Method of Defining the Tolerances of Storage Ring Lenses for Imperfections and Misalignments

Eugene Bulyak and Andrey Mytsykov
Institute of Physics and Technology 310108 KHARKOV, Ukraine

Abstract

A new method of defining the tolerance of storage ring multipole lenses for geometric imperfections and misalignments has been developed. The method is based on the estimation of multipole harmonics disturbing the beam dynamics most appreciably. Amplitudes of these harmonics are derived from the measured data on the pole shape or the magnet field of the lens by the maximum likelihood method. The PC code based on the proposed method was developed. Results of the code verification are presented.

1. INTRODUCTION

The magnet field of storage ring lattice elements should be highly perfect. For stretchers the requirements on field quality are still stricter because of the necessity of keeping specific resonant conditions during the beam slow extraction [1]. The most critical in this respect are the multipole components up to the octupole (number of pole pairs <5). These demands have necessitated the development of the method estimating of imperfections and misalignment the magnet lenses can tolerate.

2. TREATMENT OF MULTIPOLE DISTORTIONS.

For a small lens length (with reference to the orbit length), the perturbation in the transverse beam dynamics is determined by the longitudinal component of the vector magnet potential $\mathbf{A}_s$. The perturbation in the transverse Hamiltonian, $H_1=H-H_0$, is [2]:

$$H_1 = -\frac{R^2}{c|B\rho|} A_s, \qquad (1)$$

where R is the average ring radius ;

B is the magnet rigidity

c is the light velocity .

The perturbation in the transverse dynamics is related to the coefficients of potential expansion around the orbit [2] as:

$$A_s(x,z,s) = \sum_n \sum_k b_{nk}(s) x^{n-k} z^k \qquad (2)$$

Every coefficient b_{nk} in this expansion is responsible for particular effect. Of primary importance for the stretcher are the b_{40} and b_{44} coefficients, which cause nonlinearities in the betatron tunes, and the coefficient b_{30} in the case of the third-order resonant slow extraction. The betatron tune shifts are proportional to:

$$\frac{dQ_y}{d(a^2)} = \sum_j b_{4j}(s)|W_y|^4 \qquad (3a)$$

and the distortion in the hexapole harmonic is:

$$\Delta h \sim \sum b_{4j}(s) W_x^3(s) e^{(ip\vartheta)} \qquad (3b)$$

Here a_y is the transverse amplitude,

$w_x(s)$ is the Floquet function.

Summation is performed over all lattice lenses.

So, the level of the effect of the multipoles in the magnet potential expansion is determined by the particular feature of the lattice.

Thus, the estimation of tolerable imperfections and misalignment is reduced to the calculation of the most critical coefficients b_{nk} in the potential expansion.

3. QUADRUPOLE LENS IMPERFECTIONS

Below we consider two types of imperfections in the quadrupole lens. They are:

i)-the conelike slope of the poles (Fig.1)

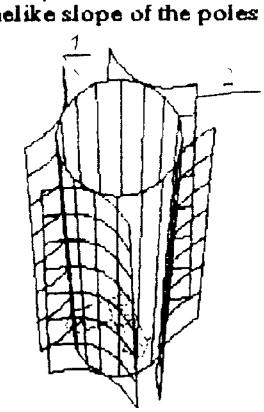

Fig.1 The conelike distortion of a quadrupole lens
1-the aperture of a lens. 2 - the pole surface.

ii) the distortion of lens symmetry of the plane perpendicular to the axis (Fig.2)

We can see that this problem requires 3-dim magnetic field representation. Neglecting the beam field and assuming large permeability of the lens yokes, one can describe the field inside the lens with the scalar

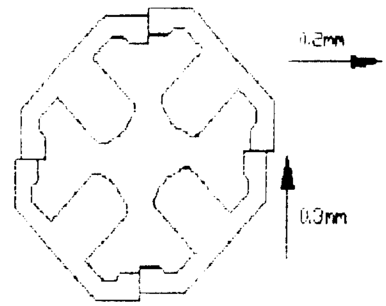

Fig.2 The distortion of lens symmetry of the plane perpendicular to the axis

potential P determined by the Laplace equation. This potential can be written as [3]:

$$P(r,\varphi,s)=\sum d_{nc}(r,s)\cos(n\varphi)+d_{ns}(r,s)\sin(n\varphi) \quad (4)$$

where (see [4]):

$$d(r,s)=\sum\frac{(-1)^k n!\, r^{2k+n}}{4^k k!\,(n+k)!}\, U(s)_n^{(2k)} \quad (5)$$

We have used the method of least squares to calculate $U_n(s)$ taking into account that on the pole surface:

$$P(r,\varphi,s)=CONST. \quad (6)$$

3.1 Two-dimensional field

In the case ($A_x=A_z=0$, $B_s=0$), we write (4) as:

$$P=\sum\frac{1}{n!}\, B^{(n-1)} r^n e^{(ip\vartheta)} \quad (7)$$

Comparing (2) and (7) one can see that the term containing $x^m z^n$ refers to the $2(m+n)$-pole lens

From (7) it follows:

$$\sum b_{nc} r^n \cos(n\varphi_i)+b_{ns} r^n \sin(n\varphi_i)=C(r_i,\varphi_i) \quad (8)$$

Formally this is the equation of the nonlinear regression:

$$r=f(\varphi,b_k). \quad (9)$$

The b_k parameters are to be estimated by the MLS method.

