Experiments with Synthetic Coloured Noise at the Heavy Ion Storage Ring ESR

F. Nolden, J. Pinkow, U. Schaaf, M. Steck
Gesellschaft für Schwerionenforschung
P.O. Box 11552
D-6100 Darmstadt

1 INTRODUCTION

Noise of controlled spectral form promises various applications in accelerator physics. In connection with a cooling mechanism the equilibrium momentum distributions can be shaped, e.g. to increase Landau damping of collective longitudinal or transverse instabilities. In stochastic extraction schemes, special shapes of the diffusion function help to control the spill time independently of the diffusion velocity around the resonance. Our generator produces voltage step sequences that are repeated periodically and consist of 4096 different single steps. The sequences are digital Fourier transforms of the desired spectral amplitudes with random phases. The spectra consist of 2048 frequency lines in the desired spectral range. Amplitudes and line spacings are such that phase space trajectories are stochastic (Chirikov's criterion) under normal conditions. A first application of synthetic noise was a measurement of the cooling force of the ESR cooler.

2 THE NOISE GENERATOR

2.1 *Hardware Configuration*

The hardware of the generator consists of a memory card with a storage capability of 4096 words, a clock, and a 12 bit DAC. The memory is read out sequentially in a recirculating mode, i.e. the first word follows the last word without break. The readout frequency is controlled by the clock and is variable in the range 0 - 80 kHz. The analog signal from the DAC consists of nearly rectangular voltage steps (rise time $\approx 2\mu s$).

2.2 *Low-Frequency Spectrum*

The principal idea is that random time sequences can be specified in a way that their spectra are of arbitrary form. We show how the frequency spectrum (the usual Fourier transform) of the signal is related to its *discrete* Fourier transform. We denote the number of voltage steps by N and the sampling (clock) rate by t_s. The sampling frequency is defined by $\omega_s = 2\pi/t_s$. The generator signal can be modelled as the convolution (denoted by $*$) of a unit rectangle $r(t)$ of length t_s with a finite series of delta impulses which is periodically repeated:

$$a(t) = r(t) * \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} a_n \, \delta\left(t - (kN+n)t_s\right). \qquad (1)$$

The Fourier spectrum $\tilde{a}(\omega)$ of $a(t)$

$$\tilde{a}(\omega) = 2\omega_r \sum_{k=-\infty}^{\infty} \sum_{m=0}^{N-1} \tilde{a}_m \frac{\sin(\omega t_s/2)}{\omega} \delta\left(\omega - (kN+m)\omega_r\right) \qquad (2)$$

consists of equidistant, discrete lines at a distance equal to the repetition frequency $\omega_r = \omega_s/N$. The sequence of coefficients $\tilde{a}_m$ is just the discrete Fourier transform (DFT) of the time sequence a_n:

$$\tilde{a}_m = \sum_{n=0}^{N-1} a_n e^{2\pi i m n/N}. \qquad (3)$$

This relation allows the easy construction of time sequences from customer modelled spectra. The fact that $a(t)$ is real leads to the condition $\tilde{a}_{N-m} = \tilde{a}_m^*$. Therefore only lines up to the Nyquist frequency $\omega_s/2$ can be chosen arbitrarily. Mirror lines appear beyond the Nyquist frequency. Furthermore, the spectrum contains a form factor $\sin(\omega t_s/2)/\omega$ which is due to the rectangular shape of the voltage steps. In order to get a sequence α_m of spectral lines up to the Nyquist frequency, the time sequence a_n must be the DFT of

$$\tilde{a}_m = \alpha_m \, e^{i\phi_m} \frac{m}{2\sin(m\pi/N)}. \qquad (4)$$

The phases ϕ_m are chosen at random. Above harmonics of the sampling frequency, the low-frequency part reappears qualitatively but is suppressed by the form factor.

2.3 *Quality of Low-Frequency Spectra*

We have measured the low-frequency spectra using an HP Dynamic Signal Analyzer. The measured amplitudes at frequencies below 100 kHz are quantitatively described by eq. (2). Figure 1 shows a chimney-like spectrum (as it could be used in stochastic extraction) with a sampling frequency of 25 kHz. Mirror frequencies and their suppression according to the form factor are clearly visible. Single lines can be seen in fig. 2. The steepness is > 35 dB between individual lines which have a typical spacing of a few Hz. It is limited by the resolution of the DAC. Depending on the application, analog low-pass filtering may sometimes be desirable in order to get rid of mirror frequencies. Because of the finite steepness of analog filters, only roughly 90 % of the Nyquist range can be effectively employed, anyway.

