Who is Afraid of a Crossing Angle?

HIRATA Kohji
KEK, National Laboratory for High Energy Physics
Oho, Tsukuba, Ibaraki 305 Japn

Abstract

The beam-beam interaction with a large crossing angle is studied by means of a fairy accurate mapping. Some simple results of the weak-strong tracking are shown: the beam-beam interaction becomes more serious when the crossing angle increases until the normalized angle Φ becomes about a half. Above this, however, it becomes less serious. When $\Phi \gtrsim 1$, the beam size becomes almost unaffected.

1 INTRODUCTION

It seems widely believed that the beam-beam effects become more serious for larger crossing angle[1], because it creates the synchro-betatron resonances[2]. This belief, however, has not been questioned seriously. To do so, first of all, we should know how beams interact with each other when colliding with an angle. Several authors have studied it either theoretically or by simulation. They, however, have introduced some rather crude approximation and simplification: it is difficult to see how valid they were. Here, we propose a new mapping which I believe is accurate enough. We apply this mapping to study the crossing angle effects.

2 BEAM-BEAM MAPPING

Here, we present the beam-beam mapping for the collision with a crossing angle. It is accurate, though not exact:

- It is symplectic in 6-dimensional sense.
- It includes all the known effects and some new effects. For example; the hourglass effets[3], change of energy due to the traversing electric field with a slope[4, 5].
- It is **local**[6]. That is, it is to be applied at one point in a ring (called $s = 0$) and therefore it can be used regardless to the other part of the ring. This is the necessary condition for the mapping.

The method is valid both for weak-strong and strong-strong cases. We will, however, restrict ourselves to the former case, for the sake of simoplicity. The basic idea is as follows: we divide the difficulty of treating complicated geometry of colliding beams in an angle into two pieces.

- We apply the Lorentz boost[4, 7] to make the collision head-on. The beam-beam kick is evaluated in a Lorentz frame in which the collision appears to be head-on but they are tilted horizontally. The Lorentz transformation is treated in an exact manner within the ultrarelativistic approximation.
- For the head-on collision, we use the mapping, called the synchro-beam mapping (SBM). This is formulated only for the head-on collision[8]. The bunches are cut into many pieces longitudinally. We apply the SBM for each collision between a slice and a test particle. Piwinski[2] did not use the slicing and used the effective beam size. It could be useful in theoretical studies but might have ignored some important factors.

Thanks to the 6-dimensional nature of the SBM, it is relatively easy to combine SBM and the Lorentz boost. The idea may be read off from Fig.1.

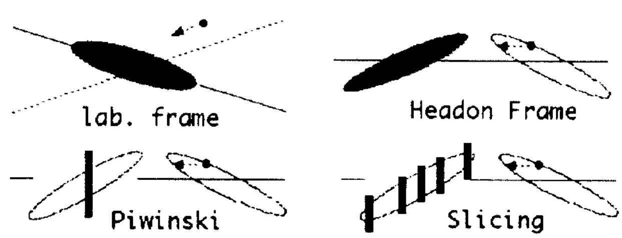

Figure 1: Collision with an angle can be seen as a headon collision with tilted beams when seen from the boosted frame. The latter can be seen as successive collisions with many slices.

The detailed description of the mapping is shown in Refs.[8, 9, 10]. The way I apply it to the weak-strong simulation is also shown in Refs.[9, 10].

3 SIMULATION

Model We use a set of parameters listed in Tab. 1. Only one IP is assumed. We track 50 particles for 10000 turns and accumulate data for luminosity, beam sizes and so on. For these parameters, 5 slices seem to be enough. The effective luminosity, $\hat{L}$, is a luminosity when the strong bunch is not affected and always keeps its Gaussian shape.

