Study On A Longitudinal Damper System*

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1 INTRODUCTION

If the beam is injected in a circular accelerator with longitudinal errors $\left(\frac{\Delta E}{E_s}\right)_0$ and $\Delta\varphi_0$, the beam will execute a coherent oscillation in longitudinal phase space and will be diluted within a time interval of about $\frac{1}{\Delta\nu_s}$ turns, even if it is properly matched to the characteristics of the machine unless there is an effective damper system to prevent this. Here $\Delta\nu_s$ is the tune spread of the synchrotron oscillation of the bunch and $\Delta\varphi_0 = \varphi_0 - \varphi_s$ where φ_0 the initial phase of the bunch center, φ_s synchronous phase.

Using the small amplitude approximation, the longitudinal admittance of the accelerator can be expressed as an ellipse whose equation is as follows:

$$S^2 = \frac{\nu_{s0}\beta^2}{h\eta}\frac{E_s}{\omega_{\rm rf}}\Delta\varphi^2 + \frac{h\eta}{\nu_{s0}\beta^2}\frac{E_s}{\omega_{\rm rf}}\left(\frac{\Delta E}{E_s}\right)^2, \qquad (1)$$

where $\nu_{s0} = \left(\frac{-Zh\eta V_a e\cos\varphi_s}{2\pi A E_s \beta^2}\right)^{1/2}$, the longitudinal tune for small amplitude, Z and A are the charge state and mass number of the ions, V_a the amplitude of the RF voltage, E_s the synchronous specific energy, $\omega_{\rm rf}$ the angular frequency of the RF system, h the harmonic number. η can be expressed as follows:

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}, \qquad (2)$$

where γ_t is the transition energy. Without a damper system, the emittance of the beam will ultimately develop into a properly centered matched ellipse with an area S^2 in phase space that is larger than S_0^2 of the injected one which is also matched but off centered by $\Delta\varphi_0$ and $\left(\frac{\Delta E}{E_s}\right)_0$.

Let the equation of the injected off centered but matched ellipse be:

$$S_0^2 = \frac{\nu_{s0}\beta^2}{h\eta}\frac{\rm E_s}{\omega_{\rm rf}}(\Delta\varphi - \Delta\varphi_0)^2 + \frac{h\eta}{\nu_{s0}\beta^2}\frac{\rm E_s}{\omega_{\rm rf}}\left[\frac{\Delta \rm E}{\rm E_s} - \left(\frac{\Delta \rm E}{\rm E_s}\right)_0\right]^2 \qquad (3)$$

and the equation of the centered matched ellipse is Eq. (1). The dilution factor S^2/S_0^2 can be expressed as follows:

$$\frac{S^2}{S_0^2} = \left(1 + \frac{S_{c0}}{S_0}\right)^2 \qquad (4)$$

where

$$S_{c0}^2 = \frac{\nu_{s0}\beta^2}{h\eta}\frac{E_s}{\omega_{\rm rf}}\Delta\varphi_0^2 + \frac{h\eta}{\nu_{s0}\beta^2}\frac{E_s}{\omega_{\rm rf}}\left(\frac{\Delta E}{E_s}\right)_0^2 \qquad (5)$$

We take RHIC[1] as an example, for gold ions, if we require an emittance growth less than 20%, the maximum allowable injection errors are $\left(\frac{\Delta E}{E_s}\right)_0 = 10^{-4}$, $\Delta\varphi_0 = 7^o$.

Next, we present an analysis of the requirements for the longitudinal damper system to damp the coherent oscillation from injection errors or coherent instabilities following the consideration used in analyzing the transverse damper system.[2-4] The damper system consists of beam monitors, signal processing electronics, an amplifier station and a kicker device. A signal proportional to bunch energy error or bunch center phase angle is amplified and transported to another location where it is applied across the damping kicker for the correction. The summation of the outputs of two beam position monitors at locations with the same dispersion function and betatron phase advance $(2\rm m + 1)\,\pi$ is proportional to the energy error of the bunch and avoids the signal due to coherent betatron oscillation. Comparing the phase of RF signal and that of the bunch induced in the monitor, one can get the bunch center phase angle.

For a given damper system, we will consider two modes of operation: (i) the kicker voltage is proportional to the beam signal and (ii) the kicker voltage is adjusted to a constant value having a sign which depends on the sign of the beam signal.

