EQUIVALENT CIRCUIT OF A 4-ROD RFQ

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Abstract

The 4-rod RFQ structure consists of four modulated quadrupole electrodes and spiral or straight support stems, which together form the rf resonator. In this paper an equivalent circuit of a 4-rod RFQ resonator is presented on the basis of a well proofed equivalent circuit model of spiral resonators. The 4-rod RFQ resonator is simulated by a lumped circuit chain with both neighbour and second neighbour coupling. The calculated rf modes and perturbation of frequency and voltage are presented together with a comparison with experiments.

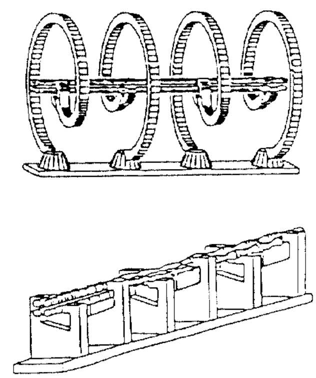
1. INTRODUCTION

Since the RFQ was successfully tested at LASL in 1980, RFQs have been widely developed [1,2,3]. The 4-rod RFQ developed in Frankfurt is especially attractive for low frequency operation. Its rf structure consists of four modulated quadrupole electrodes and spiral or straight suppport stems for low or high operating frequency respectively (figs. 1a, 1b). This kind of RFQ has been constructed in a wide operating frequency range from 200 MHz down to 26 MHz, and also a variable frequency 4-rod RFQ has been built.

It is well known that the rf performance of a resonator cavity can be simulated very well by a lumped equivalent circuit [4]. Results of model calculations for coupled spiral resonators and integral split ring resonators show, that the calculated rf performance, such as disperion, mode frequencies and voltage distribution, agree with experiment very well [5]. The 4-rod RFQ structure is similar to the coupled spiral resonator structure. They have the same spiral support stems. The only difference is the coupling load of spiral stems. In the 4-rod RFQ the coupling by the quadrupole electrodes replaces the coupling by drift tubes in coupled spiral resonators.

From the rf point of view the spiral stems correspond to coupled resonant lines or the coupled lumped circuit. The 4-rod electrodes correspond to two coupled resonant lines or its lumped equivalent circuit, and drift tubes correspond to a series of coupled capacitances. In the following on the basis of the equivalent circuit model of coupled spiral resonators [5] an equivalent circuit of a 4-rod RFQ will be discussed.

(a)



(b)

Fig.1 4-rod RFQ with spiral (a) and straight (b) support stems

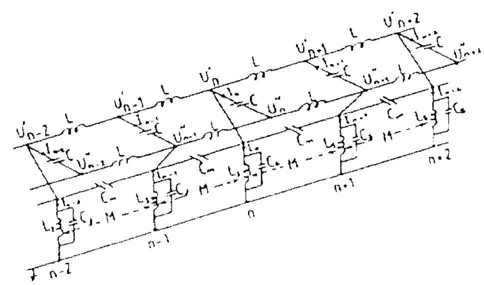


Fig 2 Equivalent curcuit of the 4-rod RFQ

2. EQUIVALENT CURCUIT

A lumped equivalent circuit of a 4 rod RFQ is shown in fig. 2. For simplicity the 4 rod electrodes are regarded as two inductance chains coupled by the capacitance C, and all the losses and current sources are neglected. In the circuit C_m stands for the coupled capacitance between neighbour support stems. The circuit equations for the currents I in the electrodes and currents J in the stems can be obtained:

$$N I_n + I_{n-1} + I_{n+1} = -PJ_n; \quad 2J_n - J_{n-1} - J_{n+1} = MI_n \qquad (1)$$

$$N = 2 + 2Y_c/Y_l + Y_m(2+Y_c/Y_l)/Y_l; \quad P = Y_c/Y_l$$

$$M = Y_s(2+Y_c/Y_l)/Y_c; \quad Y_c = j\omega C; \quad Y_l = 1/j\omega L$$

$$Y_m = j\omega C_m + 1/j\omega L_m; \quad L_m = M(1/K^2-1)$$

$$K = M/L_s; \quad Y_s = j\omega C_s + 1/j\omega L'_s; \quad L'_s = L_s(1+K)$$

The equations (1) describe the coupled quadrupole electrode and support stem system. From (1) the independent equations can be obtained:

$$k_oI_n + k_1(I_{n-1} + I_{n+1}) + k_2(I_{n-2} + I_{n+2}) = 0 \qquad (2)$$
$$k_oJ_n + k_1(J_{n-1} + J_{n+1}) + k_2(J_{n-2} + J_{n+2}) = 0 \qquad (3)$$
$$k_o = -Y_s - 2Y_m - 2Y_1(1 + 2Y_c/Y_1)/(2 + Y_c/Y_1)$$
$$k_1 = Y_m + 2Y_c/(2 + Y_c/Y_1); \quad k_2 = Y_1/(2 + Y_c/Y_1)$$

