NONLINEAR BEAM DYNAMICS EFFECTS IN HEAVY ION TRANSPORT LINE

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ABSTRACT

Beam optics in the extraction system of the ECR ion source is examined both analytically and numerically, by taking nonlinear effects due to aberration of einzel lens and space charge forces of the beam into account. Simple criteria has been derived to estimate the significance of nonlinear effects on beam emittance distortion. Maximum transported beam current and value of acceptance of the channel were estimated.

1 INTRODUCTION

New RIKEN low energy beam transport line between 18GHz ECR ion source and RFQ linac [1] consists of einzel lens, bending magnet and solenoid coil (see fig. 1). Parameters of the structure are listed in Table 1. Beam dynamics study was done to analyze nonlinear effects of space charge and focusing field of einzel lens on beam parameters and to suggest possible ways to improve beam quality through the beam line.

Table 1. Parameters of beam transport line

Maximum ion magnetic rigidity	76 kGs cm
Extraction voltage	8....10 kV
Einzel lens voltage	10.....13 kV
Maximum magnetic field in dipole	1.6 kGs
Maximum magnetic field in solenoid	6.3 kGs

2 LINEAR BEAM OPTICS: MAXIMUM BEAM CURRENT AND ACCEPTANCE OF THE CHANNEL

Intensity of the transported beam in the considered beamline is limited by dipole gap and solenoid aperture which have the same value of R_{max} = 3.5 cm. Evolution of beam radius in drift space in the case of linear space charge forces is described by KV (Kapchinsky-Vladimirsky) equation

$$\frac{d^2R}{dz^2} - \frac{E^2}{R^3} - \frac{P^2}{R} = 0 \ , \qquad (1)$$

where E is a beam emittance; $P = \sqrt{2I/(I_o \beta^3 \gamma^3)}$ is a space charge parameter, I is a beam current and $I_o = 4\pi\varepsilon_o mc^3/q$ is a characteristic value of current. Equation (1) has an approximate solution for $R'_o = 0$:

$$\frac{R}{R_o} = 1 + 0.5\, z^2 \left(\frac{E^2}{R_o^4} + \frac{P^2}{R_o^2}\right) \ . \qquad (2)$$

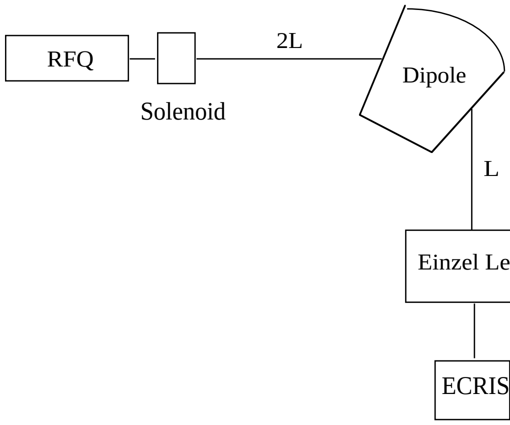

Fig. 1. Layout of extraction beamline of 18 GHz ECR ion source.

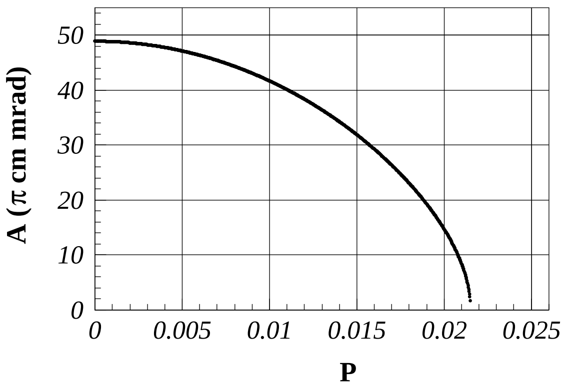

Fig. 2. Acceptance of the channel A as a function of space charge parameter of the beam $P = \sqrt{2I/(I_o \beta^3 \gamma^3)}$.

From symmetry point of view it is clear, that beam has a minimum size R_o (waist) in the middle point z=L=110 cm between dipole and solenoid. Suppose, the value of P is fixed. Differentiation $\partial E^2/\partial R_o = 0$ gives the following expressions for optimal relation $R_o(R_{max})$ and maximum emittance of the beam (acceptance of the channel A):

$$R_o = \frac{3}{8} R_{max} \left[1 + \sqrt{1 - \left(\frac{4P}{3}\frac{L}{R_{max}}\right)^2}\ \right] \ ; \qquad (3)$$

$$A = E_{max} = \frac{\sqrt{2}}{L} \sqrt{R_o^3 R_{max} - R_o^4 - 0.5(L R_o P)^2} \ . \qquad (4)$$

In fig. 2 the dependence of channel acceptance against space charge parameter of the beam P is represented. From eqs. (3), (4) it follows, that the maximum value of space charge parameter is P_{max} = 0.0215. Combining expression for parameter P and beam velocity β as a function of extraction voltage U_{ext} , the maximum transported beam current is

$$I_{max} = I_o \sqrt{2}\, P_{max}^2 \sqrt{\frac{Z}{A}}\, \left(\frac{e\,U_{ext}}{m_p c^2}\right)^{3/2} , \qquad (5)$$

where $m_p c^2/q$ =938.3 MV is a proton rest energy.

