

# CYCLIC ION ACCELERATOR WITH BENDING AND FOCUSING BY ELECTROSTATIC FIELD

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At present cyclotron type accelerators are widely applied to heavy ions acceleration. Particles bending is made by magnetic field perpendicular to median plane. Ion orbits pass inside accelerating dees electrodes. The development and distribution of such accelerators is constrained, mainly, availability large-sized, heavy, power-intensive and expensive magnetic system. For example, at proton energy 12-16 MeV magnet weight makes about 200 t. This report is devoted to creation practical opportunity discussion of ion deviation and focusing electrostatic system in a cyclic accelerator.

The cyclic accelerator scheme with electrostatic deflector is shown on fig. 1.

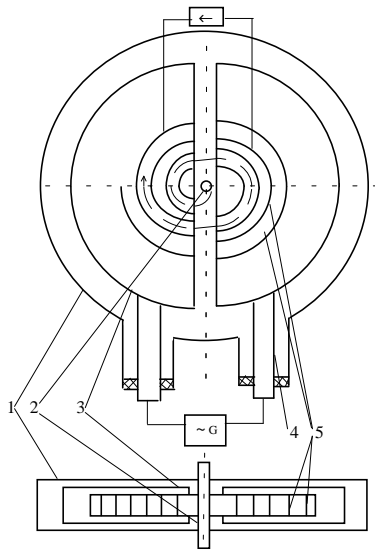


Fig.1. The Accelerator Scheme.

This deflector is made in a kind of flat capacity plates, located along ion beam trajectory inside dees. Figure 1 designates vacuum chamber, 2 - ion source, 3 - one of RF accelerating dees electrodes, connected by means of the rod 4 with RF generator output[1].

Two dees are shown on fig. 1, though the electrodes number can be more. It depends on accelerator purpose and chosen working mode. Deflector electrodes 5 are connected with DC source poles, so the potential difference between them creates centrifugal force for ions. The arrows show ion beam motion direction.

The particle acceleration is carried out as in usual cyclotron. The main difficulty consists of determination

of electrodes shape, which permits to give possibility to keep revolution constant frequency, necessary for of RF field constant frequency mode, on the one hand, and betatron oscillations stability (radial and vertical focusing), on the other hand [1].

Revolution radius of nonrelativistic particle with charge  $Z$  and energy  $W$  is defined by expression :

$$r = \frac{2W}{ZE} \quad (1)$$

where  $E$  - electrostatic field strength.

Provided this condition the centrifugal force on radius  $r$  is counterbalanced by Lorentz force. Thus, at RF field constant frequency, the electrostatic field tension  $E$  should be increased with particle energy and revolution radius  $r$  growth proportionally to  $\sqrt{W}$ .

The electrodes system, offered by the authors for ions bending and focusing, is shown on fig. 2. The electrostatic field structure in such system provides dipole component, providing particles bending, and quadrupole component, providing both vertical and radial focusing[2]. Alternating-sign focusing is reached by alternation of electrodes 2,a and 2,b along beam orbit. These electrodes are bent on a spiral pursuant to the equilibrium orbit. The dependencies of deflector electrodes internal and external radii on cyclotron parameters are submitted in paper[1].

Now we shall discuss betatron oscillations research in a offered focusing system. We shall consider vertical particles oscillations in the direction  $y$ , that is perpendicular to the equilibrium orbit plane. The electrical field on radius  $r + x$  can be submitted in a kind :

$$E_r = E_0 + x \frac{\partial E_0}{\partial r} \quad (2)$$

where  $E_0 = E_r$  for  $y=0$  and  $r=r_0$ .

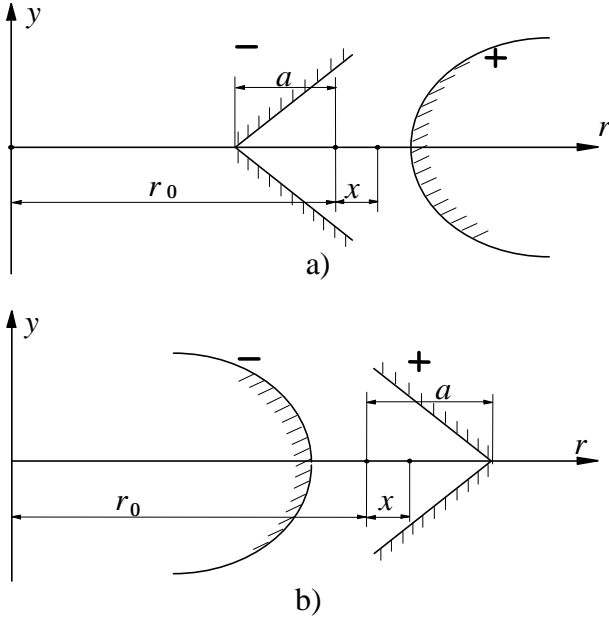


Fig.2. The Electrodes System.