Weak-Current In order to see the resonance structures, let us see the case with a small value of the nominal beam-

(ϵ_x, ϵ_y)	$(2\times10^{-8}, 2\times10^{-10})$ m
(β_x^0, β_y^0)	$(1, 0.01)$ m
$\sigma_z, \sigma_\epsilon$	$(0.01\text{m}, 1\times10^{-3})$
ν_z	0.08
T_x, T_y, T_z	2000,2000,1000

Table 1: Test parameters. Here, ϵ is the emittance, β the (nominal) betatron function at the IP, σ_z the bunch length, σ_ϵ the energy spread, T the damping time measured by the revolution time.

beam parameter, $\eta_y = 0.02$: $\hat{L}$ is shown in Fig.2 for cases with and without crossing angle.

Several differences between $\phi = 0$ and $\phi = 7$mrad cases are remarkable: Some third order resonances appears. Major difference, however, is that the resonance $\nu_x \pm 2\nu_y \sim$ integer appears. The latter two resonances are not the SB resonances and are stronger for larger ϕ. These were induced by the nonlinear terms in the Lorentz boost.

Strong Current Let us put $\eta_{x,y} = 0.05$ and compare several values of ϕ. See Fig.3. It appears that, roughly speaking, when ϕ increases, $\hat{L}$ decreases first but becomes to increase for larger ϕ. This comes from the fact that the beam sizes becomes almost nominal for large ϕ so that the loss of luminosity becomes only due to geometrical effects. As a result, $\hat{L}$ has second peak at some vbalue of ϕ.

For two values of ν_y, we show the simulation results more in detail. See Fig.4.

4 DISCUSSION

Effective beam-beam force. It seems useful to consider the geometrical luminosity L_g and effective beam-beam parameter ξ in the boosted frame. Including the hourglass[3] and the beam tilt effects, but excluding the dynamical effects, we define R_L (luminosity reduction factor) and $R_\xi^{x,y}$ (beam-beam reduction factor):

$$R_L = \frac{L_g}{L_0} = \sqrt{\frac{2}{\pi}} a e^b K_0(b), \qquad (1)$$

$$a = \frac{\beta_y^*}{\sqrt{2}\sigma_z^*}, \qquad b = a^2 \left[1 + \left(\frac{\sigma_z^*}{\sigma_x^*}\tan\phi\right)^2\right],$$

$$R_\xi^{x,y}(z) = \xi_{x,y}/\eta_{x,y} \qquad (2)$$

$$= \int dz^\dagger \rho(z^\dagger) \frac{\beta_{x,y}(S)}{\beta_{x,y}^*} F_{x,y}(z^\dagger \tan\phi, \sigma_x^*(S), \sigma_y^*(S)),$$

$$S = (z - z^\dagger)/2.$$

where L_0 is the luminosity without hourglass nor tilt effect, K_0 is a Bessel function and $F(x, \sigma_x, \sigma_y)$s are Montague's reduction factor[11] of ξ for an off center particle, which falls down with ϕ quite rapidly. These are shown in Fig.5. For small ϕ, R_ξ is even larger than 1 due to the hourglass effect which makes the beam-beam interaction more serious. This decreases rapidly for larger ϕ. At the same time,

