A Technique for Aligning Sextupole Systems Using Beam Optics

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Abstract

A technique for beam based alignment of sextupole systems is developed exploiting the enhancement effect of orbit differences by the sextupoles. This technique can in principle be applied to sextupole or sextupole strings with controlled orbit pattern and BPM configuration. This paper will discuss the theoretical basis, special optimization considerations and expected accuracy. Application to the SLC final focus is also discussed.

1. INTRODUCTION

In beam transport systems such as the SLC Final Focus System (FFS) where higher order optical aberrations critically affect the beam quality and hence the overall performance of the machine, the importance of alignment cannot be overstated. Originally for the purpose of cancelling the chromaticity at the interaction point (IP) a system of eight sextupoles was installed in two sections (the chromaticity correction section, or CCS) with phase advance of 180 degrees each. They are divided into two groups and powered in series by separate supplies (Figure 1). Changing the strengths of these sextupoles during machine tuning has been seen to lead to focusing and coupling changes at the IP, an indication of misalignment. The misalignment by 500 micron of a single sextupole can cause as much as 2 mm of unwanted dispersion at the IP [1], let alone the confusion it introduces in the tuning process. Therefore accurate measurement of such misalignments is very important for ensuring a small beam size at the IP. This report discusses a technique motivated by this requirement.

2. MATHEMATICAL FORMULATION

It would be of great convenience to develop an expression of the beam orbit at any given point as a function of the initial orbit, the optics, and compounded effects due to offsets in the optical elements. For a linear optical system, this is relatively straightforward [2]. When sextupole offsets are involved, the linear formula must be extended to include higher order effects, which proliferate quickly with increasing number of sextupoles. The formulation provided here can help keeping track of such effects to arbitrary order:

Take the complete offset operator for a displacement vector a similar to that used in quantum mechanics:

$$\begin{aligned}\mathrm{Off}(\hat{a}) &= \exp\left(a_i \cdot \frac{\partial}{\partial x_i}\right)\\ &= 1 + a_i \frac{\partial}{\partial x_i} + \frac{1}{2} a_i a_j \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} + \cdots\cdots \qquad (1)\\ \hat{a} &= (\Delta x, \Delta x', \Delta y, \Delta y')\end{aligned}$$

To first order a linear optical matrix transforms under this "offset" operation as:

$$\begin{aligned}&\mathrm{Off}(\hat{a}) \cdot R \cdot \mathrm{Off}(-\hat{a}) \cdot \hat{x}\\ &= R \cdot \hat{x} + (I - R) \cdot \hat{a} \qquad (2)\end{aligned}$$

which is just the linear offset effect [2].

To extend this formulation to the next order and retain a matrix representation we create a 14-element "pseudo vector" **P**:

$$\mathbf{P} = [\ x, x', y, y',\ x^2, xx', xy, xy',\ x'^2, x'y, x'y',\ y^2, yy', y'^2\ ]$$

P is not a real vector in the sense that all its elements are not independent. This 14-vector can be shown to transform correctly under a 14 by 14 matrix **T** which is properly extended from the 4 by 4 linear optical matrix **R** up to second order, as is done in various optics codes.

The following facts can be easily established:

1) The second order effect of an offset in **P** by a vector **a** can be consistently obtained by applying the offset operator (1) directly on **P** to second order:

$$\begin{aligned}\mathbf{P}_a &= \mathrm{Off}(\hat{a}) \cdot \mathbf{P}(\hat{x})\\ &= \mathbf{P}(\hat{x} \to \hat{x} + \hat{a})\end{aligned}$$

2) The above offset effect, to second order, can be summed up with the following *matrix* equation:

$$\mathrm{Off}(\hat{a}) \cdot \mathbf{P} = \mathbf{P} + \mathbf{A}(\hat{a}) \cdot \mathbf{P} + \mathbf{B}(\hat{a})$$

where **A** is a 14 by 14 matrix and **B** a 14-vector.

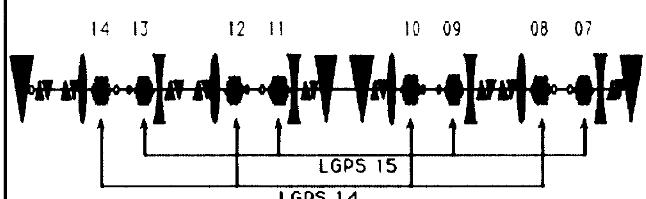

Figure1. The CCS in the Final Focus. Hexagons are the sextupoles and LGPS the power supplies.