3 BEAM ABERRATIONS IN AXIAL-SYMMETRIC LENS

After being extracted from ECRIS, particles pass through the focusing lens. In thin lens approximation, the nonlinear transformation from old variables (x_o, x'_o) to new variables (x, x') after crossing the lens is described by focal length of the lens f as well as spherical aberration coefficient C [2]:

$$x = x_o \quad ; \qquad (6)$$

$$x' = x'_o - \left(1 + \frac{C}{f^3} x_o^2\right) \frac{x_o}{f} \quad . \qquad (7)$$

Suppose, initial (unperturbed) beam emittance has a value E and is described by ellipse:

$$\frac{x_o^2}{R^2} + \frac{x_o'^2}{E^2} R^2 = 1 \quad , \qquad (8)$$

where R is a beam radius. After lens the shape of beam emittance is deformed as follows:

$$\frac{x^2}{R^2} + \frac{R^2}{E^2} \left[\left(x' + \frac{x}{f}\right)^2 + 2\left(x' + \frac{x}{f}\right)\frac{C\,x^3}{f^4} + \frac{C^2 x^6}{f^8} \right] = 1 \quad . \qquad (9)$$

Let us introduce action-angle variables I, φ instead of x, x':

$$\frac{x}{R} = \sqrt{I} \cos\varphi ; \qquad \left(x' + \frac{x}{f}\right) \frac{R}{E} = \sqrt{I} \sin\varphi \quad . \qquad (10)$$

In new variables the shape of beam emittance at phase plane is as follow:

$$I + I^2\, 2\delta \sin\varphi \cos^3\varphi + I^3 \delta^2 \cos^6\varphi = 1 \; ; \qquad (11)$$

$$\delta = \frac{C\,R^4}{E\,f^4} \quad . \qquad (12)$$

Without nonlinear perturbation δ=0 equation (11) describes ellipse in phase space. If δ≠0, equation (11) describes S-shape figure of beam emittance, which is typical for nonlinear aberration. It is easy to verify by numerical integration , that phase space area, enclosed by the curve (11) , is conserved for any value of δ:

$$\text{Area} = \frac{E}{2\pi} \int_o^{2\pi} I(\varphi)\, d\varphi = E \quad . \qquad (13)$$

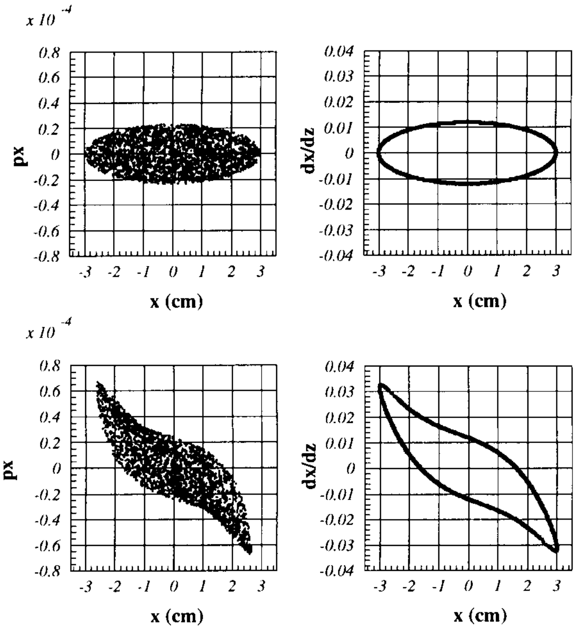

Fig. 3. Distortion of beam emittance shape due to aberration in einzel lens with parameter δ =1.5: numerical modeling (left) and calculated from formula (11).

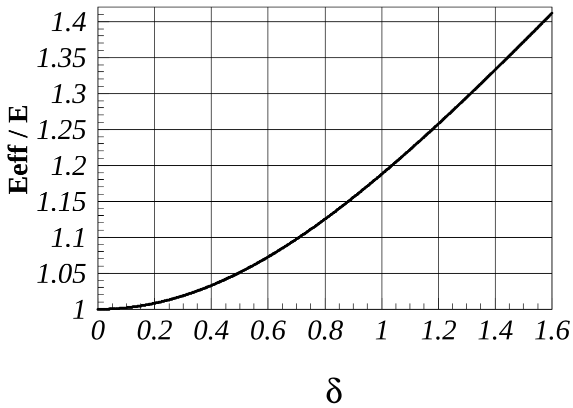

Fig. 4. Emittance growth due to aberration in einzel lens as a function of parameter δ, calculated from formulas (11), (14).

