Improving Regular Acceleration in the Nonlinear Interaction of Particles and Waves*

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Abstract

In this work we study the effects arising from the inclusion of a stationary extraordinary mode in the resonant interaction of a particle beam in a magnetized plasma and perpendicularly propagating electrostatic waves. It is found that for a stationary mode frequency of the order of the Larmor frequency and with a suitably chosen amplitude one is able to suppress the resonance which drives the weakly relativistic dynamics into chaos. Improved regular acceleration of initially low energy particles is thus attained. Analytical estimates of the optimal stationary mode amplitude and a study of the topological effects due to resonance suppression are presented. Main results are verifyed by numerical simulations.

1 INTRODUCTION

With the advent of powerful radiation-generation systems such as free-electron lasers, cyclotrons autoresonance masers, gyrotrons and ion-channel lasers, a good deal of effort has been directed to the study of the interaction of low energy particles and large-amplitude waves [1-5]. Whenever wave-particle exchange is likely to occur, particles can be highly accelerated, which is of importance in particle acceleration and in current drive techniques of controlled thermonuclear research.

In this paper we study how can one improves the regular acceleration in the resonant interaction of magnetized particles and a tranversal electrostatic wave. We analyse the introduction of a stationary extraordinary mode. The main idea is to generate a resonance that destructively interferes with the wave-particle resonance that drives initially low energetic particles into chaos. It is shown that for a judicious choice of the stationary mode amplitude the dynamics of these particles can undergo strong modifications, varying from completely diffuse to regular with highly increased acceleration. Numerical results obtained by direct integration of the equation of motion are shown in order to test the validity of the method.

2 MODEL

Consider a relativistic electron beam immersed in a low density, cold, magnetized plasma, with background magnetic field given by $\mathbf{B}_0 = B_0\hat{z}$, perturbed both by a transversal electrostatic wave and a stationary extraordinary mode. The vector potential related to $\mathbf{B}_0$ is written as $\mathbf{A}_0 = B_0\, x\, \hat{y}$.

The electrostatic wave has an amplitude A_w and propagates in the x-direction with wave vector k and frequency ω_h. It is assumed to be a magnetized Langmuir wave with Debye length sufficiently small that one can consider the frequency to be independent of the wave vector. Assuming $\omega_p \ll \omega_{c0}$, we have for the wave frequency $\omega_h^2 = \omega_{c0}^2 + \omega_p^2 \approx \omega_{c0}^2$, with ω_p as the plasma frequency and $\omega_{c0} \equiv |e|B_0/mc$ as the nonrelativistic electron cyclotron frequency.

The stationary extraordinary mode have frequency ω_X and wave vector $k_X \parallel \hat{x}$ related to each other by the cold dispersion relation. Let us choose the frequency in order to satisfy the following relation, $\omega_X^2 - \omega_h^2 \approx \omega_p^2$, such that it is near the right-hand cut-off frequency. If this is the case, the mode is approximatelly circularly polarized, and the relation $ck_X/\omega_X \approx \omega_p/\omega_{c0} \ll 1$ is valid. Taking into account the above relation and the fact that we will be interested in the low energy particles we can safely assume $k_X r_L \ll 1$ (with r_L as the particle Larmor radius), which enables one to write $\mathbf{A} \equiv \mathbf{A}_0 + \mathbf{A}_X$ as

$$\frac{e\mathbf{A}}{mc^2} = B_0 x\,\{-\varepsilon\sin(\omega_X t)\hat{x} + [1+\varepsilon\cos(\omega_X t)]\hat{y}\},$$

where $\varepsilon \equiv k_X E/\omega_X B_0$, with E the eletric field amplitude.

Introducing canonical guiding-center variables ($P_x = \sqrt{2I}\cos\theta$, $x = \sqrt{2I}\sin\theta$) and scaling time and distance to ω_{c0} and ω_{c0}/c, the dimensionless particle Hamiltonian is given by

$$\begin{aligned} H &= \{1 + 2I + 4\varepsilon I[\sin^2\theta\cos(\omega_X t) \\ &- \cos\theta\sin\theta\sin(\omega_X t)]\} \qquad (1) \\ &+ A_w \sum_{n=-\infty}^{+\infty} J_n(k\sqrt{2I}) cos(n\theta - \omega_h t), \end{aligned}$$

where the term proportional to ε^2 is discarded and, as we are considering particles with very low initial energies, we set $P_z = 0$ and, for simplicity, $P_y = 0$. Since the differences between the frequencies involved in the above system, ω_{c0}, ω_X and ω_h, are all of the order of $\omega_p^2 \ll 1$, it will be assumed in the following that $\omega_h = \omega_X = \omega_{c0}$ (or in adimensional form $\omega_h = \omega_X = 1$).