2 PROPORTIONAL MODE

In the proportional mode, the correction effect is proportional to the bunch error ΔE_n. Let $\Phi_n = \binom{\Delta\varphi_n}{\Delta E_n}$ being the vector representing the longitudinal coherent oscillation at the n-th crossing of the beam monitor. The kicker voltage $V_{\rm kn}$ during n-th crossing of the kicker is proportional to ΔE_n with the sign chosen to effectively reduce it

$$eV_{kn} = -K_0\Delta E_n \qquad (6)$$

where K_0 is a constant which depends on the gain and other electrical properties of the damper system. Then the change of ΔE during n-th crossing of the kicker is expressed as

$$\delta E_n = \frac{ZeV_{kn}}{A} = -K\Delta E_n, \qquad (7)$$

where

$$K = \frac{ZK_0}{A} \qquad (8)$$

Then the vector $\Phi_{\rm n+1}$ at the (n+1)-th crossing is

$$\phi_{\rm n+1} = M_{cp}\; M_c\; M_{kc}\; M_k\; M_{pk}\; \phi_n \qquad (9)$$

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where M_c and M_k are the 2×2 matrices representing the effect of RF cavity and damping kicker respectively, M_{cp}, M_{kc} and M_{pk} are the 2×2 transfer matrices from RF cavity to monitor, from damping kicker to RF cavity and from monitor to damping kicker respectively. We have

$$M_c = \begin{pmatrix} 1 & 0 \\ \frac{ZeV_a \cos\varphi_s}{A} & 1 \end{pmatrix}, \quad (10)$$

$$M_k = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - K \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} M_{pk}^{-1}, \quad (11)$$

The transfer matrix of drift space can be expressed as follows. For example,

$$M_{cp} = \begin{pmatrix} 1 & \frac{2\pi h\eta}{E_s\beta^2}\frac{L_{cp}}{L} \\ 0 & 1 \end{pmatrix}, \quad (12)$$

where L_{cp} the central orbit length from cavity to monitor and L the total length.

Then

$$\Phi_{n+1} = \left[M_0 + M_{cp}\, M_c\, M_{kc} \begin{pmatrix} 0 & 0 \\ 0 & -K \end{pmatrix} \right] \Phi_n \quad (13)$$

M_0 is the unperturbed transfer matrix at the beam monitor and

$$M_0 = \begin{pmatrix} 1 - \mu_{s0}^2 \left(\frac{L_{cp}}{L}\right) & \frac{2\pi h\eta}{E_s\beta^2}\left[1 - \mu_{s0}^2 \frac{L_{cp}}{L}\left(1 - \frac{L_{cp}}{L}\right)\right] \\ \frac{ZeV_a \cos\varphi_s}{A} & 1 - \mu_{s0}^2\left(1 - \frac{L_{cp}}{L}\right) \end{pmatrix} \quad (14)$$

where μ_{s0} is the unperturbed synchronous oscillation phase advance per revolution $\mu_{s0} = 2\pi\nu_{s0}$. The determinant of M is

$$\mathrm{Det}\, M = 1 - K \neq 1 \quad (15)$$

and $K > 0$ for damping.

The eigenvalues λ are

$$\lambda = \left(1 - \frac{\mu_{s0}^2}{2} - \frac{K}{2} + \frac{K}{2}\mu_{s0}^2\frac{L_{kc}}{L}\right) \pm \sqrt{\left(1 - \frac{\mu_{s0}^2}{2} - \frac{K}{2} + \frac{K}{2}\mu_{s0}^2\frac{L_{kc}}{L}\right)^2 - 1 + K} \quad (16)$$

There are two cases

(i) $K_1 \leq K \leq K_2$ where K_1 and K_2 are the solutions of the following equation

$$\left(1 - \frac{\mu_{s0}^2}{2} - \frac{K}{2} + \frac{K}{2}\mu_{s0}^2\frac{L_{kc}}{L}\right)^2 - 1 + K = 0 \quad (17)$$

Considering $\mu_{s0}^2 \frac{L_{kc}}{L} << 1$,

$$K_1 \simeq -2\mu_{s0} - \mu_{s0}^2\left(1 + 2\frac{L_{kc}}{L}\right), \quad (18)$$

$$K_2 \simeq +2\mu_{s0} - \mu_{s0}^2\left(1 + 2\frac{L_{kc}}{L}\right). \quad (19)$$

For damping, $K > 0$ is required, so that

$$0 \leq K \leq K_2 \quad (20)$$

In this case the eigenvalues of the system are complex quantities

$$\lambda = \sqrt{1 - K}e^{\pm i\mu} \quad (21)$$

Apart from a phase factor μ, the fractional damping rate is given by $\frac{1}{2}K$ per turn.