Nevertheless, the effective area, occupied by the beam, is increased. Let us denote the value of effective beam emittance as square of product of min and max values of action variable:

$$E_{eff} = \sqrt{I_{max}\, I_{min}} \quad . \qquad (14)$$

Values I_{max}, I_{min} are found numerically from eq. (11). At fig. 4 the dependence of emittance growth as a function of parameter δ is presented. Dependence can be approximated by parabola:

$$\frac{E_{eff}}{E} = 1 + K\,\delta^2 \quad , \qquad (15)$$

where parameter K≈0.16. Numerical calculations indicate that growth of the rms beam emittance

$$E_{rms} = 4 \sqrt{<x^2><x'^2> - <xx'>^2} \quad (16)$$

has the same dependence as δ^2 with coefficient K=0.08...0.5 for different beam distributions. Above analysis shows, that to prevent substantial emittance distortion, the parameter δ should be limited $\delta < 0.8$.

4 BEAM EMITTANCE GROWTH IN DRIFT SPACE

Beam emittance is affected by self nonlinear space charge forces as well. Let us consider a cylindrically-symmetric beam in drift space with initial Gaussian distribution:

$$\rho\,(r_o) = \frac{2I}{\pi R^2 \beta c} \exp(-2 \frac{r_o^2}{R^2}) \ . \quad (17)$$

For space charge dominated beam a single particle trajectory is described by equation [3]:

$$\frac{d^2 r}{dz^2} = \frac{2\,I}{I_o \beta^3 \gamma^3} \frac{f\,(r_o)}{r} \ , \quad (18)$$

where function $f(r_o)$ depends only on initial conditions:

$$f\,(r_o) = 1 - \exp(-2 \frac{r_o^2}{R^2}) \approx 2 \frac{r_o^2}{R^2} - 2\frac{r_o^4}{R^4} + ... \quad (19)$$

Taking only two lowest order terms in function $f(r_o)$, the approximate solution of the problem is given by

$$r = r_o + P^2 \frac{z^2 r_o}{R^2} (1 - \frac{r_o^2}{R^2}) + z\,(\frac{dr}{dz})_o \ . \quad (20)$$

From eq.(20) it follows, that radius of particle increases as z^2, while the slope of trajectory is proportional to z. At the initial stage of beam emittance growth we can assume , that particle radius is unchanged, while the slope of the trajectory is changed. It gives the nonlinear transformation, similar to (6), (7):

$$r = r_o \ ; \quad (21)$$

$$r' = r'_o + \frac{2zP^2}{R^2} r_o - \frac{2zP^2}{R^4} r_o^3 . \quad (22)$$

Transformation (21),(22) results in the same equation for distorted beam emittance shape as (11). The difference is that the axial-symmetric lens provides focusing, while space charge forces of the beam are always defocusing. It creates different orientation of distorted phase space ellipse at the phase plane. The parameter δ for nonlinear space charge problem is

$$\delta = 4 \frac{z}{E} \frac{I}{I_o \beta^3 \gamma^3} \ . \quad (23)$$

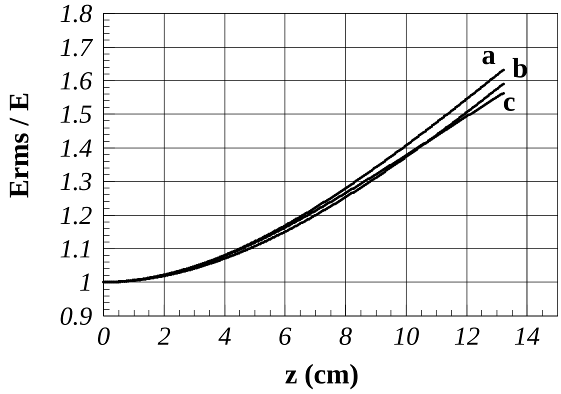

Fig. 5. Emittance growth of the beam with A/Z=5, β =1.9 10^{-3}, E= 1.5 π cm mrad for different values of initial beam radius: a) R = 2cm; b) R = 1cm; c) R = 0.5 cm.

Once the equation (11) is the same for both cases, the dependence of beam emittance growth rate on parameter δ should be the same. Numerical analysis shows, that for Gaussian beam initial growth of rms beam emittance can be approximated by formula:

$$\frac{E_{rms}}{E} = 1 + 0.014\,\delta^2 \ . \quad (24)$$

As follows from (23), (24), initial emittance growth does not depend on initial beam radius R. In fig. 5 results of beam emittance growth for different values of initial beam radius are presented. The quadratic dependence of beam emittance growth on distance z is clear while dependence on initial beam radius is negligible.

5 CONCLUSIONS

Nonlinear effects associated with beam emittance distortion in low energy beam transport line were analyzed. Two important phenomena: effect of spherical aberrations in axial -symmetric focusing lens and influence of nonlinear space charge forces on beam emittance growth were analyzed both analytically and numerically. Nonlinear mapping describes the distortion of beam emittance shape in phase plane. Simple analytical criteria has been derived to estimate importance of beam parameters as well as lens parameters on beam emittance growth.

6 REFERENCES

[1] N.Inabe et. al , "Design of a beam transport line between the 18GHz ECRIS and the RFQ linac", RIKEN Accelerator Progress Report 28 (1995), 166.

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[3] D.Bruhwiler, Y.Batygin, "Beam transport for uniform irradiation: nonlinear space charge forces and the effect of boundary conditions", Proc. of the 1995 Particle Accelerator Conference, Dallas, 3254.