3 ANALYSIS OF THE RESONANCES

A. Pendulum-Like Electrostatic Resonances

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Let us by now focus on the perturbations of the particle motion due to the electrostatic wave. The appearance of primary wave-particle resonances is related to each of the harmonics in the last term of the Hamiltonian (1). Their location in phase space can be estimated from $d_t(n\theta - t) \sim 0$. This leads to an approximate expression, valid to zero order, for the action at the n^{th} resonance

$$I_n = (n^2-1)/2, \qquad n \geq 2. \tag{2}$$

For $n = 1$ the above relation is no longer valid because for small I, terms proportional to A_w cannot be disregarded in $d_t\theta$ [4,5], leading to several changes in the particles motion. This case will be treated in detail in the next sub-section.

These resonances are of the pendulum type, presenting n hyperbolic fixed points and n elliptic fixed points appearing for $n\theta = 2m\pi$, with $m \leq n$ an integer. The maximum action excursion around I_n for particles trapped in the resonance is given by

$$\Delta I_n^{pend} = 2\sqrt{F/G} = 2\sqrt{n^3\, A_w\, J_n(k\sqrt{2I_n})}. \tag{3}$$

B. Non-Pendulum-Like Electrostatic Resonance

As quoted before, in the low energy case we cannot neglect wave terms even in zero order calculation and particle trajectories significantly differ from a pendular one. To see this we analyze the Hamiltonian (1) disregarding the extraordinary mode and taking into account the leading contributions for $I \lesssim 1$. The important term in the summation (1) is the one with $n = 1$. Performing a time-removal canonical transformation - $\theta - t \to \theta$, $I \to I$ and $H \to h^{(1)} = H - I$ - the Hamiltonian assumes the form

$$h^{(1)}(I,\theta) = \sqrt{1+2I} - I + A_w\, J_1(k\sqrt{2I})\, \cos\theta. \tag{4}$$

This Hamiltonian has been extensively studied in the limit $I \ll 1$ in Ref. 7. Trajectories described by (4) may be either trapped or untrapped. Trapped ones present a triangular shape instead of the typical pendulum-like one.

Maximum action excursion, I_1^{max}, for particles trapped in this resonance is given by

$$\sqrt{1+2I_1^{max}} - I_1^{max} + A_w\, J_1(k\sqrt{2I_1^{max}}) = 1, \tag{5}$$

which gives the maximum I value on the boundary (wich has finite rotating frequency). Considering a resonance overlapping criterium, one finds that the threshold amplitude for a $n = 1$ and $n = 2$ overlapping is $A_{w,th} = 0.135$ for $k = 1$. However, the introduction of a stationary extraordinary mode reduces second island amplitude, preventing the premature overlap and improving the regular energization of particles.

C. Stationary Mode Resonances

In order to study the resonances caused by the stationary mode, one takes $A_w = 0$ and expand the Hamiltonian (1) for small ε. Considering only first order terms one realizes that the only perturbing term that resonantly interact with the particles is the one containing the harmonic $2\theta - t$. This stationary wave-particle resonance is a pendulum-like one.

4 RESONANCE SUPPRESSION

Let us begin by analyzing the optimal stationary mode amplitude, ε_{op}, in order to suppress the second electrostatic wave-particle resonance. Comparing the perturbation amplitudes of the electrostatic wave and the stationary mode, for $I = I_2$, we can obtain an approximate value for ε_{op} as

$$\varepsilon_{op} = \frac{A_w\sqrt{1+2I_2}}{I_2} J_2(k\sqrt{2I_2}). \tag{6}$$

Although at first glance the resonance suppression, as it is presented here, seems to lead to a complete cut out of the resonance, it actually leads to much more involved effects to be discussed next.

To better understand the effects of the resonance suppression, one can analyze the dynamics of the particles near the second resonance by studying the *dynamics of the fixed points* [6] of the island as ε is varied. In order to do so, let one write a $pendulum-like$ Hamiltonian , now taking into account the influence of the stationary mode and also keeping linear terms of $\hat{I} = I - I_2$ in the perturbation. The importance of the inclusion of linear terms in $\hat{I}$ will be apparent. The Hamiltonian takes the form

$$\begin{aligned} h^{(2)}(\hat{I},\theta) &= (G/2)\,\hat{I}^2 + [A_w(\alpha_0 + \alpha_1\,\hat{I}) \\ &- \varepsilon(\beta_0 + \beta_1\,\hat{I})]\cos(2\theta) \end{aligned} \tag{7}$$

where α_i and β_i are the coefficients of the Taylor expansion, around I_2, of $J_2(k\sqrt{2I})$ and of $I/\sqrt{1+2I}$, respectively. *Usual Fixed Points.* Usual pendulum-like fixed points (UFP) appear for $2\theta_{UFP} = m\pi$ and $\hat{I}_{UFP} = -s\,(A_w\alpha_1 - \varepsilon\beta_1)/G$, where $m = 1, 2$ and $s = \cos(m\pi) = \pm 1$. In order to analyze their stability, one calculates the matrix eigenvalues λ_{UFP} of the linearized motion around the UFP's. If the eigenvalues are real the surrounding orbits have an expanding direction and a contracting direction, thus the UFP is hyperbolic. Otherwise, the surrounding orbits circulate around the UFP which is therefore elliptic. It is found that unless the stationary mode amplitude is near the optimal one the stability of the UFP's is governed by the value of s and half of them are hyperbolic, half are elliptic. For ε small compared to A_w the elliptic ones are those for which $s = +1$, otherwise the $s = -1$ are stable.

