Low Emittance Lattices for Electron Storage Rings Revisited *

D. Trbojevic and E. Courant
Brookhaven National Laboratory
Upton, New York 11973, USA

Abstract

Conditions for the lowest possible emittance of the lattice for electron storage rings are obtained by a simplified analytical approach. Examples of electron storage lattices with minimum emittances are presented. A simple graphical presentation in the normalized dispersion space (Floquet's transformation) is used to illustrate the conditions and results.

1 INTRODUCTION

Electron storage rings are often used to produce high brilliance synchrotron light sources. The lower the transverse electron beam emittance the higher the brightness of the emitted light. The light is emitted in the direction of motion of the relativistic electron beam. The synchrotron light fans out tangentially to the dipole arc. At the point of emission an rms solid angle of the emitted light is inversely proportional to the Lorentz factor $(1/\gamma)$ - beam energy. The transverse horizontal emittance is:

$$\varepsilon_x = \frac{C_q \gamma^2}{J_x \rho} \langle H \rangle_{dipole}, \quad (1)$$

$$H = \gamma_x D_x^2 + 2\alpha_x D_x D_x^{'} + \beta_x D_x^{'2}, \quad (2)$$

where $C_q = 3.84\ 10^{-13}$ m, the horizontal partition factor $J_x \simeq 1 + \frac{1}{\gamma^2}\frac{R}{\rho} \simeq 1$, $\langle H \rangle = 1/L_d \int_0^{L_d} H ds$, α, β, and γ are the Courant-Snyder parameters, while D_x and $D_x^{'}$ are the horizontal dispersion function and its slope. The minimum transverse emittance is achieved by minimizing the average integral of the function H, over the length of the dipole. To illustrate this requirement it is useful to use Floquet's coordinate transformation [1] for the dispersion function as:

$$\chi = \frac{D_x}{\sqrt{\beta}} \quad and \quad \xi = D_x^{'} \sqrt{\beta_x} + D_x \frac{\alpha_x}{\sqrt{\beta_x}}, \quad (3)$$

then the function H is presented as:

$$H = \chi^2 + \xi^2, \quad (4)$$

$$\chi = \sqrt{H}\ sin\phi, \quad \xi = \sqrt{H}\ cos\phi, \quad (5)$$

where ϕ is the betatron phase advance. In the normalized dispersion coordinate system the vertical axis is chosen to be χ. The dispersion function satisfies the second order differential equation of motion:

$$\ddot{D}(s) + K_x(s) D(s) = \frac{1}{\rho}, \quad (6)$$

where $K_x(s) = \frac{1}{\rho} - \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$. The presence of a dipole in the lattice is manifested in the normalized χ and ξ dispersion space as a change along the horizontal axis of $\Delta\xi = \Delta D_x^{'} \sqrt{\beta}$, where $\Delta D_x^{'} = \theta$ (change of the slope of the dispersion function is equal to the bending angle of the dipole). The amplitude of the function H will depend on the dipole position in the lattice. Outside the dipoles ($\frac{1}{\rho}$=0) a solution of the homogeneous differential equation 6 represents a circle as equation 4 shows. The Chasman-Green lattice where the dipoles are approximated by the thin lens approximation, is presented in the normalized dispersion diagram in fig.1. Two dipoles lie on the horizontal axis. A part of the lattice with the zero dispersion is located at the origin of the χ and ξ normalized dispersion diagram. The end of the second dipole in fig. 1 is connected to the beginning of the first dipole by the semi-circle shown in equation 4. The minimum possible radius of the semi-circle corresponds to the smallest possible emittance. The size of the radius is determined by the dipole. Fig.1 shows both the idealized case where two vectors represent dipoles in the lowest possible Chasman-Green lattice and a single vector in the middle of the ξ axis representing the lowest possible emittance lattice. The real lattices are presented as well at the same fig.1 where dipoles have curved lines ($\sqrt{H}$) due to a change of the betatron phase throughout them.