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These facts can be used to extend (2) to the second order in an expression relating the initial orbit **P** and the final orbit **P'** separated by a second order optical element **T** with an offset a:

$$\mathbf{P'} = \mathrm{Off}(\hat{\mathbf{a}}) \cdot \mathbf{T} \cdot \mathrm{Off}(-\hat{\mathbf{a}}) \cdot \mathbf{P} \qquad (3)$$
$$= \mathbf{T} \cdot \mathbf{P} \qquad \text{zeroth term}$$
$$+ [\, \mathbf{A}(\hat{\mathbf{a}}), \mathbf{T}\,] \cdot \mathbf{P} + \mathbf{A}(\hat{\mathbf{a}}) \cdot \mathbf{T} \cdot \mathbf{A}(-\hat{\mathbf{a}}) \cdot \mathbf{P} \qquad \text{x-depend. term}$$
$$+ \mathbf{T} \cdot \mathbf{B}(-\hat{\mathbf{a}}) + \mathbf{B}(\hat{\mathbf{a}}) + \mathbf{A}(\hat{\mathbf{a}}) \cdot \mathbf{T} \cdot \mathbf{B}(-\hat{\mathbf{a}}) \qquad \text{constant}$$

To first order this reduces to the old result. One can now compound the effects of many such second order elements by simple minded matrix multiplication with the aid of a symbolic program such as MACSYMA, just like the linear case.

From (3) we see that in addition to terms depending only on the offset **a**, as is true in the linear case, the offset effect on the orbit also contains terms *linear* in the initial orbit **P**, the latter being one order lower in **a** than the former (**B** is quadratic in **a**), and therefore dominant. We can thus take advantage of this fact to experimentally measure the sextupole offsets by sweeping the incoming beam and looking at the outcoming orbit, as opposed to the offset of linear elements, which has to be measured by varying the magnet strengths [2].

Apart from yielding a larger signal magnitude, varying only the incoming orbit has other advantages. It is relatively easy from an operational point of view. And because we are only interested in the difference orbits, the data is immune to the following systematic errors: BPM offsets, any unknown kicks (e.g., quadrupole offsets) *after* the measured sextupole, and mismatch between energy and bend strengths. Finally (3) suggests that if we vary only the incoming orbit, the difference orbits are linear in **a** and only a linear fit is needed to recover the second order effect of **a**. Thus the obvious strategy is to generate large orbit excursions inside the sextupole and look at the correlation between the incoming and outgoing orbits.

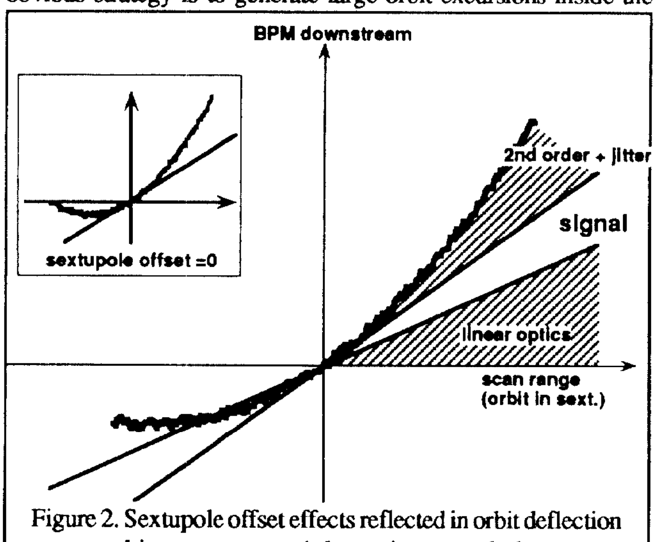

Figure 2. Sextupole offset effects reflected in orbit deflection versus orbit scan range. A beam is scanned about some predetermined reference line through a sextupole with an offset. The thick wiggly line shows the BPM reading downstream as a function of the incoming beam position or angle at a fixed point (or the strength of the corrector used for the scan). The same pattern with an aligned sextupole is shown in the box. When an offset is present, a very small linear signal and some additional wiggliness are added to this pattern which can be barely detectable.

