with realistic magnet sizes and structures. The calculation was successful to prove the basic idea of QPU. However, the analytic investigation of basic equations gives rise to the spectral formula giving the peak intensity and the resonant frequency [11].

We inserted spacers between the magnetic segments to produce longer inter magnet distance. These spacers do not generate the magnetic field, *i.e.* do not contribute to emit the radiation. This means that, if the total length of the device is the same with the conventional one, the QPU is less effective in regard to the total radiation power. This disadvantage may be recovered to some extent by changing the inclination, $\tan\alpha$, to some other irrational value.

An advantage of QPU is of course the generation of irrational harmonics in spectrum. With a combination of crystal/grating monochromator, we will be able to use purely monochromatic radiation in a wide energy range since any higher harmonic radiation from QPU is not contaminated by rational harmonics if $\tan\alpha$ is properly chosen.

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