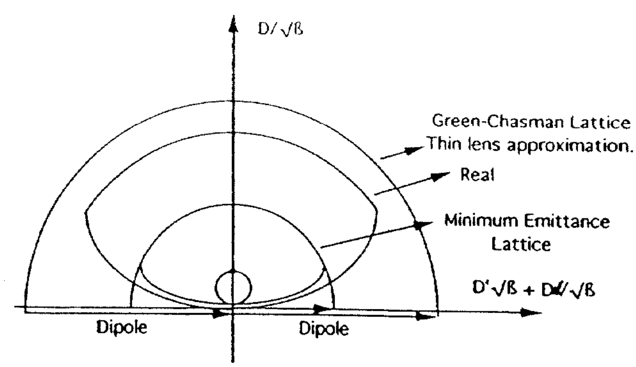


Fig. 1 Thin element approximation of the Chasman-Green minimum emittance lattice and of the lowest emittance lattice - where dipoles are horiz. vectors.

*Work performed under the auspices of the U.S. Department of Energy.

2 SIMPLIFIED ANALYTICAL APPROACH

As in [2] [4], we neglect the centripetal focusing in dipoles and approximate the horizontal β function and horizontal dispersion function D at quadratic functions of s:

$$\beta = \beta_0 + \frac{s^2}{\beta_0}; \;\; \alpha = -\frac{s}{\beta_0}; \;\; D = D_0 + \frac{s^2}{2\rho}; \;\; D' = \frac{s}{\rho}. \quad (7)$$

The mean value of H in the dipole is:

$$\langle H \rangle = \frac{1}{L}\int_{-L/2}^{L/2} \frac{D^2 + (D\alpha + D'\beta)^2}{\beta} ds. \quad (8)$$

The slope of the dispersion function D' and the betatron function α are equal to zero at the center of the dipole (a center of symmetry). Thus 8 becomes

$$\langle H \rangle = \frac{1}{\beta_0}[D_0^2 - \frac{\theta D_0 L}{12} + \frac{\theta^2 L^2}{320}] + \frac{\theta^2 \beta_0}{12} \quad (9)$$

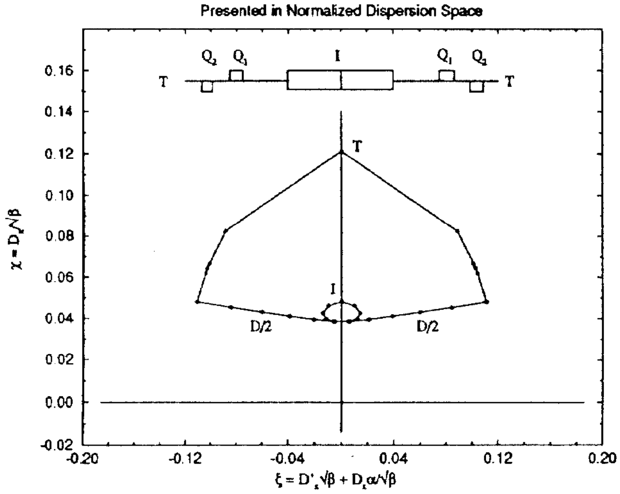
To minimize the emittance we minimize 9 with respect to the central values D_0 and β_0.(with given L and θ). We find

$$\beta_0 = \frac{L}{2\sqrt{15}}, \;\; D_0 = \frac{\theta L}{24} \;\; and \;\; \langle H \rangle = \frac{\sqrt{15}}{180} L\theta^2. \quad (10)$$