3. PRACTICAL DIFFICULTIES

Although a mathematical expression for compounded sextupole offsets can be derived using (3), in reality the formula is already very complicated with two sextupoles where 4th order terms in combinations of incoming orbit and offsets must be included. Fortunately in the SLC Final Focus the power supply configuration allows individual sextupoles to be sufficiently isolated so that we don't need to take into account the effects of adjacent sextupoles.

We are however still faced with practical difficulties as indicated in Figure 2. Shown are the orbit signals after a sextupole versus the incoming orbit. The actual signal reflecting the sextupole offset is sandwiched between two other contributions (hatched area) which obscure the real signal and complicate the analysis.

The upper hatched area consists of the following:

1) known second order sextupole effect (a parabolic curve),
2) any linear *systematic* error (e.g., quadrupole offsets *before* the sextupole),
3) effects due to any pulse to pulse uncertainty (the wiggly part, e.g., upstream beam jitter, corrector strength error from one setting to the next).

The lower hatched area consists of the following:

4) linear optical contribution due to the sweeping of the beam, known but of overwhelming magnitude over the signal.

Of the above, the second item indicates that we should eliminate all unknown systematics such as power supply errors as much as possible. We should also avoid having any intervening quadrupoles between the incoming orbit and the sextupole being measured.

The third item forces us to abandon the linear fit mentioned at the end of section 2. Unlike the linear optics case [2], where unknown incoming orbits and corrector errors are simply absorbed into the fit as an unknown linear quantity, here the difference orbit would demand unknown quadratic terms in the incoming orbits, corrector errors, and offsets. Furthermore since the orbit differences come in two forms not simply related to each other,

$$\Delta X = X_A - X_B \quad \text{and} \quad \Delta(X^2) = X_A^2 - X_B^2,$$

it appears that a quite involved nonlinear fit would be needed, likewise accuracy and simplicity compromised. If we want to restore the linear nature of the difference orbits, the *pulse to pulse* incoming position *and angle*, as well as corrector errors, must be known in advance.

The fourth item becomes a problem as it grows much faster than the signal. In order to obtain some appreciable signal, this linear "background" usually reaches such a proportion that it totally obscures the signal and may cause orbit dependent errors due to nonlinearity in the BPM's and the quadrupoles.

4. A PROPOSED SOLUTION

To attend to all the problems mentioned in section 3 except that of the pulse to pulse corrector uncertainty, an experimental scheme is proposed here taking into account the realistic SLC FFS. An orbit bump is derived which generates very large excursion inside the

