Calculation of the Luminosity Spectrum and the Differential Luminosity

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Abstract

The paper presents a computer code for solving the Luminosity Spectrum and the Differential Luminosity. Instead of using the particles-in-cell method, the code is based on analytical solutions and formulae. The user creates an input data file, which contains only a few parameters, such as the beam size, the number of particles and the center-of-mass energy. The beam, with a round or elliptical cross-section, is described by a uniform or a Gaussian density function in transverse and longitudinal direction. In all cases the program makes use of the analytical solutions for the energy loss, the deflection angle, the Luminosity Spectrum and the Differential Luminosity. The results for several Linear Collider designs and different disruption numbers are obtained and compared with other numerical solutions. Numerical examples demonstrate cleary the accuracy of this method.

1 INTRODUCTION

The Luminosity $\mathcal{L}$ in Linear Colliders during the interaction of e^+e^- beams is not constant. At the interaction point (IP) $\mathcal{L}$ is dependent on the time and will be different from $\mathcal{L}_0$.
There are two different kinds to analyse the change of Luminosity: In the first part we consider a spectral representation of the Luminosity which is investigated in dependence of the center-of-mass-energy for the colliding electron and positron beams. On the basis of the beamstrahlung effect the energy loss influences the bending of the trajectories. The whole center-of-mass-energy decreases during the collision and define an energy range of an undisturbed E_{cm0} to a center-of-mass-energy with maximum radiation loss $E_{cm} = E_{cm0} - \delta_{max}$. In this range we assign every energy value to a Luminosity number, this relation is called the Luminosity spectrum.
Instead of the spectrum the Luminosity is analysed in the timedomain, usually with the derivation $\frac{d\mathcal{L}}{dt} = f(t)$.

2 BEAM-BEAM RADIATION SPECTRUM

At the IP the bunch particles are under the influence of the electromagnetic (EM) fields provided by the oncoming beam, this effect gives rise to the bending of the particle trajectory which induces the radiation loss of the particle energy.
The spectral distribution of radiation from an electron circulating in an homogenous magnetic field, defined as synchrotron radiation, is described with an expression that was derived by Jackson [JAC83] and Schwinger [SCHWI49]. We applied this relations to the actual radius of curvature of the particles trajectory and found an expression for the beamstrahlungs spectrum

$$\frac{dI(\omega)}{d\omega} = 3\,\frac{e^2}{c}\,\frac{\omega}{\omega_c}\,\frac{\gamma_u \Delta z \left|\vec{F}_\perp\right|}{p_\parallel\; c\;\beta}\sqrt{\beta + \frac{1+\beta}{2}}\;\cdot$$
$$\cdot \int_{u^*}^{\infty} \left[2\frac{p}{u^*} - 1\right] A_i(p)dp \qquad (1)$$

with the critical frequency

$$\omega_c = 3\frac{\left|\vec{F}_\perp\right|}{cm_0\gamma_k\sqrt{\left(\frac{\rho\omega_k}{c}\right)}}\gamma_k^3\left(\frac{1+\frac{\rho\omega_k}{c}}{2}\right)^{3/2} \quad , \qquad (2)$$

for a allowable relative particle velocity from $\beta = \frac{|\vec{u}|}{c} \in [0.9\,,\,1.0]$ and a limit of integration

$$u^* = \left(3\frac{\omega}{\omega_c}\right)^{2/3} \quad . \qquad (3)$$

$p_\parallel$ in Eq.(1) is the longitudinal particle impuls, Δz is the length from the perfect circle section and $\vec{F}_\perp$ is the vertical beam-beam force component from the oncoming space-charge field. We apply our Eq.(1)-(4) to the SLC parameter set (Tab.1) and calculated the radiation spectra from an electron shortly after the entrance in a positron bunch (Fig.1). Eq.(1) was normalized to $\frac{dI(\omega)}{d\omega}\frac{c}{e^2} = f(\omega)$.

E_{cm}[GeV]	N	$\sigma_{x,y}$[m]	σ_z[m]
100	$5.0 \cdot 10^{10}$	$1.5 \cdot 10^{-6}$	$1.05 \cdot 10^{-3}$

Table 1: Set of SLC parameter

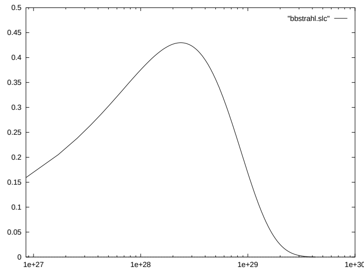

Figure 1: Radiation spectra from an electron shortly after the entrance in a positron bunch.

