Thermal Model Calculations in Superconducting RF Cavities

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Abstract

Finite element method based code (using the Modulef library) allows to solve the non-linear heat equation in 2D-axisymmetrical geometries incorporating multiple regions with appropriate boundary and interface conditions. This code can handle such complicated geometries as encountered in Superconducting RF cavities and related cryogenic equipment. Calibrations cells for special surface thermometers developed for SRF cavities operating in superfluid Helium were used to validate this modelling technique. Comparison with the experimental results shows a very good agreement. A modified LHe cooling arrangement of a monochell 1.5 GHz cavity has been studied with this code and the first results are discussed.

1. INTRODUCTION

For future applications of Superconducting Radiofrequency (SRF) cavities, in high energy particle accelerators, these resonators must exhibit accelerating gradients $E_{acc} > 15$ MeV/m and unloaded quality factor $Q_0 > 5 \times 10^9$ at an operating temperature $T_{bath} = 1.8$ K [1]. To reach these goals an important improvement of the fabrication process and a better understanding of the physical phenomena limiting the cavity performance ($E_{acc}$, $Q_0$) are necessary. The thermal behaviour of the cavity is concerned by these limitations and two topics must be considered: a) the general problem of the cavity surface thermal stability when subjected to anomalous RF losses due to local defects or impacting electrons, as well as the effect of the residual surface resistance creating a global heating even in the absence of defects. b) Thermal phenomena related to the cooling conditions of some critical cavity components (main power coupler, high order mode coupler, beam pipes, ...). At low temperatures the RF surface resistance, the thermal conductivity, the heat transfer coefficient and the thermal contact resistance are strongly temperature dependent. The steady-state heat equation with the boundary conditions is then strongly non-linear.

POISNL, a Finite Element Method (FEM) based computer code can handle such problems with multiple regions (materials) and arbitrarily shaped 2D-axisymmetrical computational domain.

Considering the expensive and large time consuming experimental tests of these devices, the disposal of such tool can greatly improve the design productivity: firstly it can be used to determine the critical parts in the system by numerical simulations and for systematic study of the effect of different parameters, secondly by considering more efficient and simple experimental models in order to validate the design principles.

2. CODE DESCRIPTION

The POISNL code was developed by R. BRIZZI at the IPN Orsay laboratory for heat transfer analysis in complex geometry devices operating in liquid helium LHe. It was built with the aid of the MODULAR Finite Element Software Library (MODULEF) developed in France by the INRIA [2] and written in Fortran 77.

MODULEF library includes more than 2000 procedures and has a large portability. Among its capabilities: automatic mesh generation (2D and 3D) using different kinds of algorithm, solution of linear systems by direct and iterative methods with dynamic memory allocation, computation of eigen-vectors (eigen-values) of 2D and 3D operators, graphic output modules, ... It is now largely used to solve engineering problems in various fields: heat transfer analysis, elasticity, dynamic problems, incompressible fluid flow, electromagnetic fields, resonant cavities ...

Briefly, MODULEF consists of modules communicating between them through some invariant data structures. A module is set of procedures performing one logical process which transforms Input Data Structure into Output Data Structure.

POISNL

The governing equation is the steady-state non-linear heat equation. For a 2D-axisymmetrical computational domain this equation takes the following general form in cylindrical coordinates $(r,z)$:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r k_r(T) \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( k_z(T) \frac{\partial T}{\partial z} \right) + \sigma_i = 0 \quad (1)$$

where the subscript $i$ concerns parameters inside the $i^{th}$ medium $k_i(T)$ being the material thermal conductivity and $\sigma_i$ the rate of heat generation per unit volume inside this subdomain. For all the cases discussed in this paper $\sigma_i = 0$.

Typical boundary conditions are: adiabatic wall, prescribed heat flux, heat transfer with LHe (Kapitza conductance $H_k$) and prescribed temperature.

Using the Garlekin method, the continuous variational formulation of the boundary value problem is derived. The domain discretization is performed by various automatic mesh generators included in the general preprocessor of MODULEF. In our case the finite elements are triangles and Lagrange polynomials of degree 1 are used for interpolation between the triangle nodes. The resulting non-linear system is solved in POISNL by an iterative method using linearisation technique. The linear system is solved by the Cholesky method with dynamic memory.
allocation. A relaxation factor is used between two successive iterations in order to improve the process convergence. The steady-state solution is obtained when the RMS value of the relative variation of the temperature distribution over the whole domain is less than $10^{-5}$ (typically $20 \mu K$ in the considered temperature range).