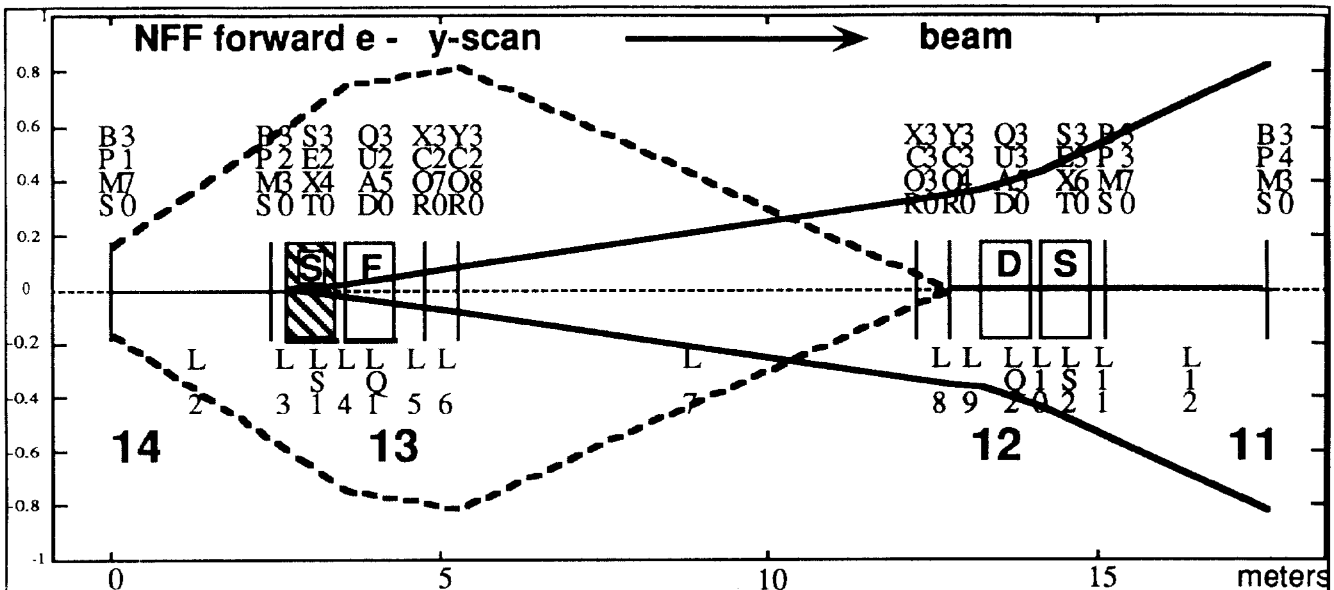

Figure 3. Optimized bump for sextupole offset measurement given realistic SLC CCS optics and hardware. The hatched block indicates the sextupole to be measured. The other blocks are focusing and defocusing quadrupoles and a powered-off sextupole. Vertical lines mark the locations of BPM's and corrector dipoles as indicated. The dashed lines are the bump envelopes in the exact shape. The solid lines are the propagation of the sextupole offset signals in the exact shape, although not to scale with the dashed lines.

sextupole but closes in both position and angle at some downstream BPM's. Figure 3 shows one such example in the y-plane. The bump is optimized in the sense that mathematically of all such closed bumps allowed by the SLC CCS optics, this is the one which gives the maximum offset signal at the BPM's. The adjacent sextupoles are turned off to avoid higher order complication. There is no intervening quadrupole between the initial BPM's and the measured sextupole. The two successive BPM's in the beginning ensure that both the position *and the angle* of every individual pulse are known and can be subtracted from the wiggly part of the total signal, thereby avoiding the more involved nonlinear fitting problem and source of error. The two quadrupoles are exploited to both help the bump closure, thereby enabling larger bump amplitudes, and magnify the signal of the sextupole offset as it propagates down to be picked up by the last two BPM's. The linear "background" orbit is totally closed before these two BPM's so it does not overwhelm the signal or cause problems of nonlinearity. The bump shape is generated by correlated scan pattern of the two y-correctors shown and three upstream y-correctors. A study of the correlation ratios and the regular operation point in the 5-dimensional region spanned by the ranges of these five correctors gives us the maximum attainable amplitude of such bumps. It is seen that with the existing correctors in the CCS, a significant signal at the last BPM, in some cases as big as the amount of the sextupole offset itself, can be generated against a flat "background".

In order to address the problem of pulse to pulse corrector uncertainty, namely, arbitrary errors in the *assumed* corrector strengths at each step of the scan, we must perform a calibration run first with the sextupoles turned off. The orbits so obtained, which may display failure of bump closure, are then subtracted from the orbits in the production run with the sextupoles turned on. In so doing we again avoid getting involved in nonlinear problems and all we require is good reproducibility of the correctors. Theoretically when the contributions from the pulse to pulse incoming orbits and corrector errors are subtracted, the wiggliness in the total signal should be eliminated. This fact can thus be used to check the quality of the data against other random errors.

Due to the independent sextupole offset effects in the x and the y planes, one single scan can yield information on offsets in both planes. However if the offset is not predominantly in one plane, the cross-talk between x and y may obscure the result if one tries to interpret the data out of a single scan. More scans in both planes may be needed in such cases.

A package has been developed to simulate the entire measurement and analysis procedure. The success of this method depends mainly on the accuracy of the BPM's. It is seen that with a CCS sextupole offset of 500 microns and BPM resolution of 10 microns, we can achieve 90% accuracy in the best case. The outcome deteriorates quickly as BPM errors increase.

5. CONCLUSION

We discussed the general principle and difficulties involved in extracting the sextupole misalignment signal and presented such a technique for the SLC Final Focus. The principle and existing computer package can be applied to similar systems elsewhere. Simulation suggests that appreciable signal magnitudes can be expected. The main potential obstacle to the success of this method is the BPM error.

6. REFERENCES

[1] Nobu Toge, private communication.

[2] Y. Chao et al, " Alignment of the SLC Final Focus System Using Beam Orbits", IEEE Particle Accelerator Conference Proceedings, San Francisco, California, May 1991, pp. 628-630.