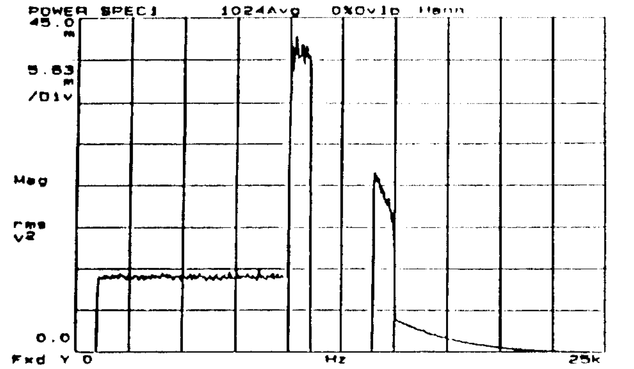

Figure 1: Typical noise spectrum for stochastic extraction

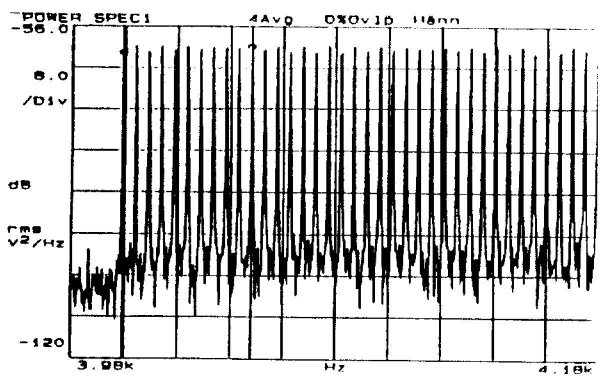

Figure 2: Single lines resolved at the edge of a spectrum

2.4 *RF Mixing*

To get a useful rf signal, a local oscillator (LO) and a (preferably SSB) mixer are needed. In order to preserve the excellent quality of the low-frequency spectra, the mixers and the following amplifiers must be driven below their nominal power. Non-linearities of the mixers or amplifiers lead to undesired spectral bands at harmonics of the LO frequency and smear out sharp edges in frequency spectra. Care must be taken either to eliminate these bands or to ensure that they do not interact with the beam at harmonics of the revolution frequency.

3 BEAM DYNAMICS WITH SYNTHETIC NOISE

When a coasting beam in a storage ring is excited by rf noise, the evolution of the momentum distribution can be described by a diffusion equation. However, this may not be true for excitation spectra consisting of individual lines. If there were only one line, the result would be synchrotron motion with a well-defined separatrix in phase space. It has been known for some time that if a dynamical system is excited simultaneously by several oscillators at different frequencies, the result is chaotic motion in phase space, if the height of the individual 'would-be separatrices' (the bucket height) is comparable to their distance. This is usually referred to as Chirikov's criterion. Chirikov's numerical experiments have shown that the random walk through phase space can be well described by means of a diffusion constant calculated from the mean power as though the spectrum were continuous [1].

The height of the 'would-be separatrices' is proportional to $(P/N)^{1/4}$, where P is the mean squared voltage seen by the beam. If the sensitivity of the kicker is constant over the spectral range of the noise, P is directly proportional to the electrical power at the input port. Let us now assume that a broad spectrum with a sharp edge at some frequency ω_e is applied. Then the question arises whether phase space is distorted beyond energies corresponding to ω_e because of the presence of discrete lines. We intend to resolve this question using numerical simulations.

4 MEASUREMENTS OF THE ELECTRON COOLING FORCE

One method to measure the cooling force of an electron cooler is to excite the beam with rf noise and then to measure the equilibrium momentum distribution Ψ, which is given by the relation

$$(\ln \Psi)' = \frac{F_{\parallel} v}{D}, \tag{5}$$

where $F_{\parallel}$ is the effective longitudinal cooling force, v the mean beam velocity, and D the diffusion function. Precise measurements of $F_{\parallel}$ over a large range of relative velocities are feasible, if the slope of $\ln \Psi$ remains significantly large. Therefore the ratio of the cooling force and the diffusion should be kept approximately constant, and because the cooling force varies with velocity, the application of *coloured* noise should enhance the precision of cooling force measurements. Up to now, such measurements have only been performed with band-limited white noise.

It is useful to describe the cooling force in a frame which is co-moving with the mean longitudinal velocity v of the electron beam. Because of the acceleration of the electron beam by a dc voltage, its velocity distribution ψ_e becomes non-isotropic. It is commonly written as a Bi-Maxwellian:

$$\psi_e(v_s, v_r, v_z) \propto \exp\left(-\frac{v_s^2}{2v_{\parallel}^2} - \frac{v_r^2 + v_y^2}{2v_{\perp}^2}\right) \tag{6}$$

The longitudinal velocity $v_{\parallel}$ can be smaller than the transverse velocity $v_{\perp}$ by orders of magnitude. From the theory of magnetized cooling [3] we expect that the cooling force rises linearly from zero to a maximum at an electron velocity close to $v_{\parallel}$. Hence $v_{\parallel}$ is an important measure of the quality of the electron cooling device.