3 - APPLICATION TO SURFACE THERMOMETRY ON SRF CAVITIES STUDIES

We have developed special thermometers (superfluid "epoxy" thermometers) for accurate temperature measurement of HeII cooled cavity surface. These thermometers are employed as diagnostic probes on SRF cavities in order to detect and locate anomalous RF losses on the cavity surface and electron emission trajectories [3 - 4]. POISNLL code was useful for the study of the first test cell, allowing us to analyse the influence of different parameters on the thermometer's thermal response. Moreover, it was possible to simulate and design a new thermometer (superfluid "vacuum" thermometer) with improved efficiency. The results predicted by the numerical model were well confirmed by the experimental data [4].

![Fig. 1 - Calculated isotherms (POISNLL) of the calibration cell for SRF cavity surface thermometry](image)

A new calibration cell was designed consisting of a niobium plate-heater assembly machined from a high purity niobium ingot (RRR = 270). Three resistive thermometers ($Th_3 - Th_2$) are located on the heater post for in situ thermal conductivity measurements. Four other resistive thermometers ($Th_5 - Th_4$) are used to measure the radial temperature distribution on the Nb plate heated side. The vacuum thermometer to be calibrated is installed at the center of the He II cooled side of the Nb plate (Fig. 1). The calibration procedure was already discussed [4] so we will focus on the simulation results.

The thermal conductivity $k(T)$ is measured with this cell so the Kapitza conductance $H_k$ at Nb/HeII interface remains the only unknown model parameter of the problem. By using the POISNLL code, its value can be adjusted by fitting the experimental temperature distribution to the numerical results. A trial and error method was adopted for this purpose: starting from typical $H_k$ values published in previous works [5 - 6], the temperature field within the niobium specimen is calculated. Then $H_k$ was adjusted by considering the temperature at the vacuum thermometer location and the temperature distribution on the heated side of the niobium specimen and comparing the experimental data with the calculated values and so on until a good fit.

![Fig. 2 - Radial temperature distribution on the Nb plate heated side](image)

This method has been used at different bath temperature ($1.5 K \leq T_{bath} \leq 1.9 K$) for different heater power ($10 mW \leq Q \leq 120 mW$). In Fig. 1 are displayed the isotherms calculated by POISNLL. In the heater post region the isotherms are horizontal plans giving a high confidence on the $k(T)$ experimental determination. The agreement between experimental and calculated temperature in this region is better than 15 %. The radial temperature distributions $\Delta T(R)$ are plotted in Fig. 2 for two different $T_{bath}$ ($1.5 K$ and $1.9 K$). Concerning $\Delta T(R)$, the difference between experimental and calculated temperatures is less than 0.3 mK which is very close to the measurement resolution ($\pm 0.2 mK$).

Finally after $H_k$ adjustment using this method, for different $T_{bath}, a$ least square fit yields the expression:

$$H_k(W/cm^2.K) = h_0.T_{bath}^{n} = 0.0258 T_{bath}^{4.4}$$

(2)

The previous reported data for niobium samples [5 - 6], of different purity and surface treatment shows the following ranges for the $h_0$ and $n$ values: $3.18 \leq n \leq 4.65$ and $0.0072 \leq h_0 \leq 0.043$. In conclusion, this adjusting method gives results within the range of the previous experimental data obtained with different measurement techniques [5 - 6].

4 - APPLICATION TO 1.5 GHz SRF SINGLE CELL CAVITY WITH MODIFIED LHe COOLING

In order to simplify the design of cryostats for large electron accelerators, it is planned to surround each cavity with its own LHe vessel. The vessel will not contain the couplers and beam tubes, thus permitting to have all critical connections (RF couplers) inside the insulation vacuum. This should allow an increased safety margin against cavity vacuum leaks. We have checked in the experiment described below, that the behaviour of the cavity at high field level is not affected by the absence of direct LHe cooling of the beam tubes.