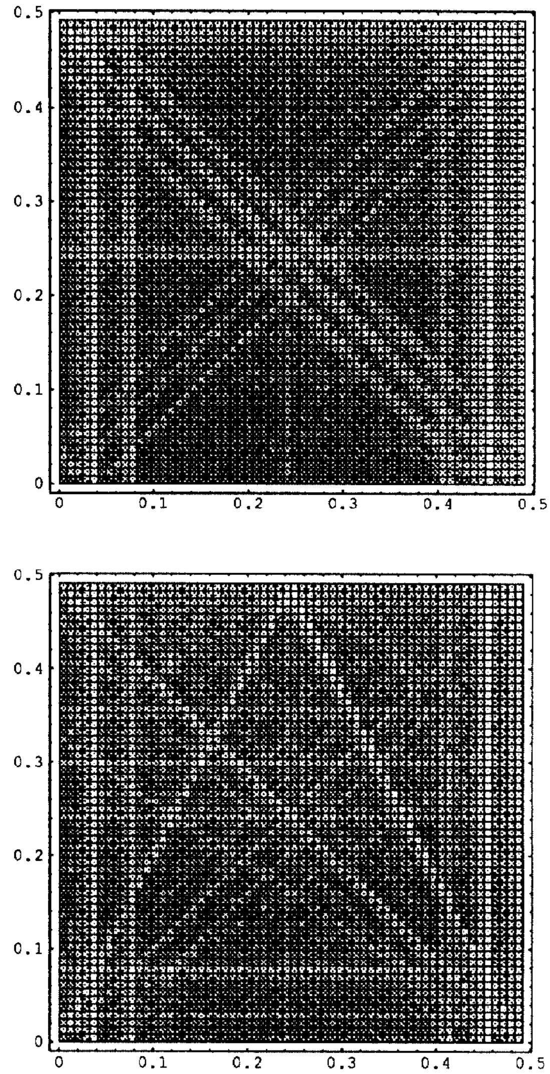

Figure 2: The effective luminosity $\hat{L}$ as a function of ν_x (vertical axes) and ν_y horizontal axes for $\phi = 0$ (up) and $\phi = 7$ mrad (down). The darker means larger luminosity. (Normalization is different between two cases).

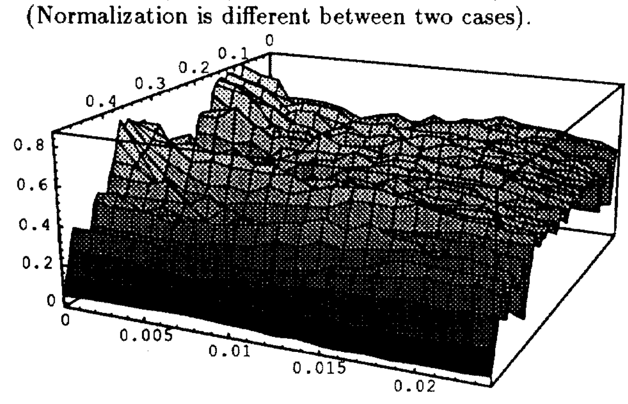

Figure 3: The ϕ and ν_y dependence of $\hat{L}/L_0$ for $0 \le \phi \le 25$ mrad and $0 \le \nu_y \le 0.5$. $\nu_x = 0.2$ is assumed.

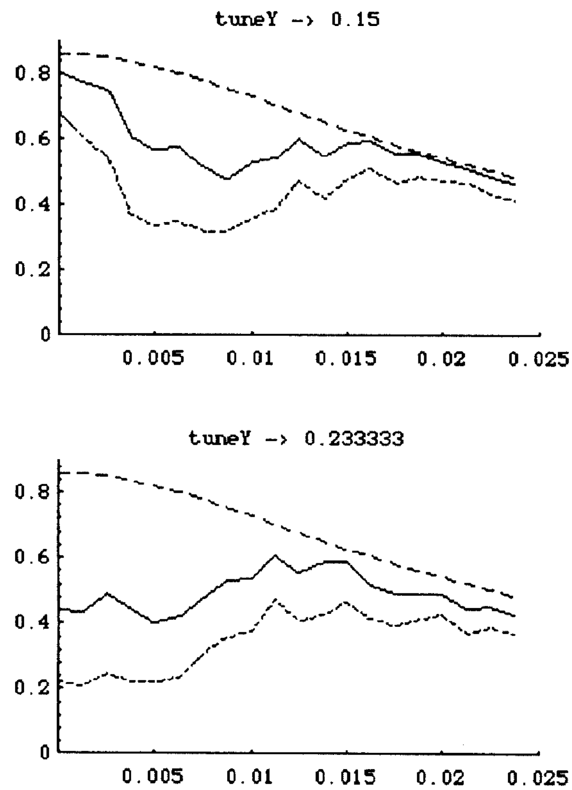

Figure 4: The ϕ dependence of luminosities. The dashed line is the geometrical luminosity L_g which falls down for larger ϕ. The solid line is $\hat{L}$ and dotted line is the Gaussian Luminosity (calculated by the data of $\sigma_{x,y}$'s) for ν_y = 0.15 (up) and 0.2333 (down). $\nu_x = 0.2$ is assumed. The recovery of the luminosity is clearly seen.