3 BEAM-BEAM LOSS

Knowledge of the beamstrahlung spectrum allows us now to calculate the classical energy loss of a particle by integrating (1) over all frequency ω.

$$\frac{d\delta_c}{dz} = \frac{1}{\Delta z} \int_0^\infty dI(\omega) d\omega \tag{4}$$

The total particle energy is

$$E = mc^2 = \gamma_u m_0 c^2 \approx \frac{p_\parallel}{|\vec{u}|} c^2 \quad , \tag{5}$$

so it can be related to the radiation loss

$$\frac{d\delta_c}{dz} = \frac{1}{E}\frac{d\delta_c}{dz} = \frac{|\vec{u}|}{p_\parallel c^2}\frac{d\delta_c}{dz} \quad . \tag{6}$$

Inserting the integral (4) into this expression, we find a purely analytical solution by using the airy integrals and after some transformations we get

$$\frac{d\delta_c}{dz} = \frac{2}{3}\frac{\hbar \alpha_s p_\parallel \left|\vec{F}_\perp\right|^2 c^2}{(m_0 c^2)^4} \left(\frac{1+\frac{|\vec{u}|}{c}}{2}\right)^2 \left(\frac{|\vec{u}|}{c}\right)^{-3} \tag{7}$$

with $|\vec{u}| = \rho\omega_k$, the instantaneous particle velocity. To derive the average loss for both colliding beams Eq.(7) is integrated along the length and the beam cross-section. In this paper the beam is described by a Gaussian density function in transverse and longitudinal direction, further more we calculated some other cases for the computer code. The beam length refers to the inertial system of the bunch which gives rise to the force field and must be recalculated with the Lorentzfactor.

$$L = 2\sqrt{3}\sigma_L = 2\sqrt{3}\gamma_v\, \sigma_z \quad . \tag{8}$$

The factor $2\sqrt{3}$ in the effective lenght L is generated in comparison with the energy loss from a equivalent uniform distribution. We insert in Eq.(7) the vertical beam-beam force for a Gaussian particle distribution and after integrating over the bunchlength L, we get the average relative energy loss.

$$\delta_{c,G_rG_z} = \frac{4}{3}\sqrt{\frac{3}{\pi}}\, 8 \ln\left(\frac{9}{8}\right) \frac{r_e^3 N^2 p_\parallel c}{BLm_0c^2} \left(\frac{1+\beta}{2}\right)^2 \beta^{-3} \quad . \tag{9}$$

The following tables (3,4) present the average relative energy loss for several LC designs calculated with Eq.(9). The results are compared with other numerical solutions.

	σ_x[m]	σ_y[m]	σ_z[m]
TESLA1	$6.39 \cdot 10^{-7}$	$1.01 \cdot 10^{-7}$	$1.0 \cdot 10^{-3}$
TESLA2	$4.95 \cdot 10^{-7}$	$0.64 \cdot 10^{-7}$	$1.0 \cdot 10^{-3}$
TESLA3	$10.0 \cdot 10^{-7}$	$0.64 \cdot 10^{-7}$	$1.0 \cdot 10^{-3}$
SLC	$1.5 \cdot 10^{-6}$	$1.5 \cdot 10^{-6}$	$1.05 \cdot 10^{-3}$
NLC	$1.7 \cdot 10^{-7}$	$6.5 \cdot 10^{-9}$	$0.11 \cdot 10^{-3}$

	E_{cm}[GeV]	$N[10^{10}]$
TESLA1	500	5.14
TESLA2	500	2.50
TESLA3	500	5.14
SLC	100	5.00
NLC	500	1.67

Table 2: Set of LC parameter

		SLC	NLC
HESHBEAM	δ_c[%]	$4.01 \cdot 10^{-2}$	61.7
SCHROEDER	δ_c[%]	$4.5 \cdot 10^{-2}$	78.0

Table 3: Beam-Beam radiation loss from SLC and NLC [1]

		TES.1	TES.2	TES.3
HESHBEAM	δ_c[%]	7.76	1.52	1.77
ABEL	δ_c[%]	7.7	3.5	1.8
MACPAR	δ_c[%]	9.3	3.8	3.0
TRACKIT	δ_c[%]	9.5	3.7	2.9
RBEAM	δ_c[%]	11.3	-	3.3

Table 4: Beam-Beam radiation loss from TESLA1,TESLA2 and TESLA3 [2]

4 LUMINOSITY

The spectral representation of the Luminosity $\mathcal{L}$ is a function of the center-of-mass energy E_{cms}. In section 3 we found, that the center-of-mass energy E_{cms} decreases during the penetration process ($E_{cms} \leq E_{0cms}$). This entails a spread of the Luminosity spectra to smaller center-of-mass