3. LATTICE DESIGN PROCEDURE

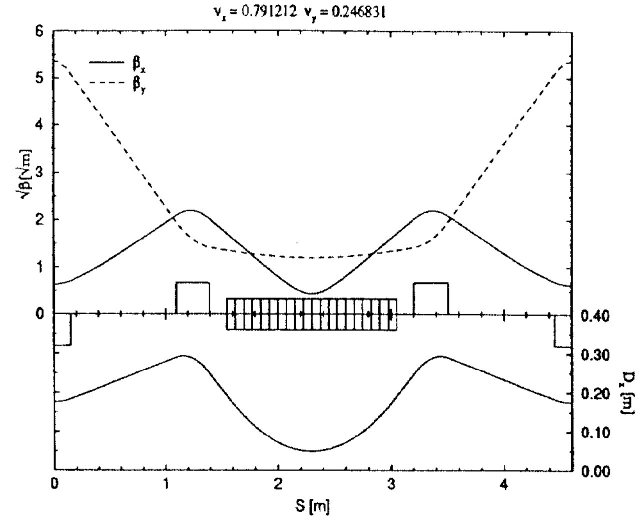
To obtain the values 10 at the center of a dipole we design a symmetric module, half of which consists of half of the dipole (length L, bending angle θ) plus a pair of quadrupoles. The strengths of the quadrupoles and their locations are free parameters; we may adjust these so that the central values of the functions are 10. Since there are more free parameters than constraints, we may also adjust the central value of the vertical β function and the overall length of the module.

Fig.2 Lattice Module of the Lowest Possible Emittance
Presented in Normalized Dispersion Space



The basic module of the lowest possible emittance lattice is presented in the normalized dispersion space in fig.2 while the betatron functions are presented in fig.3.

Fig.3 Basic Module of the Lowest Emittance Lattice
$\nu_x = 0.791212 \;\; \nu_y = 0.246831$



3 ZERO DISPERSION FOR THE INSERTION DEVICES

It is often desirable to have straight sections in the ring with zero dispersion, to accommodate RF cavities and insertion devices. A lattice has been devised in which a second module containing a zero dispersion section is added with suitable matching of the orbit functions. The lattice required a shorter dipole to provide the zero dispersion at its end.

4 CHROMATIC CORRECTION

The linear chromaticity was corrected by the standard two families of sextupoles close to the focusing and close to the defocusing quadrupoles at positions where the horizontal dispersion has maximum values. The position of the sextupoles was chosen to minimize the second order tune shift introduced by them as well. An additional module was added to the lattice with the same design procedure as described above. The length of the second module was varied to allow the for the betatron phase difference between the sextupoles to be close to a multiple of 90^0. The amplitude distortion functions [6]), could be presented as vectors which rotate in the betatron phase space with phase angles between the setupoles [7] as: $\mathbf{B_1}(\phi_x)$, $\mathbf{B_3}(3\phi_x)$, $\mathbf{B}_-(2\phi_y - \phi_x)$, and $\mathbf{B}_+(2\phi_y + \phi_x)$. The second order tune shift induced by the sextupoles was:

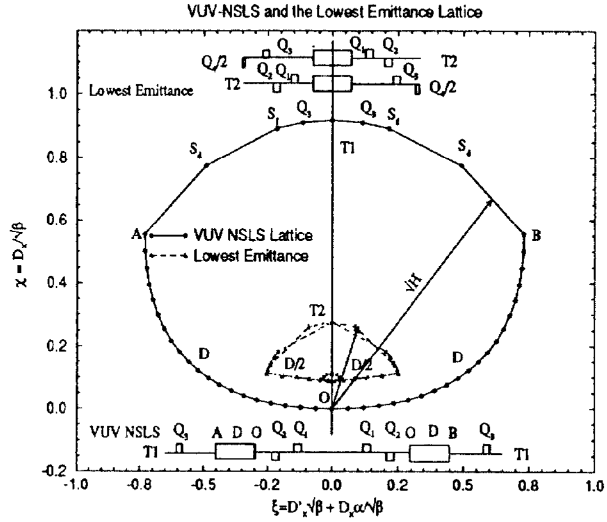
$$\nu_x = 6.350 + 170.0 \;\; \varepsilon_x - 6.8 \;\; \varepsilon_y \quad (11)$$

$$\nu_y = 2.312 - 6.8 \;\; \varepsilon_x - 91.0 \;\; \varepsilon_y. \quad (12)$$

5 EXAMPLES OF THE MINIMUM EMITTANCE LATTICES

An example of a minimum emittance lattice is shown in fig.4. There are two basic modules constrūcted by using the same initial conditions.

