Methods of Reducing the Azimuthal Momentum Created During Axial Injection into a Strong Axial Field

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Abstract

For the proposed Moscow-TRIUMF-Spectrometer it is desired to inject axially, with the minimum possible creation of azimuthal momentum, a beam of 4.1 MeV μ^+ into a 3 T field produced by coaxial coils. It is shown that an external coil, acting as a converging lens, with axial field antiparallel to the axial field of the spectometer, imparts to a low divergence input beam azimuthal momentum of opposite sense to that produced by the spectrometer field. A coil located inside the iron shield of the spectrometer, adjacent to the shield end cap, shifts part of the spectrometer radial field into a region of low axial field where the converging rays from the exterior lens are close to the symmetry axis and, so, experience a weak radial field. It is shown that such a configuration reduces the transverse momentum at the experimental target to about 1/6 the value obtained when the two extra coils are turned off.

I. INTRODUCTION

The proposed Moscow-TRIUMF Spectrometer (MTS) is an axisymmetric, warm bore configuration of five coaxial superconducting coils symmetric about the midplane of the middle coil. The system is presented in a TRIUMF Research Proposal [1] and further details are presented in [2]; the subject of this paper is treated in greater detail in two reports [3] and [4]. Magnetic fields and particle trajectories were calculated using PE2D [5]. The particle beam enters the system at large positive values of z (z = 500 cm) and travels toward the z = 0 plane which is a symmetry plane for the coils and the iron shield of the MTS. The z axis is the axis of rotational symmetry. The coordinate system used is the (r, θ, z) cylindrical coordinate system; the units used are Tesla and centimeter. Unless otherwise stated, the muons are chosen to be monoenergetic with kinetic energy 4.1 MeV (29.72 MeV/c).

The particle beam experiences a transition from a region of negligible axial field to a region of very high axial field. If we consider a source-free cylinder, centered on the axis of axial symmetry, radius R, stretching from $z = -\infty$ to $z = 0$, the net magnetic flux through the cylindrical surface and the two end discs is zero, resulting in an integrated B_r field that is approximately a linear function of R and insensitive to the details of the axial dependence of the B_z field. The z component of the particle velocity interacts with this B_r field to produce an azimuthal component of momentum.

2. THE TOTAL SYSTEM

The total system is presented in fig. 1; the fields produced are presented in fig. 2 and 3.

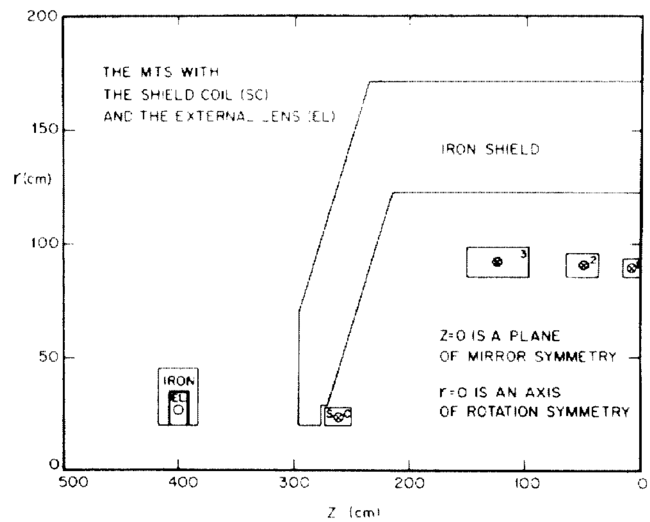

Figure 1. The complete system. Coils 1, 2 and 3 are superconducting (3968 A/cm^2); the shield coil (SC, 650 A/cm^2) and the external lens (EL, 480 A/cm^2) are normal conducting. Currents for coils 1, 2 and 3 and for the shield coil flow into the plane of the diagram; the current for the external lens flows out of the plane of the diagram. There is an iron shield around the external lens. Exact system dimensions are presented in [3] and [4]. Trajectories start at z=500 cm on or near the z-axis, parallel or nearly parallel to the z-axis; the experimental target would be located on the z-axis at z=0.