At the ESR cooler ring [4], we excited a $^{129}Xe^{54+}$ beam (energy 243 MeV/u, 10^6 particles) with synthetic rf noise.

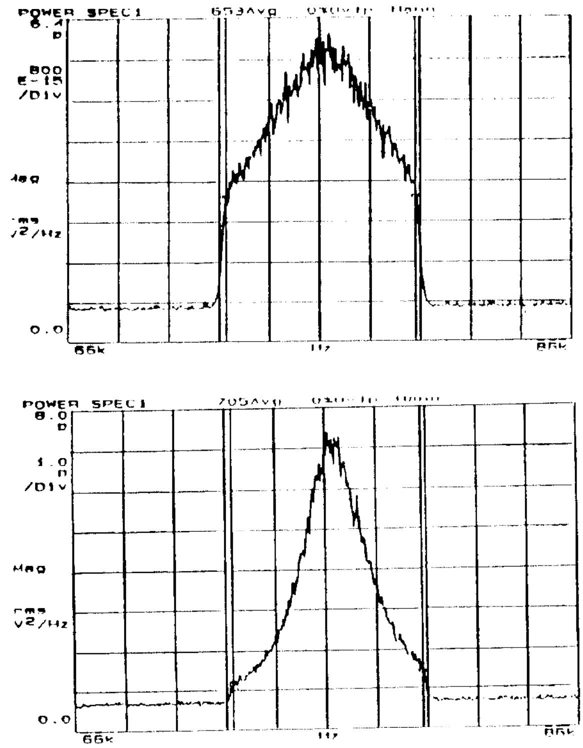

Figure 3: Schottky spectra with white and coloured noise

The noise spectra were symmetric around a frequency which was close to the 26$^{\text{th}}$ harmonic of the mean particle revolution frequency. The Schottky spectra were measured at the 49$^{\text{th}}$ harmonic. Figure 3 allows the comparison of a spectrum resulting from an excitation with band limited white noise (above) and from a power spectrum of triangular shape (below).

The spectrum obtained with coloured noise allows a significantly more precise evaluation of the cooling force, because the slope of the distribution function is larger. We determined $\ln \Psi$ after subtraction of background noise and by performing a polynomial fit (see figure 4). The cooling force was then calculated according to eq. (5). We draw the following conclusions from the resulting curve:

- The force has both a distinct maximum and a minimum at velocities v_s in the co-moving frame of -7100 m/s and +6200 m/s, respectively. (We define v_s to be zero at the zero of the cooling force.) The corresponding electron energies are approximately 0.13 meV. This confirms earlier measurements [2], and shows that we operate the ESR cooler in the regime of magnetized cooling.

- The height of the maximum at negative velocity is larger than the absolute height of the minimum at positive velocity. This tendency has also been observed at the LEAR cooler [5], and at the TSR of Heidelberg [6]. We do not have a straightforward explanation for this observation.

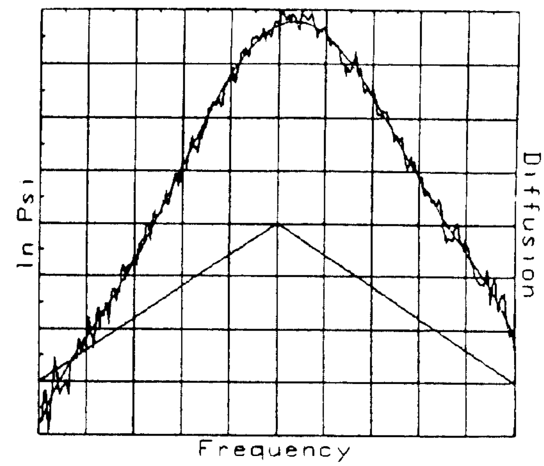

Figure 4: $\ln \Psi$ and D from lower spectrum of fig. 3

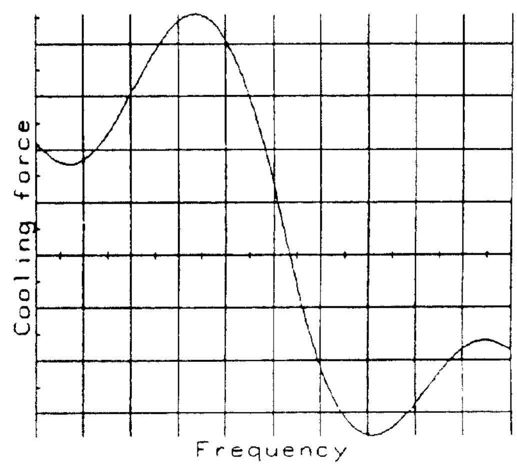

Figure 5: $F_{\|}$ in arbitrary units from fig. 4

- Near the edge of the noise spectrum the statistics is too poor to allow an interpretation. Measurements with a broader excitation spectrum, somewhat higher beam current and larger measuring time are expected to yield better information about the cooling force at higher velocities.

5 REFERENCES

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