Fig.4 Low Emittance Lattices in the Normalized Dispersion
VUV-NSLS and the Lowest Emittance Lattice



The electron energy was chosen to correspond to the energy of the UV ring of the National Synchrotron Light Source (NSLS) at Brookhaven National Laboratory. To be able to compare this example with the other similar lattices the dipole length and number was chosen to be the same as in the UV NSLS ring (L_d=1.5 m and N_d=8). The basic module cosists of the dipole surounded by two triplets where the central quadrupole is the defocusing quadrupole. In the second module which starts at the middle of the dipole there are two defocussing quadrupole sepatated by a drift. The chromaticity of this example is higher than the one of the existing UV ring in NSLS. But the sextupole correctors have a very reasonable strength of KSF=4.38 m^{-2} and KSD=-3.77 m^{-2}. The quadrupole gradients were chosen not to exceed 15 T/m. The transverse emittance of the present UV ring at NSLS was reduced to the mimimum possible value of 8.68 10^{-7} mrad, compared to the existing emittance of 1.3 10^{-6} mrad. By the same procedure few other examples have been designed with the parameters of the X-ray ring at NSLS and of the X-ray ring of Argonne National Laboratory (ANL). The emittance of the X-ray ring at the NSLS was reduced to the minimum possible emittance by a factor of 8.8, while an example of the X-ray ring at the ANL was reduced by a factor of 11.27. The examples to be compared with the X-ray NSLS and to the ANL X-ray had the same lattice elements, most importantly the same dipoles. In the X-ray ring example to be compared to the NSLS ring an additional qaudrupole was introduced.

6 CONCLUSION

We repeated the analysis [4] for obtaining the conditions for the minimum emittance lattice of the electron storage rings. Conditions for the minimum emittance show that dispersion function in the middle of the dipoles should have a small value but not zero $D_0 = \frac{\theta L_d}{24}$ and that the horizontal betatron function should have a mimimum at the middle of the dipole with a value of $\beta_0 = \frac{L_d}{2\sqrt{15}}$. These initial conditions where applied and few examples of the minimum possible emittance were designed and presented in detail. We used Floquet's coordinate transformation to ilustrate the design procedure. It is possible to reduce the sextupole second order tune shift with amplitude by constructing lattice modules where betatron phase differences between sextupoles is mutiple of 90^0. We show that the transverse emittance of the existing electron storage rings can be easily reduced to the minimum possible value by either different settings of the existing quadrupoles or, in some examples, by introducing an additional quadrupole or by repositioning the existing quadrupoles. We hope that this work will help in the future electron storage ring design as well as existing rings.

7 REFERENCES

[1] E. D. Courant and H.S. Snyder, Ann. Phys. 3, 1 (1958).

[2] L.C. Teng, TM-1269, Fermilab Internal Report, June 1984,p.1-9.

[3] L.C. Teng, TM-1269-A, Fermilab Internal Report, December 1984 1984,p.1-6.

[4] S.Y.Lee and L.C.Teng, Conference Record of the 1991 IEEE Particle Accelerator Conference, May 6-9, 1991, San Francisco, California, p. 2679-2681.

[5] D. Trbojevic, D. Finley, R. Gerig, and S. Holmes, in Proceedings of Second European Particle Accelerator Conference, Nice, 1990, edited by P. Martin and P. Mandrillon, pp. 1536-1538.

[6] Tom L. Collins, "Distortion Functions", Fermi National Laboratory, Fermilab-84/114, Internal Report, October 23, 1984, 1-45 pp.

[7] S.Y. Lee, K.Y. Ng and D. Trbojevic, Fermi National Accelerator Laboratory, Fermilab-FN-595, October 1992, 1-65 pp.

[8] S.Y.Lee, K.Y. Ng, and D. Trbojevic, Physical Review E, Volume 48, No 4, October 1993, pp. 3040-3048.