Absolute Measurement of Beam Energy by Compton Scattering*

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Abstract

We propose that the energy of a relativistic electron beam can be measured by observing the Compton radiation it scatters from a resonating cavity. The radiation is to be observed at the frequency at which the intensity does not depend upon the phase of the beam relative to the rf. The measurement will give $\delta E/E$ to within 10^{-4} in about 10 seconds for a 1 mA beam. The number of photons can be significantly increased, and the measuring time decreased proportionately, by use of an rf wiggler in place of the cavity.

1. INTRODUCTION

The absolute measurement of the beam energy to high precision is of great importance in accelerator operations, and various systems have been proposed or implemented in recent years.[1]-[3]

We have considered using a cavity resonator as a wiggler. The idea is to increase the energy of the photons of initial angular frequency ω_0 by a Compton-scattering process. The maximum frequency of the scattered photons is $4\gamma^2\omega_0$ for a fully relativistic beam. If ω_0 can be precisely measured, then a careful measurement of the maximum final frequency will yield an equally precise value for γ^2, and the beam energy can be inferred. The method is attractive because the only measurements involved are optical (to find the final wavelength) and frequency (ω_0). Precision of order 10^5 or better is routinely achieved in both types of measurements. The proposed method, compared with wigglers, offers some advantages in precision; it is much easier to maintain high stability of the frequency in a resonant circuit than in the spacing of the component parts of a wiggler.

At least three problems can arise: First, the final energy depends both on the initial frequency of the photon and also upon its direction, so the signal can be contaminated by unwanted waves in any real cavity. Second, the input wavetrain is of finite length, so the initial frequency is not precisely known. Third, the spectrum depends upon the phase of the beam relative to the cavity rf. It turns out that the variation of the spectrum with phase provides a marker that can be used to determine the beam energy.

2. THE ACCELERATING FIELD

To be definite, we will discuss scattering from a square cavity of sides L in the $x-y$ plane (z dimension arbitrary), oscillating in its TE_{nn0} mode. The electric field vector inside the cavity can be expressed as a function of position and time as

$$\mathbf{E} = \mathbf{E}_0 \sin(\frac{n\pi x}{L}) \sin(\frac{n\pi y}{L}) \cos(\omega_0 t), \tag{1}$$

and is 0 outside; $\mathbf{E}_0$ is parallel to the z axis. This can be written

$$\mathbf{E} = -\frac{\mathbf{E}_0}{4}\left(e^{i\frac{n\pi}{L}(x+y)} + e^{-i\frac{n\pi}{L}(x+y)} - e^{i\frac{n\pi}{L}(x-y)} - e^{-i\frac{n\pi}{L}(x-y)}\right) \sin(\omega_0 t). \tag{1.a}$$

Thus, the mode is a superposition of four plane waves that move parallel to the diagonals of the square. All four waves have the same frequency, and all four contribute to the scattering. We can cause three of the waves to be unimportant by directing the beam also along a diagonal, so it is parallel to one wave, antiparallel to another, and perpendicular in the laboratory frame to the other two. On scattering, the maximum frequency in the laboratory of the antiparallel wave is very nearly $4\gamma^2\omega_0$; that of the parallel wave is ω_0; and that of the two perpendicular waves is $2\gamma^2\omega_0$. The important part of the spectrum of the high-frequency backscattered waves is so far from that of the others that we can neglect them.

3. COMPTON SCATTERING

The number of photons scattered in unit time is

$$\frac{dN}{dt} = \left(\frac{N}{V}\right) I\sigma\sqrt{2}L, \tag{2}$$

where I is the number current, that is, the number of incident electrons per unit time, N/V is the number of photons per unit volume in the cavity, σ is the Thompson cross section, and L is the cavity side. Some reasonable numbers for a facility such as CEBAF are instructive: $I = 6 \times 10^{14}\,\mathrm{sec}^{-1}$ ($100\,\mu$A beam current), $E = 10^6$ volts/meter, $\omega_0 = 2\pi \times 1.5 \times 10^9\,\mathrm{sec}^{-1}$ (rf frequency), $\sqrt{2}L = 0.20$ meter. These give approximately 10^8 photons scattered per second. This number represents the scattering rate of photons at all energies between 0 and $\hbar\omega_{max}$. The number scattered into a frequency interval $\delta\omega_{max}/\omega_{max} = 2\delta\gamma/\gamma$ is $\left[dN/dt\right]\delta\omega_{max}/\omega_{max}$, approximately, because the Compton scattering cross-section is nearly enough flat for our computations. Thus, the upper-end counting rate in a relative energy band 10^{-3} is something like 2×10^5 per second.