The field inhomogeneity ($|\Delta B/B|$) is $< 1.2 \times 10^{-4}$ over the cylindrical region, $0 \le r \le 15$ cm, $-13 \le z \le 13$ cm.

The B_r field created by the transition in the B_z field strength is approximately proportional to the coordinate r; the external lens focusses the beam to a small size in this transition region. In the external lens, the v_z velocity component interacts with the B_r field to produce an azimuthal velocity component, this azimuthal velocity component interacts with the B_z field to produce a radially inward force. The focussing strength of such a lens [6] is proportional to

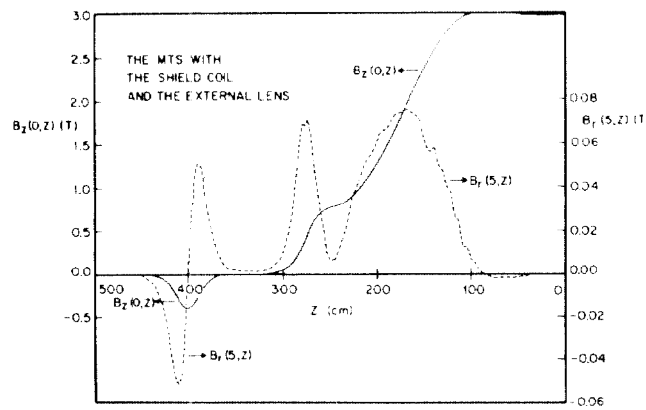

Figure 2. For the complete system: the axial field along the z-axis and the radial field along the r=5 cm line.

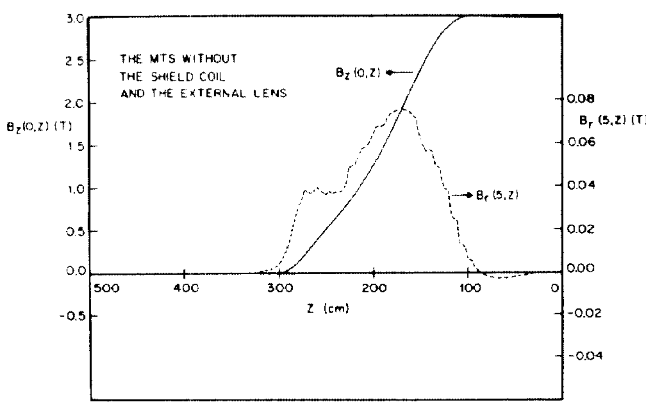

Figure 3. The MTS with the shield coil and external lens coil turned off: the axial field along the z-axis and the radial field along the line r = 5 cm.

the integral of $B_z^2(0,z)$. If the r component of the trajectory has mirror symmetry about the z = 400 cm plane through the center of the lens, the azimuthal momentum gained on the entry to the lens is lost when the ray exits the lens. However, for the input beams presented below, the input rays are parallel or very nearly parallel to the z-axis while the exit rays are converging toward the symmetry axis; the azimuthal momentum gained on approach is not entirely lost on exit. As will be shown later, with the lens and MTS axial fields antiparallel, this reduces the value of $v_\perp/v$ at z = 0.

The shield coil is located inside the shield, adjacent to the shield end cap. Its purpose is to move some of the B_r field from the B_z transition region to a region of small B_z. (Compare figs. 2 and 3). In the region of small B_z the trajectories are essentially straight lines. The external lens causes these trajectories to pass close to the symmetry axis in the region where the shield coil produces a local maximum, with respect to z, in the B_r field. Since B_r is approximately proportional to r, these trajectories experience only a small azimuthal force in this region.

Various shapes and excitations have been tried for the shield coil; the only firm conclusion reached is that the coil should be relatively long so that the system is insensitive to exactly where the incoming rays have their minimum distance from the axis, so long as it is somewhere near the middle of the B_r spike produced by the shield coil.

This is a highly non-linear problem and the results presented here for the system of fig. 1 may not represent an optimum. Certainly, there are configurations that yield worse results, but among the systems considered that yielded better results there was no indication of direction in system parameter space that trended towards an optimum. It is concluded that, if an optimum exists, it is a very shallow optimum.