Entering an index of electrical field decrement :

$$n = -\frac{r_0}{E_0} \left( \frac{\partial E}{\partial r} \right) \Big|_{x=y=0},$$

expression (2) can be rewritten in the kind :

$$E_r = E_0 \left( 1 - \frac{nx}{r_0} \right) \quad (3)$$

To determine electrical field components E close equilibrium orbit we shall use Maxwell equation  $\text{div } \mathbf{E} = 0$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{\partial}{\partial y} E_y = 0 \quad (4)$$

Pursuant to expressions (1) and (2) we have:

$$E_y = E_0 (n-1) \frac{y}{r_0} \quad (5)$$

We shall write equation of vertical oscillations in a following kind:

$$ZE_y = -m_0 \frac{d^2 y}{dt^2} \quad (6)$$

where  $m_0$  - ion rest mass.

Passing to independent variable  $s$  - particle coordinate along the equilibrium trajectory, as well as taking into account stability condition of particle motion on a circle in a radial field with velocity  $V_0$  :

$$ZE_0 = \frac{m V_0^2}{r_0} \quad (7)$$

we shall receive :

$$\frac{d^2 y}{ds^2} = (1-n) \frac{y}{r^2} \quad (8)$$

where  $s = V_0 t$ . Then the matrix of focusing gap in direction  $y$  has the kind :

$$M_F = \begin{vmatrix} \cos \sqrt{n-1} \alpha & \frac{r_0}{\sqrt{n-1}} \sin \sqrt{n-1} \alpha \\ \frac{\sqrt{n-1}}{r_0} \sin \sqrt{n-1} \alpha & \cos \sqrt{n-1} \alpha \end{vmatrix} \quad (9)$$

Where  $\alpha_0$  - angular extent of the gap,  $\alpha = S_f / r_0$ ,  $S_f$  - extent of the gap along coordinate  $s$ . The matrix of the defocusing gap will have a following kind:

$$M_D = \begin{vmatrix} \text{ch} \sqrt{1-n} \alpha & \frac{r_0}{\sqrt{1-n}} \text{sh} \sqrt{1-n} \alpha \\ \frac{\sqrt{1-n}}{r_0} \text{sh} \sqrt{1-n} \alpha & \text{ch} \sqrt{1-n} \alpha \end{vmatrix} \quad (10)$$

We shall proceed to calculation of particles radial motion. Particle kinetic energy varies in electrostatic field at radial deflection. The ion velocity on radius  $r_0 + x$  is defined by expression :

$$V_x^2 = V_0^2 \left( 1 - 2 \frac{x}{r_0} \right) \quad (11)$$

At conclusion of this equality we have taken into account, that on radius  $r_0 + x$  particle potential energy larger, than on radius  $r$ , on the value  $ZE_{\rho x}$  and, accordingly, kinetic energy smaller on the same value  $ZE_{\rho x}$ .

The equation of radial oscillations in linear approximation will be recorded in a following kind:

$$\frac{1}{r_0^2} m_0 \frac{d^2 r}{dt^2} = \frac{m V_x^2}{r_0 + x} - ZE_r \quad (12)$$

After transformations, taking into account of the formula (2), (3) and (11), we shall receive equation:

$$\frac{d^2 x}{ds^2} = -x(3-n) \frac{1}{r_0^2} \quad (13)$$

From equations (8) and (13) follows, that simultaneous vertical and radial focusing is possible at  $1 < n < 3$ . The limiting case  $n = 1$  corresponds to the system of coaxial cylinders, at which focusing in a vertical direction is absent.  $n = 2$  corresponds to concentric spherical surfaces, at which weak focusing in vertical and radial directions is provided [3]. For electrodes, shown on fig. 2, it is possible to make field quadrupole structure with large field decrement index, which value is defined by relation  $n = \pm r_0/a$ . We shall write matrixes of focusing and defocusing gaps for radial motion:

$$N_F = \begin{vmatrix} \cos \sqrt{3-n} \alpha & \frac{r_0}{\sqrt{3-n}} \sin \sqrt{3-n} \alpha \\ \frac{\sqrt{3-n}}{r_0} \sin \sqrt{3-n} \alpha & \cos \sqrt{3-n} \alpha \end{vmatrix} \quad (14)$$

$$N_D = \begin{vmatrix} \operatorname{ch}\sqrt{n-3}\alpha & \frac{r_0}{\sqrt{n-3}}\operatorname{sh}\sqrt{n-3}\alpha \\ \frac{\sqrt{n-3}}{r_0}\operatorname{sh}\sqrt{n-3}\alpha & \operatorname{ch}\sqrt{n-3}\alpha \end{vmatrix} \quad (15)$$