A single cell cavity has been modified for this purpose (Fig. 3). Close to one of the iris, the beam tube has been cutted and a niobium disk has been electron beam welded in order to isolate the beam tube from the LHe bath. The beam tube is now located in a separated vacuum chamber closed with stainless steel flanges.
A manganin wire heater wrapped around the beam tube and located at \( z = -11.5 \text{ cm} \) from the center of the cavity is used to simulate the coupler RF losses. The resulting temperature gradient along the beam tube is measured by means of nine thermometers located at the periphery of the tube at three different positions A, B, C (\( z_A = -10.2 \text{ cm}, z_B = -8.7 \text{ cm}, z_C = -7.2 \text{ cm} \)). At each position 3 thermometers, disposed radially and separated by 120°, measure the mean temperature at that location. The nine thermometers were calibrated in a separate experiment by comparison to two secondary standard thermometers (Germanium and Carbon Glass type) mounted in the same OFHC thermometric copper block. An overall absolute accuracy of ±10 mK was obtained in the temperature range 1.8 K < T < 16 K.

Using the POISNL code, the special cavity assembly was simulated with the pertinent boundary conditions. The RF power losses \( q \) on the cavity surface were introduced by means of an analytical formula deduced by fitting the URMEL code solutions for the magnetic field distribution:

\[
H(r,z) = \text{cte for } |z| \leq 3.86 \text{ cm}
\]
\[
H(r,z) \propto \exp(-\beta_I z) \quad \beta_I = 0.95 \text{ cm}^{-1}
\]
\[
|z| \leq 5.6
\]
\[
\beta_2 = 0.61 \text{ cm}^{-1} \quad 5.6 \leq |z| \leq 15.0
\]

and \( q \propto R_s(T) H^2 \).

In all the numerical runs the RF surface resistance \( R_s(T) \) includes the BCS term and a residual term \( R_{res} = 20 \text{ nOhm} \).

The critical part of this special cavity, from the thermal point of view is the joining region between the body of the cavity and the niobium flange welded to the beam tube. This flange acts like a fin "draining" a large part of the heat flux towards LHe bath hence reducing the heat load to the cavity body. A close view of the mesh in this region is presented in Fig. 4.

![Fig. 4 - Close view of the triangular mesh in the vicinity of the cavity iris - Heat balance in the critical region](image)

In Fig. 5 a typical POISNL result is displayed showing the isotherms in the vicinity of the niobium flange. Notice the isotherms deformation in this region and the effect of HeII cooling on the Nb flange wetted wall. The calculated temperatures at the three different points (A, B, C) were compared to the measured values for different heater power. Concerning the temperature distribution, the discrepancy (≤5.5%) between the two results, which is much larger than the experimental errors, must be attributed to the niobium thermal conductivity curve \( k(T) \), which can slightly differ from the niobium effectively used.

Furthermore for an input power \( Q = 3 \text{ W} \), the temperature remains lower than the niobium critical temperature. Moreover the hottest points are located in the region of vanishing surface magnetic field. Using the measured temperature distribution we have calculated the heat flux flowing in the beam tube, the discrepancy between the numerical results and experimental data was less than 24% in the heater power range 0.5 W < \( Q < 3 \text{ W} \). This error is principally due to uncertainty on \( k(T) \).

From the POISNL computed temperature field in the critical region and the \( H_x \) value used in the model a heat balance was deduced. The results are summarized in Fig. 4, for an input power \( Q = 3 \text{ W} \), 1.94 W are drained by the niobium flange towards LHe, the remaining heat flux (1.06 W) is extracted by LHe in the entrance region of the cavity (4.1 cm < |z| < 5.55 cm).

In conclusion \( \frac{3}{2}Q \) is drained by the Nb flange and \( \frac{1}{2}Q \) is extracted from the cavity body in the region of attenuated RF surface magnetic field (|z| > 3.9 cm). These results were well confirmed by experimental tests: 1E5 = 19 MeV/m was reached with \( Q_{exp} > 10^{10} \). These values were reproduced up to \( Q = 3 \text{ W} \) with just a slight \( Q_{exp} \) decrease at 3 W, which was certainly due to the bath temperature increase.

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