(ii) If $\left(1 - \frac{\mu_{s0}^2}{2} - \frac{K}{2} + \frac{K}{2}\mu_{s0}^2\frac{L_{kc}}{L}\right)^2 - 1 + K > 0$, there are two real eigenvalues. It can be proved that, if

$$K_2 \leq K \leq \left(2 - \frac{\mu_{s0}^2}{2}\right) / \left(1 - \frac{\mu_{s0}^2 L_{kc}}{2L}\right), \quad (22)$$

both $|\lambda_1|$ and $|\lambda_2|$ are smaller than 1 and

$$\lambda_1 \geq \sqrt{1 - K_2} \quad (23)$$

This means that, if Eq. (22) is fulfilled, the system is also a damping system.

Because the fractional damping rate of the coherent oscillation amplitude is $K/2$ per turn, the required maximum kicker voltage V_{k0} for damping the oscillation amplitude from ΔE_0 to $\Delta E_0 e^{-1}$ within N turns is given by

$$V_{k0} = \Delta E_0 \frac{2}{N}\frac{A}{Ze}. \quad (24)$$

3 CONSTANT VOLTAGE MODE

In the constant voltage mode the bunch energy error signal is used only to trigger the correction by applying a constant kicker voltage. It also controls the sign of the voltage according to the actual sign measured of the instantaneous bunch energy error. This method still provides effective damping but with a considerably reduced peak power requirement.

The Courant-Snyder invariant of the longitudinal coherent oscillation at the n-th turn is given by

$$S_{cn}^2 = \frac{\nu_{s0}\beta^2 E_s}{h\eta\omega_{\mathrm{rf}}}(\Delta\varphi_n)^2 + \frac{h\eta}{\nu_{s0}\beta^2}\frac{E_s}{\omega_{\mathrm{rf}}}\left(\frac{\Delta E_n}{E_s}\right)^2 \quad (25)$$

Let δE_n be the bunch energy change in the n-th turn under the action of the damping kicker, then assuming the phase $\Delta\varphi_n$ is unchanged and neglecting the quadratic terms of $\frac{\delta E_n}{E_s}$, we have a change in S_{cn}^2 given by

$$\Delta S_{cn}^2 = 2\frac{h\eta}{\nu_{s0}\beta^2}\frac{E_s}{\omega_{\mathrm{rf}}}\frac{\delta E_n}{E_s}\frac{\Delta E_n}{E_s} \quad (26)$$

Using explicitly the equation for ΔE_n

$$\frac{\Delta E_n}{E_s} = -S_{cn}\left(\frac{\nu_{s0}\beta^2\omega_{\mathrm{rf}}}{h\eta E_s}\right)^{\frac{1}{2}} \sin\theta_n \quad (27)$$

where θ_n is the phase of the longitudinal coherent oscillation during n-th crossing the kicker which varies from turn to turn and

$$\theta_n = (n\mu_{s0} + \theta_0), \tag{28}$$

we obtain

$$\Delta S_{cn} = -\left(\frac{h\eta E_s}{\nu_{s0}\beta^2\omega_{\rm rf}}\right)^{\frac{1}{2}} \frac{\delta E_n}{E_s} \sin\theta_n. \tag{29}$$

For the correction kicker we take

$$V_{kn} = V_0 \frac{|\sin\theta_n|}{\sin\theta_n} \tag{30}$$

where V_0 is a constant and V_{kn} is the kicker voltage during n-th turn which is a constant but with a sign opposing to that of ΔE_n. We have finally

$$S_{cn} = S_{c0} - \sum_{n=1}^{n} \left(\frac{h\eta E_s}{\nu_{s0}\beta^2\omega_{\rm rf}}\right)^{\frac{1}{2}} \frac{ZeV_0}{AE_s} |\sin\theta_n|. \tag{31}$$