3.2 The code Bu4My

We have developed a PC code based on the above described method. This code is designed to calculate the multipole components in the vector potential, using the data of pole shape measurements. The algorithm is as follows.

i)- Interactive manual fitting of the shapes by choosing the appropriate value of the coefficients b_n.

ii)- Auto MLS fitting of the chosen coefficients.

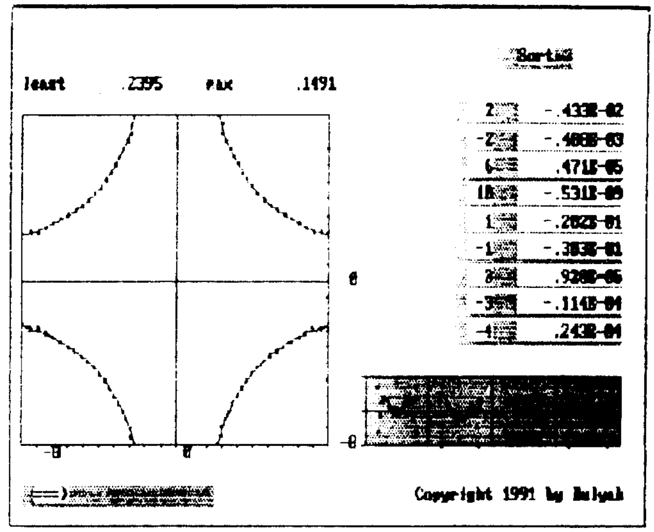

Fig.3 The BU4MY panel for the case (Fig.2)

With this code we have estimated the quadrupole of the PSR-2000 lattice [1]. The lens geometry was distorted as it is shown in Fig.2. The BU4MY panel for this case is depicted in Fig.3. The octupole component value obtained for the distorted lens exceeds that of the ideal lens by 4 orders of magnitude.

3.3 Conelike lens distortion

We consider the lens the poles of which are placed on the surface of a cone (see Fig.1). For ϕ=const the radius of the aperture r is related to the longitudinal coordinate s as:

$$r=(1+\lambda s)R \quad (10)$$

It can be suggested that the potential at the (r,ϕ,s) point is the sum of 'partial' potentials P_n. Since this kind of perturbation does not change the degree of symmetry, the equipotential line of Pn at s=const is the curve of order n+2n+3n+.... So, the multipoles, we interested in, n=2,3,4 project into multipoles of order 6,9,12 and higher. Therefore, we may calculate only the 'flat' multipoles. For the fulfilment of the condition (6) it is sufficient that:

$$\frac{n!\, r^{2k+n}}{4^k k!\,(n+k)!}U(s)_n^{(2k)}=CONST. \quad (11)$$

then:

$$U(s)_n^{(2k)}=u_n\frac{(n-1+2k)!}{(n-1)!\,R^n}\,\lambda^{2k}[1+\lambda s]^{-(n+2k)} \quad (12)$$

can be rewritten as

$$d_{nc}(r,s)=\frac{u_n}{R^n}\sum[(1-NR_k\frac{\lambda^2 r^2}{[1+\lambda s]^2})*$$

$$*NF_k\frac{\lambda^{2k} r^{2k+n}}{[1+\lambda s]^{2k+n}}] \quad (13)$$

where.

$$k=0,2,4,6......NR_k=\frac{(n+2k)(n+2k+1)}{4(k+1)(n+k+1)},$$

$$NF_k=\frac{n(n+2k-1)}{4^k k!(n+k)}$$

Let us consider the case of small λ. By neglecting the high order-terms in (13) we obtain [4]:

$$F=\sum\frac{1}{n!}B^{(n-1)}r^n e^{(ip\vartheta)}[\frac{1}{[1+\lambda s]^n}-\frac{n}{4}\frac{\lambda^2 r^2}{[1+\lambda s]^{2+n}}] \quad (14)$$

The pole shape described by (14) is in good agreement with the measured one as is shown in Fig.4.

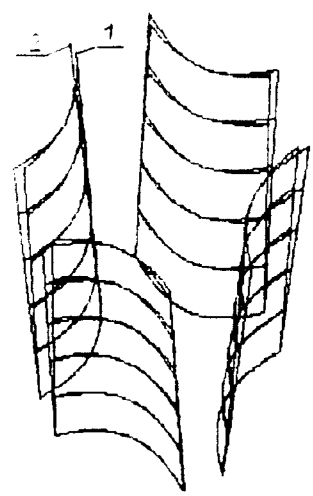

Fig.4. The comparison of measured equipotential surface - 1 and calculated one - 2.

It is of interest to consider the expansion of $(1+\lambda s)^{(n+2k)}$ up to second order, because the expression for the potential satisfies the Laplace equation.

$$F=\frac{1}{n!}B^{(n-1)}r^n e^{(ip\vartheta)}(1-2\lambda s+3\lambda^2 s^2-\frac{n}{4}\lambda^2 r^2) \quad (15)$$

This field is connected with the vector potential (Cartesian frame):

$$A_z=\frac{u_2}{R^2}(x^2-y^2)(1-2\lambda s+3\lambda^2 s^2)-$$

$$-\frac{n}{4}\frac{u_2}{R^2}(x^4-y^4)\lambda^2 \quad (16)$$

So, we have obtained the expression describing the conelike perturbation in the lens geometry. Similarly, the expression for an arbitrary multipole would be obtained. This perturbation leads the 'quasi octupole' terms to occur terms in the Hamiltonian ($A=x^3y+xy^3$) (compare to x^3y-xy^3 for the true octupole)

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