3. TRAJECTORY RESULTS

A 4.1 MeV paraxial muon travels an axial distance of 21 cm in a 3 Tesla axial field. In the transition from zero to full field in the system of fig. 1, this particle executes ~ 5 Larmor periods.

The trajectory parameters of greatest interest are the velocity components v_r, v_θ, and $v_\perp$, where

$$v_\perp = \sqrt{v_r^2 + v_\theta^2} \qquad (1)$$

For rays with $v_\perp = 0$ at z = 500 cm, the value of $v_\perp/v$ at z = 0 is a non-linear, monotonically increasing, function of the initial radial coordinate. For rays with specified values of r and $v_\perp/v$ at z = 500, the value of $v_\perp/v$ at z = 0 is a sensitive function of the value of the initial v_θ/v_r : for the rays with initial coordinates at z = 500 cm of r = 2 cm, $v_\perp/v = 0.002$ and $0 \le \arctan v_\theta/v_r < 360°$, the final values of $v_\perp/v$ at z = 0 range over $0.045 \le v_\perp/v \le 0.615$.

In Fig. 4 we present, for various system excitations, $v_\perp/v$ vs. z for the particular ray with r = 2 cm, $v_\perp = 0$ at z = 500 cm. It can be seen that Curve a, corresponding to the excitations presented in fig. 1, represents the best result.

4. BEAM RESULTS

Computer routines have been written [7] to generate PE2D input data for multiple rays with randomly chosen initial conditions. Results are reported for the same set of 500 rays uniformly distributed at z = 500 cm over $0 \le r^2 \le (2\text{ cm})^2$, $0 \le v_\perp/v \le 0.002$, and $0 \le \arctan(v_\theta/v_r) < 360°$; the source disc has a mean radius of 1.359 cm and the beam has a mean divergence of 1.0 mrad. These source parameters represent a beam that could be produced by collimation of the beam transported from the muon source to the MTS location. With the external lens and the shield coil off, the beam at z = 0 has r_{mean} = 0.525 cm and $v_\perp/v]_{mean}$ = 0.140; with the external lens and the shield coil excited as indicated in fig. 1 (curve a of fig. 4) the beam at z = 0 has r_{mean} = 0.144 cm and $v_\perp/v]_{mean}$ = 0.024; a reduction in $v_\perp/v]_{mean}$ by a factor of 1/6.

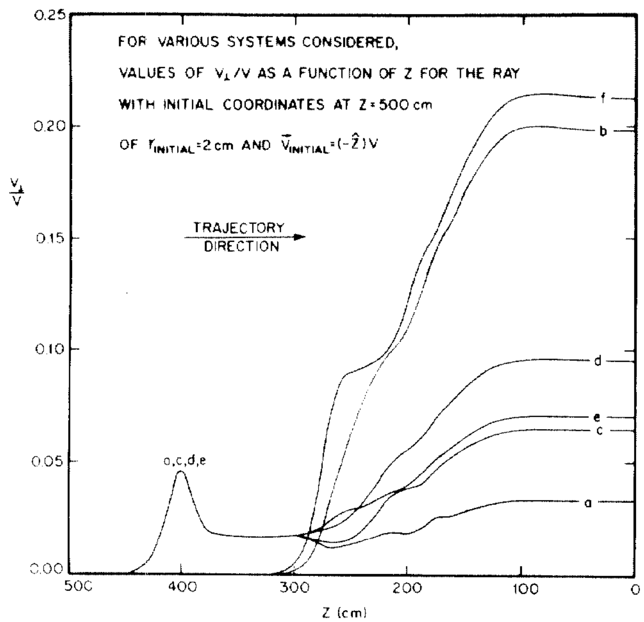

Figure 4. Values of $v_\perp/v$ vs. z for different external lens and shield coil excitations. The current senses are relative to those of coils 1, 2 and 3 of fig. 1; Curve a corresponds to the indicated current senses of fig. 1.

	Current Sense	
Curve	EL	SC
a	opposite	same
b	off	off
c	opposite	off
d	same	off
e	same	same
f	off	same

5. REFERENCES

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