The equations (2,3) describe the currents in quadrupole electrodes and support stems, respectively. Note that the voltage between the electrodes V_n is proportional to I_n:

$$V_n = U'_n - U''_n = (-1)^n I_n/Y_c$$

Therefore, equation describes the voltage distribution V_n along the electrodes. The equation (2) reveals that the 4-rod RFQ is a coupled structure system with not only nearest neighbour, but also second neighbour coupling. k_1 and k_2 are the first and second neighbour coupling coefficients, as shown in the circuit in fig. 2.

It can be pointed out that if $Y_1 = 0$, then $k_2 = 0$, $k_1 = Y_m$, $k_o = -Y_s - 2Y_m$, and equation (2) will degenerate to the equation of coupled spiral resonators [5].

3. RF-MODES

Equation(2) describes a system of coupled oscillators. However, its basic performance can be simulated by a simple coupled system as in fig. 3. k_1 and k_2 are the coefficients of mutual inductance for neighbour and second neighbour coupling, respectively. The curcuit equations are

$$k_oI_o + 2k_1I_1 + 2k_2I_2 = 0$$
$$k_1I_o + k_oI_1 + k_1I_2 + k_2I_3 = 0$$
$$k_2I_{n-2} + k_1I_{n-1} + k_oI_n + k_1I_{n+1} + k_2I_{n+2} = 0$$
$$(n = 2, \ldots, N-2)$$
$$k_2I_{N-3} + k_1I_{N-2} + k_oI_{N-1} + k_1I_N = 0$$
$$2k_2I_{N-2} + 2k_1I_{N-1} + k_oI_N = 0$$
$$k_o = 1 - \omega_r^2/\omega^2, \quad \omega_r^2 = 1/LC$$

The solution is approximately

$$I_n^{(q)} = A\cos n\varphi q; \quad \varphi_q = \pi q/N \quad (n, q = 0, 1, \ldots, N)$$

with the dispersion relation:

$$k_o + 2k_1\cos(\pi q/N) + 2k_2\cos(2\pi q/N) = 0$$

which shows that the coupled resonant circuit has N+1 modes.

It can be concluded that the performance of the 4-rod RFQ can be simulated by the coupled circuit shown in fig. 3, if the same end conditions are established. Therefore, mode voltage of the 4-rod RFQ can be expressed as

$$V_n^{(q)} = (-1)^n A I_n^{(q)} = (-1)^n A\cos(n\pi q/N), \qquad (4)$$

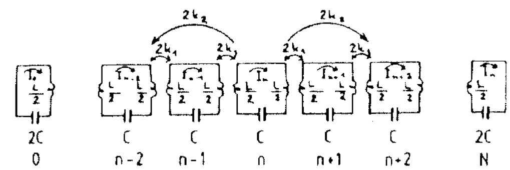


Fig.3 A simple coupled resonant curcuit with first and second neighbor coupling

(a)

(b)

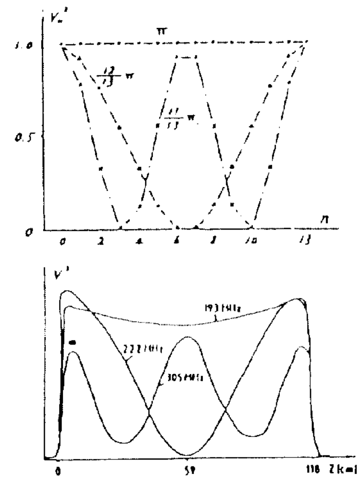


Fig.4 Calculated (a) and experimental (b) voltage distribution for a 4-rod RFQ (N=13)

As an example fig.4a shows the calculated V_n^2 for three modes (N = 13). Fig. 4b shows the corresponding experimental values V^2 for comparison.