The spectrum of Compton-scattered photons in the electron rest frame, which is almost identical with the center-of-mass (c.m.) frame, is

$$I = A(1 + \cos^2\Theta), \tag{3}$$

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where Θ is the c.m. polar angle. Klein-Nishina corrections are neglected.

In the laboratory, the energy of the final photon is connected to the c.m. angle by the Lorentz relation

$$\hbar\omega \approx \gamma\hbar\omega'(1 - \cos\Theta) \tag{4}$$

(ω' is the c.m. frequency, $2\gamma\omega_0$). Therefore, the energy spectrum in the laboratory is

$$I(\omega) = A\left[1 + (1 - \frac{\omega}{\gamma\omega'})^2\right] \tag{5}$$

between the limits $\omega = 0$ and $(\omega/\gamma\omega') = 2$, and is 0 elsewhere.

A rather severe limitation arises because of the finite size of the cavity. The signal that accelerates the electrons is

$$\begin{aligned} f(t) &= \sin(\omega_0 t), \quad 0 < t < 2\pi n/\omega_0, \\ &= 0 \quad \text{elsewhere.} \end{aligned} \tag{6}$$

The intensity found from the Fourier transform $F(\omega)$ of this function is (absorbing all constant factors into B)

$$| F(\omega) |^2 = B \frac{\cos^2}{\sin^2}\left(\frac{n\pi\omega}{2\omega_0}\right)\frac{1}{(\omega^2 - \omega_0^2)^2} \quad \begin{matrix} n \text{ even} \\ n \text{ odd.} \end{matrix} \tag{7}$$

If the relative phase is changed by $\pi/2$, so

$$f(t) = \cos(\omega_0 t), \quad 0 < t < 2\pi n/\omega_0, \tag{8}$$

then the intensity becomes

$$| F(\omega) |^2 = B \frac{\cos^2}{\sin^2}\left(\frac{n\pi\omega}{2\omega_0}\right)\frac{\omega^2}{(\omega^2 - \omega_0^2)^2} \quad \begin{matrix} n \text{ even} \\ n \text{ odd.} \end{matrix} \tag{9}$$

In the remainder of this note, we will express the ratio ω/ω_0 as v.

The observed spectrum is the fold of (7) or (9) with the Compton distribution. One comment concerning the fold is necessary: Because the total cross section for Compton scattering is independent of the initial photon energy, $\int_0^v A\left[1 + \left(1 - 2v_1/v\right)^2\right] dv_1$ must be independent of v, so the amplitude A must vary as $1/v$. The observed spectrum is thus either

$$I(v_1) = B\int_{v_1}^{\infty}(v^2 + 2vv_1 + 2v_1^2)\frac{1}{v^3(1 - v^2)^2}\frac{\cos^2}{\sin^2}\left(\frac{n\pi v}{2}\right) dv \tag{10a}$$

or

$$I(v_1) = B\int_{v_1}^{\infty}(v^2 + 2vv_1 + 2v_1^2)\frac{1}{v(1 - v^2)^2}\frac{\cos^2}{\sin^2}\left(\frac{n\pi v}{2}\right) dv, \tag{10b}$$

according as to whether $f(t) = \sin\omega_0 t$ or $\cos\omega_0 t$. These spectra have been evaluated numerically by use of the IMSL routine QDAGI.

Given a particular mode for the cavity, the spectrum depends upon the phase of the beam relative to the rf. The scattered intensities are equal (the curves describing the spectra cross) at a point in the vicinity of $v = 0.5$ to $v = 1$. This crossing point is easy to detect by measuring the intensity of the scattered radiation as the phase is swept through all possible values. The intensity will be observed to change with the phase at all frequencies except at the crossing, which can be found by seeking a null point as the phase of the cavity relative to the beam is varied. Calculation then gives the γ-factor, with a precision limited by our ability to determine the crossing frequency. In the TE_{110} mode, the value of v at the crossing point is found to be 0.5132. The Compton spectra for the extreme phases in the TE_{100} are shown in Figure 1. Figure 2 then shows a schematic layout of a system to detect the crossing frequency.

This method can be tried most conveniently with a beam energy of about 500 MeV and a cavity frequency of 300 MHz. This will put the crossing frequency in the middle of the visible part of the spectrum, where conventional optical techniques can be used for transmitting and measuring the signal. The frequency is a factor 5 lower than that used for calculation of the counting rate at a CEBAF-like facility, so the rate in this case is higher. For a $200 - \mu$A beam, the photon flux in a band of width 0.5 nm will be approximately 10^7 per second. The statistical noise on this signal will be about 3×10^3, which implies that it should be possible to achieve a precision of 0.001 in v in about 1 second. In 10 seconds, precision can be increased to the point that $\delta E/E$ is 3×10^{-4}.