We shall consider the most simple structure of focusing period FD, the resulting matrix of which has the kind  $M = M_F N_D$ . We believe that the values  $n$  are identical on focusing and defocusing gaps. We shall write diagonal elements of resulting matrix :

$$\begin{aligned} a_{11} &= \cos\sqrt{n-1}\alpha \operatorname{ch}\sqrt{n-3}\alpha + \frac{\sqrt{n-3}}{\sqrt{n-3}} \sin\sqrt{n-1}\alpha \operatorname{sh}\sqrt{n-3}\alpha \\ a_{22} &= \cos\sqrt{n-1}\alpha \operatorname{ch}\sqrt{n-3}\alpha - \frac{\sqrt{n-1}}{\sqrt{n-3}} \sin\sqrt{n-1}\alpha \operatorname{sh}\sqrt{n-3}\alpha \end{aligned} \quad (16)$$

Betatron oscillation stability condition is expressed as follows:

$$\operatorname{Sp}M = \frac{a_{11} + a_{22}}{2} < 1$$

We shall suppose  $n \gg 1$ . This condition is simply ensured at the significant radius  $r_0$ . Then

$$\operatorname{Sp}M \cong \cos\sqrt{n}\alpha \operatorname{ch}\sqrt{n}\alpha \quad (17)$$

The stability areas are close to points  $\pi / 2 \cdot i$ , where  $i=1,3,5,\dots$ . First, the widest area, corresponds to the case :  $\sqrt{n}\alpha = \pi / 2$ .

We shall make estimation of accelerator parameters. We shall take the value  $n = 100$ . Then focusing system element angular extent  $\alpha = 9^\circ$ . At trajectory radius  $r_0 = 100 \text{ cm}$  the element linear extent along coordinate  $s$  is equal  $15 \text{ cm}$ . Then the value  $a = \frac{r_0}{n}$  corresponds  $1 \text{ cm}$ .

One of important parameters is a spiral step of particle trajectory. We shall suppose spiral step  $\Delta r = 4 a = 4 \text{ cm}$ . At voltage between dees  $U = 100 \text{ kV}$  the proton energy on radius  $r$  is defined by the formula [4]:

$$W = \frac{U r_0}{\Delta r} = 2.5 \text{ MeV} \quad (18)$$

Field tension, ensuring circular trajectory with radius  $r = 100 \text{ cm}$  at proton energy  $W = 2.5 \text{ MeV}$ , can be got from expression (7)

$$E_0 = \frac{2W}{r_0} = 50 \text{ kV} / \text{cm}$$

Electrical field gradient  $G = E_0 / a = 50 \text{ kV} / \text{cm}^2$ .

We shall define potential difference between electrodes  $V$ , supposing that the distance between them  $A = 2 a$  :

$$V = \int_0^{2a} E_r dx = 2G a = 100 \text{ kV} \cdot$$

Thus, it is shown, that the considered cyclotron model without magnet has practically feasible parameters for ion energy up to  $10 \text{ MeV}$ . At higher energies the beam orbit output radius and, accordingly, all installation size,

become excessively large, that, in turn, results in growth of RF generator power.

It is known, that in classical cyclotron the particle revolution frequency in magnetic field does not depend on particle energy (for nonrelativistic particles). In this connection self-phasing mechanism does not act at RF field constant frequency in cyclotron. For offered cyclotron without magnet and with electrostatic deflection self-phasing mechanism takes place. Really, we shall rewrite forces balance for a equilibrium particle (6) in the kind :

$$m_0 \omega r_0 = ZE \quad (19)$$

where  $\omega$  - circular frequency of particle revolution, equal to RF generator frequency. The particles, moving on trajectory, close to equilibrium (in the same average field  $E$ ) and having energy larger, than equilibrium particle, will rotate with frequency smaller, than the RF field frequency, and on the contrary, for particles with smaller energy revolution frequency will be higher, than RF field frequency. Thus, equilibrium particle stable longitudinal oscillations are supplied on the decreasing part of RF voltage.

According to preliminary estimations, supposed construction can be successfully realized for accelerators on small and average energies with considerably smaller financial costs, than classical cyclotron. At the same time, it can be successfully used for modernization of working cyclotrons to decrease energy consumption of magnet, which is very large.

## REFERENCES

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