If, on the other hand, ε is near its optimal value, such that the condition

$$|A_w\alpha_0 - \varepsilon\beta_0| < \left|(A_w\alpha_1 - \varepsilon\beta_1)^2/G\right| \tag{8}$$

is satisfied, λ_{UFP} is always imaginary and all the UFP's are of the elliptic type irrespective of s value.

Extra Fixed Points. A more detailed inspection of the equations of motion for $\hat{I}$ and θ, reveals that an extra set of fixed points, which shall be called EFP, may appear. If the condition (8) holds, a different set of real roots of the motion equations are found for $\hat{I}_{EFP} = -(A_w\alpha_0 - \varepsilon\beta_0)/(A_w\alpha_1 - \varepsilon\beta_1)$ and $2\theta_{EFP} = \cos^{-1}[G\,(A_w\alpha_0 - \varepsilon\beta_0)/(A_w\alpha_1 - \varepsilon\beta_1)^2]$. For increasing ε these points are

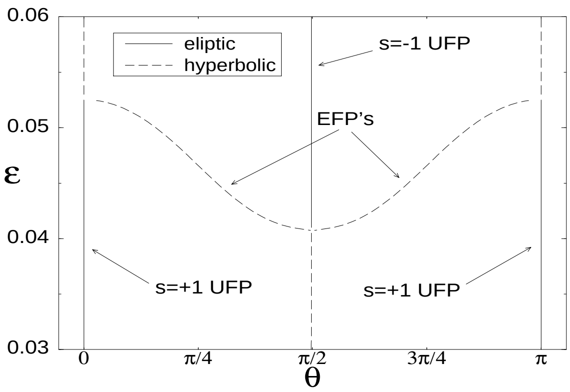

Figure 1: Fixed point dynamics as ε is varied.

initially located at the same position occupied by the unstable $s = -1$ UFP. Then they start migrating in the direction of the $s = +1$ UFP, colliding with them and being extinguished. Analyzing the EFP stability one finds that the matrix eigenvalues λ_{EFP} are always real during the $existence$ interval of the fixed points.

5 NUMERICAL VERIFICATIONS

Let us begin by analyzing the behavior of the second island as we introduce the stationary extraordinary mode. In order to do so we study a small amplitude case, where the structure of the islands is not too deformed. The dynamics of the fixed points is shown in detail in Fig. 1. By means of a Newton-Raphson algorithm [8] the dynamical periodic orbits are followed and their linear stability determined as one varies ε. Stable periodic orbits are represented by solid lines, while unstable ones by dotted lines. One can notice a great agreement between the fixed point dynamics presented in this figure and that described in Sec. 4. The resonance suppression interval (the interval of existence of the EFP's) is approximately $\varepsilon \in [0.041, 0.053]$ which is in good agreement with the predicted optimal value.

Now let us turn to the case of higher wave amplitude, where large acceleration of initially low energy particles is expected to occur. In Figs. 2(a) and 2(b), it is compared the Poincaré plots of a system without and with the stationary mode, respectively. The amplitude is $A_w = 0.4$. For $\varepsilon = 0$ (Fig. 2(a)) a completely chaotic phase-space is presented. All major stable fixed points of both the first and second resonant islands have already undergone infinite cascades of periodic doubling and are not present. In fact, no structure is apparent anymore. One can expect some relatively fast particle diffusion for this deep stochastic regime.

On the other hand, in Fig. 2(b), when the stationary mode is turned on with an optimal amplitude $\varepsilon = \varepsilon_{op} = 1.544 \times 10^{-1}$ the Poincaré plot is dramatically changed. Some stable fixed points of the first two islands are present again. The whole structure of the first non-pendulum island is restored, which leads to high regular acceleration of initially low energy particles.

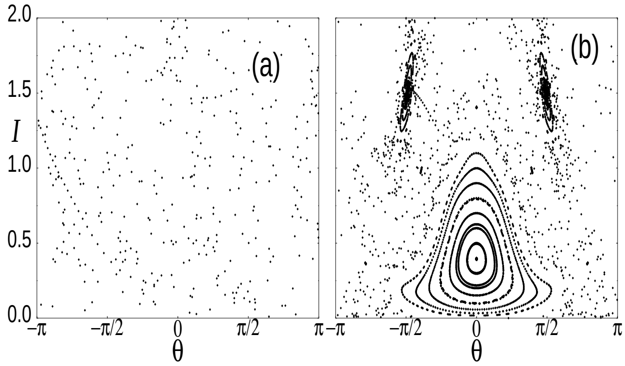

Figure 2: Poincare Plots for $\varepsilon = 0$ (a) and $\varepsilon = \varepsilon_{op}$ (b)

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