The operating frequency ω_π can be deduced approximately:

$$\omega_\pi^2 = \omega_s^2[1 + 4K/(1-K)]/(1 + 4C_m/C_s + 4C/C_s),$$
$$\omega_s^2 = 1/L'_sC$$

With $k_o + 2k_1 + 2k_2 = -Y_s$ the 0-mode frequency can be deduced: $Y_s = 0$, then $\omega_o^2 = \omega_s^2$. Using the results to calculate the losses the shunt impedance and the Q-value can also be determined[8].

4. PERTURBATIONS

The equations for the currents can be written as:

$$\vec{T}_o I = 0$$

If a cell frequency in the oscillator chain is perturbed: $\omega_n^2 = \omega_r^2 + \delta\omega_n^2$ (n = 0, 1, . . , N), the $\pi q/N$ mode, the mode frequency, the voltage and the matrix will be modified:

$$\omega_q^{2\prime} = \omega_q^2 + \delta\omega_q^2; \; I_n' = I_n + \delta I_n; \; T^{(q)} = T_o^{(q)} + \delta T^{(q)}$$

The matrix equation with perturbation is

$$[\vec{T}_0 + \delta\vec{T}][I + \delta I] = 0$$

For operating mode q = N the solutions are:

$$\delta\omega_N^2/\omega_N^2 = (1/N)\sum_{n=0}(\delta\omega_n^2/\omega_r^2)$$

$$\delta I_n^{(N)}/I_n^{(N)} = -\sum_{m,n=0}^{N} M_{mn}(\delta\omega_m^2/\omega_r^2),$$

$$M_{mn} = (-1)^{m+n}(2/N)\sum_{p=0}^{N-1}[W_r\cos(n\pi p/N)\cos(m\pi p/N)/(1 - \omega_N^2/\omega_p^2)]$$

As an example of N = 11, a group of M_{mn} is shown in fig. 5 [7]. It demonstrates that if $\delta\omega^2/\omega^2 > 0$, then $\delta I_m^{(N)}/I_m^{(N)} < 0$. This characteristic has been used to make the voltage distribution flat. Fig.6 shows an example. The original distribution for a RFQ with straight stems is shown at 193 MHz. Then using shorting bars to make $\delta\omega_m^2 > 0$ (m = 0, 1, 2/11, 12, 13). According to fig.5 the corresponding $\delta I_m^{(13)}/I_m^{(13)} < 0$. Therefore, voltage distribution becomes flat. At the same time the frequency is raised to 202 MHz.

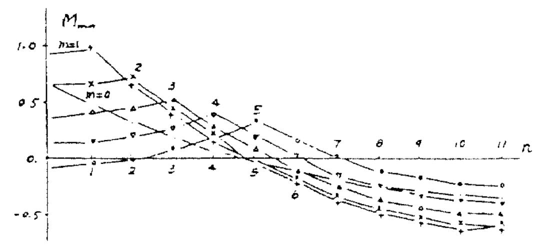


Fig.5 Influence of a frequency perturbation on the voltage distribution

5. CONCLUSIONS

The agreement between calculated and experimental rf performances of the 4-rod RFQ shows that this lumped equivalent circuit is a well suited model to study a 4-rod RFQ cavity, describing it as a coupled system with first and second neighbour coupling.

6. ACKNOLEDGEMENTS

One of the authors, J.X. Fang, would like to thank the W.E. Heraeus Foundation and Prof. H. Klein for valuable support.

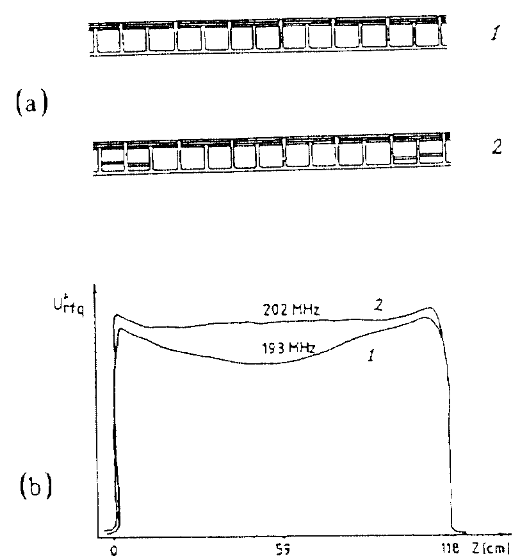


Fig.6 Changing of cell frequencies (a) and the influence on the voltage distribution (b)

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