We can replace $\sum_1^N |\sin\theta_n|$ by $N\overline{|\sin\theta_n|} = N\frac{2}{\pi}$ and

$$S_{cN} = S_{c0} - N\frac{2}{\pi}\left(\frac{h\eta E_s}{\nu_{s0}\beta^2\omega_{\rm rf}}\right)^{\frac{1}{2}} \frac{ZeV_0}{AE_s}. \tag{32}$$

The required constant kicker voltage for damping the coherent oscillation amplitude from ΔE_0 to $\Delta E_o e^{-1}$ within N turns is

$$V_0 = \Delta E_0 \left(1 - e^{-1}\right) \frac{\pi}{2} \frac{A}{ZeN} \simeq \frac{\Delta E_0 A}{ZeN} \tag{33}$$

Comparing Eqs. (24) and (33), we find that the required voltage for the constant voltage mode is only half of the maximum kicker voltage required for proportional mode. The constant voltage mode is proposed to allow a reduction by a factor of 4 of maximum power requirement. When the residual coherent oscillation amplitude become small enough one may switch to the proportional mode.

If we use $\Delta\varphi_n$ signal to control the damping kicker voltage, the $\Delta\varphi_n$ signal should be delayed by 1/4 synchronous oscillation period. This is because the phase difference between ΔE_n and $\Delta\varphi_n$ is $\pi/2$. Without this delay, the maximum allowable K value is only about $\frac{E_s\beta^2}{2\pi h\eta}\mu_{s0}^2$ and the maximum damping rate is only $1/2\left(\mu_{s0}^2 - \frac{\mu_{s0}^4}{4}\right)$ per turn, which is much smaller than the previous analyzed case.

4 RHIC PARAMETERS

Taking RHIC as an example, in RHIC due to nonlinear effect, for gold ions $\Delta\nu_s = 10^{-4}$ and for protons $\Delta\nu_s = 4\times10^{-5}$. We take gold ions case for it requires more kicker power. If $\Delta\varphi_0 = 0$, $\left(\frac{\Delta E}{E_s}\right)_0 = 10^{-4}$, [or $\Delta\varphi_0 = 7°$, $\left(\frac{\Delta E}{E_s}\right)_0 = 0$], then from Eq. (33), the required total kicker peak voltage V_k is

$$V_k = \frac{A}{Ze}\Delta E_0 \times 10^{-4} = 280 \text{ volts}. \tag{34}$$

The required total peak kicker power P_k is

$$P_k = \frac{V_k^2}{n_k R_k} \tag{35}$$

where n_k is the number of kickers with $\frac{V_k}{n_k}$ volts on each, R_k the kicker impedance. Let's select $R_k = 50\Omega$, then

$$P_k \simeq \frac{1600}{n_k} \text{ watts}.$$

If we select two kickers $n_k = 2$, then the total peak power is about 800 watts. For proton beam, this system can damp an initial error $\left(\frac{\Delta E}{E_s}\right)_0 = 2.4\times10^{-4}$ within 2.5×10^4 turns. The rise and fall time of the wide band damper system should be less than 100 nsec each to allow damping of the coherent motion of 2×57 bunches individually.

For damping the coherent oscillation induced by coupled bunch longitudinal instabilities we don't need damping bunches individually. It is only necessary to damp some dangerous modes. The coherent oscillation amplitude induced by instability is determined by the sensitivity of the beam monitor which should be less than the possible injection error. The damping time of the damping system is inversely proportional to the coherent oscillation amplitude. For the above analyzed damping system of RHIC, oscillation amplitude $\left(\frac{\Delta E}{E}\right)_0 = 10^{-4}$, the damping time is 10^4 turns or 128 nsec for gold ion. In order to fulfill the requirement for damping coupled bunch instability, the damping time should be less than 60 msec[1] so that, the sensitivity of beam monitor should be better than $\left(\frac{\Delta E}{E_s}\right) = 5\times10^{-5}$ or $\Delta\varphi = 3.5°$. Otherwise we have to increase the peak kicker power because the damping time also inversely proportional to the kicker voltage or the square root of the peak kicker power.

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6 REFERENCES

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