[1] SCHROEDER[SCHR90]

[2] ABEL[YOK85], MACPAR[RIT84], TRACKIT[D.SCHU93] and RBEAM from R.Brinkmann cp.[D.SCHU93]

energies. An expression of $x = \frac{E_{cms}}{E_{0cms}}$ which consists of two areas was determined.

$$\mathcal{L}(x) = \begin{cases} \mathcal{L}_1 \text{ fuer } 1-\delta_{max}/2 \leq x < 1 \\ \mathcal{L}_2 \text{ fuer } 1-\delta_{max} \leq x \leq 1-\delta_{max}/2 \end{cases} \quad (10)$$

$$\mathcal{L}_1(x) = \mathcal{L}_0\big[(1-x) + (1-x)\cdot \cdot\big(\ln(\delta_{max}) - \ln(1-x) - 2\ln 2\big)\big] \quad (11)$$

$$\mathcal{L}_2(x) = \mathcal{L}_0\big[\delta_{max} - (1-x) - (1-x)\cdot \cdot\big(\ln(\delta_{max}) - \ln(1-x)\big)\big] \ , \quad (12)$$

where δ_{max} is the maximum energy loss of the beam-beam radiation. The spectra in figure 2 was calculated with the equation and the TESLA parameter set from [CHE92]. We compared our solution with the shape of the curve which is presented in [CHE92], we discovered a good correspondence.

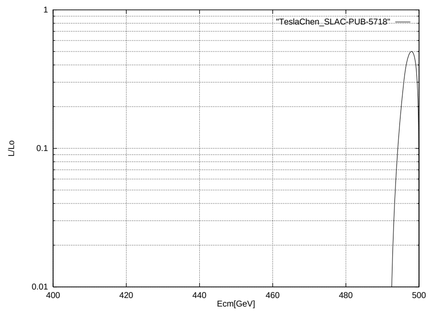

Figure 2: Luminosity spectrum $\frac{\mathcal{L}}{\mathcal{L}_0} = f(E_{cm})$ for LC design TESLA taken out of [CHE92]

5 DIFFERENTIAL LUMINOSITY

When the beam-beam loss is included, the effective Luminosity $\mathcal{L}$ will be different from $\mathcal{L}_0$ and a Luminosity enhancement factor H_D can be defined as

$$H_D = \frac{1}{\mathcal{L}_0}\mathcal{L} \ . \quad (13)$$

The differential Luminosity enhancement factor $\frac{dH_D}{dt}$ is defined by

$$\frac{dH_D}{dt} = \frac{1}{\mathcal{L}_0}\frac{d\mathcal{L}}{dt} \ . \quad (14)$$

In the calculation we distinguish between two regimes, the weak-focusing regime corresponding to the range $(0 < D \leq 1)$ and the transition region $(1 < D \leq 10)$. For the transition region we derived the equation

$$\frac{dH_D}{dt} = \frac{c}{\sqrt{\pi}\sigma_z}\frac{e^{-\frac{c^2t^2}{\sigma_z^2}}}{(t-t_f)^2\frac{9}{16}\frac{c^2}{\sigma_z^2}\frac{D}{2} + \frac{16}{9}A^2\frac{2}{D}} \ . \quad (15)$$

Figure 3 shows $\frac{dH_D}{dt}$ as a function of time for the disruption $D = 0.7$, the time t is in units of σ_z/c.

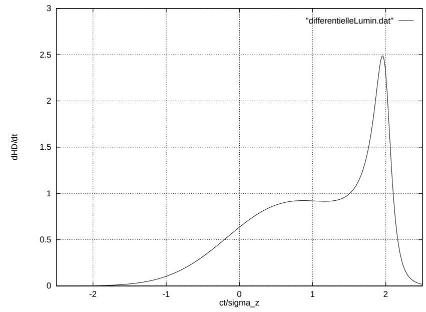

Figure 3: Differential enhancement factor $\frac{dH_D}{dt} = f(ct/\sigma_z)$ for $D = 0.7$ and $H_D = 2.35$, comparison with [CHE88]:Fig.5.

6 CONCLUSION

In this paper we have presented some formulas and results from our programm HESHBEAM. The results obtained from ABEL,SCHROEDER,HESHBEAM for TESLA1, SLC, NLC are in agreement, but there are some discrepancies with the results of RBEAM,TRACKIT,HESHBEAM for TESLA1-3. More detailed analysis of our results is available [H.SCHU95], but due to space limitation is not included here.

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