4. REFERENCES

[1] Watson et al., " Precision Measurements of the SLC Reference Magnets," SLAC-PUB-4908, March 1989.

[2] B. Bevins, " Precision Beam Energy Measurement at CEBAF Using Synchrotron Radiation Detectors," CEBAF TN 91-054.

[3] Ivan Karabekov, " Specification for Construction of Absolute Energy Monitors for CEBAF," CEBAF TN 91-045.

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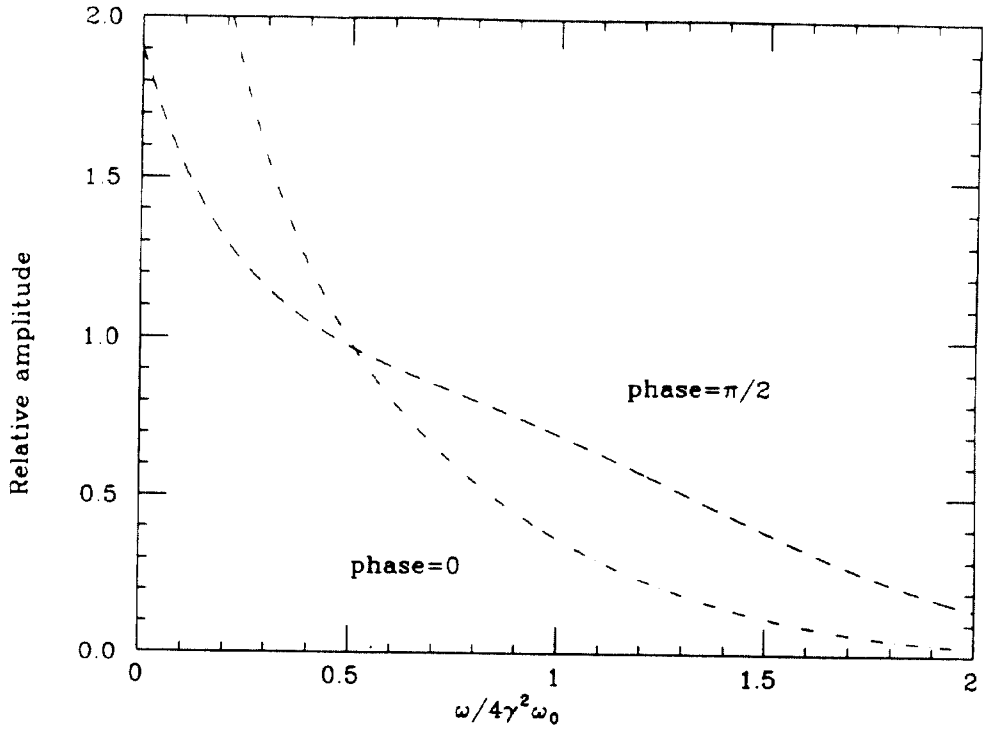

Figure 1. Observed Compton backscattering in TE_{100} mode.

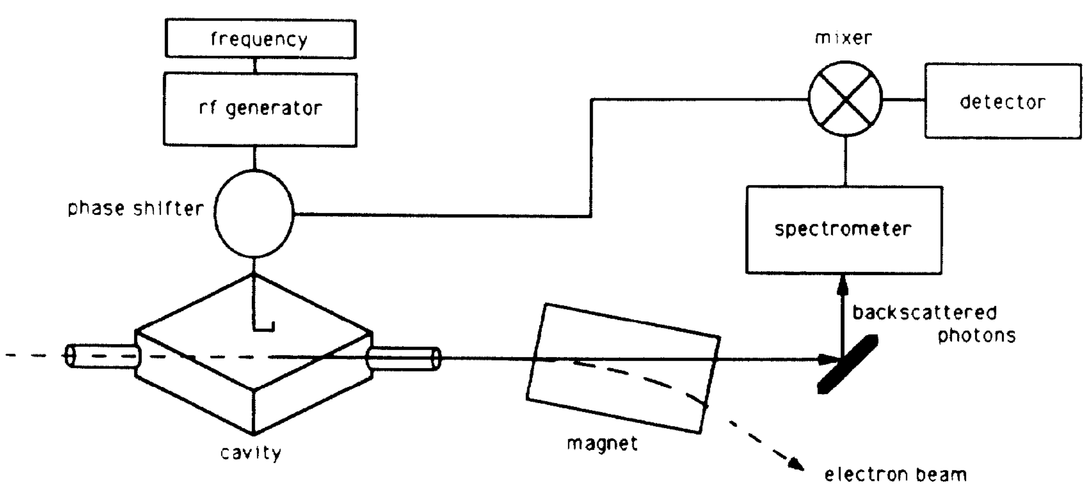

Figure 2. Schematic layout of system to detect crossing frequency.