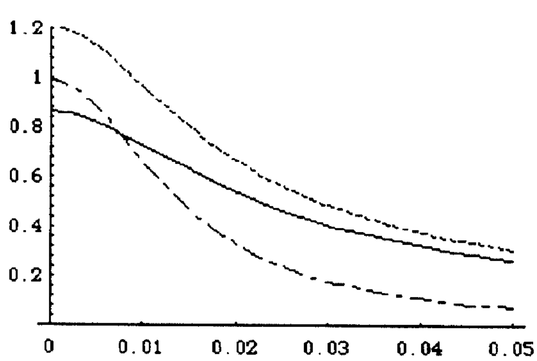

Figure 5: The (geometrical) luminosity reduction factor R_L (solid) and tune shift reduction factors R_ξ^y (dotted) and R_ξ^x (dot-dash).

R_L also decreases but less rapidly. By looking at Fig.5, we can vaguely understand why the luminosity recovers for large value of ϕ: the beam-beam force becomes weeker.

One of the important differences from Piwinski's formalism is the inclusion of the bunch length effect by using several slices. In fact, when we use only one slice, the effect grows almost proportional to ϕ and does not begin to decrease. From Eqs.(1) and (2), it seems that two parameters are important: $R = \sigma_z/\beta_y^0$ and $\Phi = \phi\sigma_z/\sigma_x$ (Piwinski angle). For $R \gtrsim 1$, the hourglass effect is important even for $\phi = 0$ [12]. When $\Phi \gtrsim 1$ the tilt effect is important. Piwinski's formalism worked well for DORIS where $R \ll 1$ and $\Phi \simeq 1/2$ (DORIS used vertical crossing so that σ_x is to be replaced by σ_y in Φ). For the present parameters, $\phi = 10$ mrad corresponds to $\Phi = 0.707$.

Am I afraid of a Crossing Angle? Now that we have fairly accurate mapping for the collision at an angle, the fear on the crossing angle dissapears. I do not mean that the crossing angle has ignorable effect. It has still remarkable effects. Whenever we know how to analyze it, there is no need to be afraid of it.

The author wishes to thank M. Furman, H. Moshammer, R. Siemann and K. Oide for valueable discussions.

5 REFERENCES

[1] For example, N. Toge, in "Proc. of Int'l Workshop on B Factories", KEK, 1992 Eds. E. Kikutani and T. Matsuda, KEK Report, KEK Proceedings 93-7 (1993).

[2] A. Piwinski, DESY Report DESY 77/18 (1977).

[3] G. E. Fischer, SLAC report SPEAR-154 (1972). SPEAR Storage Ring Group, IEEE Trans. Nucl. Sci. **NS-20**, 3, 838 (1973). M. Furman, Proc. 1991 IEEE Particle Accelerator Conference p.422 (1991).

[4] J. Augustin, Orsay, 36-69 (1969).

[5] M. Bassetti, *Longitudinal Energy Changes in a Bunch*, LNF-T-105 (1978).

[6] É. Forest and K. Hirata, *A Contemporary Guide to Beam Dynamics*, KEK Report 92-12 (1992).

[7] K. Oide, private communication (1990).

[8] K. Hirata, H. Moshammer and F. Ruggiero, Part. Accel. **40**, 205 (1993).

[9] K. Hirata, SLAC report SLAC-PUB-6375 (1993).

[10] K. Hirata, Proc. 6-th ICFA Beam Dynamics Workshop, Madeira 1993; KEK-Preprint 93-190 (1993).

[11] B. W. Montague, CERN report CERN/ISR-GS/75-36 (1975).

[12] Krishnagopal and Sieman, Phys. Rev. **D41**, 2312 (1990).