Possible Approach to the Creation of Systems for High Energy Proton Beam Losses Measurements (part 2. Mathematical Aspects and Experimental Results)

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1 INTRODUCTION

The second part of our work presents the mathematical model of beam losses measurements. This model is based on the physical picture of the beam losses distribution described in part 1 and mathematical methods of the interpretation and processing of experimental data.

2 MATHEMATICAL MODEL OF THE SCHEME OF MEASUREMENTS

The physical picture of the development of beam losses and their registration by beam losses measurement system (BLMS) is represented in the physical part of this work (part 1). It can be described in the form of the linear mathematical model (the model of a linear regression) where the data of each subsequent (with respect to the beam) beam losses monitor (BLM, radiation monitor or RM) are the linear superposition of the data of previous monitors plus a free member:

$$y_k(i) = y_{k0} + a_{k1}y_1(i) + \dots + a_{k,k-1}y_{k-1}(i) + \nu_k(i) \quad (1)$$

Here $y_k(i)$ are the data of k-th BLM in i-th (i=1,2, ...M) measurement (cycle), k=1,...N, N is the total number of radiation monitors, $\nu_k(i)$ - random values which define the errors of the data of k-th BLM [1].

In the frames of the physical model y_{k0} signifies the part of the data of k-th BLM which is not connected with the data of previous k-1 monitors and is exclusively defined by beam losses in the area that is controlled by k-th radiation monitor. It is natural, therefore, to call y_{k0} as "own" data of k-th BLM. Coefficients a_{km} define a possible dependence of the data of corresponding monitors.

We will not discuss here shortcomings of proposed model (1) pointing out that the results we obtained with its help do not conflict with the physical picture of the studied process.

The most common practice is to solve equation (1) with the use of the technique based on the method of least squares (LSM). Attempts of the direct use of LSM may, however, cause serious problems connected with the bad condition [8] of the so-called design matrix (see further) corresponding to system (1). In this case LSM estimates become unstable in the sence that relatively small errors of input data lead to considerable errors of the obtained solution and the accuracy of these estimates is very low. It is necessary, therefore, to have some criterion of the stability of initial system (1). We used the simple numerical criterion that is based on the well-known estimates of D.W.Marquardt [2] connected with the comparing of matrix eigenvalues and invariant estimates of the condition of a matrix [8].

If this criterion indicates that the design matrix of system (1) is well conditioned then well-known LSM estimates (see, for example, [7]) of the parameters of system (1) permit to make clear (with the corresponding accuracy) the disposition of losses sources caused by the charged particles which were lost from the transported beam.

If the design matrix is badly conditioned then for the construction of stable estimates one should use special mathematical methods. Amongst such methods we point out the regularization of A.N.Tikhonov [6], the technique based on the methodes of the principal components [5]. In our case it was convenient to use some results following from the method of the reduction of Y.P.Pytiev [1]. One of the main advantages of the reduction consists in the correct and from a mathematical point of view relatively simple use of an additional information about a studied physical phenomenon. It was particularly important for us in connection with our plans to continue and evolve this work in the future. Let us expound some ideas based on our physical model and developed on the basis of the reduction.

The concept of the reliability of a model [1] (the technique of the use of this concept is presented in details, for example, in [3],[4]) permits to reject effectively the experimental data that are not the realizations of process (1) but random outliers at measurements. We interpreted these random cycles as equipment transient errors at the injection or extraction of a beam. The problem of the robastness [7] of estimates are solved, therefore, automatically.

One can write system (1) in the matrix form:

$$y_k = A_k x_k + \nu_k, \quad (2)$$

where $x_k=(y_{k0},a_{k1},\dots,a_{k,k-1})$, $y_k=(y_k(1),y_k(2),\dots,y_k(M))$, $\nu_k=(\nu_k(1),\nu_k(2),\dots,\nu_k(M))$, matrix elements of A_k are easily defined from (1). Convert now (2) with the use of a linear operator R_k:

$$R_k y_k = R_k A_k x_k + R_k \nu_k =$$

$$= x_k + (R_k A_k - I_k) x_k + R_k \nu_k. \quad (3)$$

Here I_k is the unit $k \times k$ matrix.

If now R_k is considered as a regularising operator and $R_k y_k$ is interpreted as a solution of (2) then such estimate will be accompanied by errors of two kinds: a noise part $R_k \nu_k$ and a systematic error $(R_k A_k - I_k) x_k$ or so-called "false signal". The systematic error on the operator difference $(R_k A_k - I_k)$ and the unknown vector x_k. If there are not a proper additional information about x_k then one has a natural wish to reduce "false signal" to zero. However, it is not reasonable if the corresponding noise level is very high. Moreover, allowing some relatively small systematic error one can effectively reduce the noise and (as a result) balance the influence of these two kinds of errors on the solution [1]. The corresponding stable solution of (2) may be presented in the form:

$$R_k = A_k^* \left(A_k A_k^* + \omega_k I_k\right)^{-1} = \left(A_k^* A_k + \omega_k I_k\right)^{-1} A_k^* \quad (4)$$

where A_k^* is the transposed matrix A_k, $A_k^* A_k$ is the design matrix for system (1), parameter ω_k is defined by the relation of levels of the resulting noise and "false signal" [1].

Let us assume now that certain a priori information allows to surmise that some BLMS consisting of L (less than N) specially disposed BLM permits to find all possible losses sources. It is evident that in accordance with our model and in the frames defined by the corresponding errors the sum of "own" BLM data (we use the abbreviation SOD for this sum) must be the same for all such systems (we call them "hermetic"). This fact, on one hand, can be used as an additional information to obtain unknown parameters of (1) and, on the other hand, can be used to identify various "hermetic" BLMS.

On the basis of the results mentioned above we have created the special package of application programs. These programs running on the computers of the IHEP (Protvino) distributed computer network execute the data acquisition, permit to solve system (1) and represent the obtained results in the form of various histograms, pictures and tables.

3 RESULTS OF DATA PROCESSING

The proposed method was tested with the use of the experimental data obtained from the part of the channel for the transportation of an intensive (approx. 2×10^{13} protons/cycle) beam from the U-70 IHEP accelerator to an experimental area. BLM were disposed at the places of the most probable (on the basis of the relations of apertures and envelopes) beam losses. The matter of beam profile monitors (BPM) was used as beam losses sources.

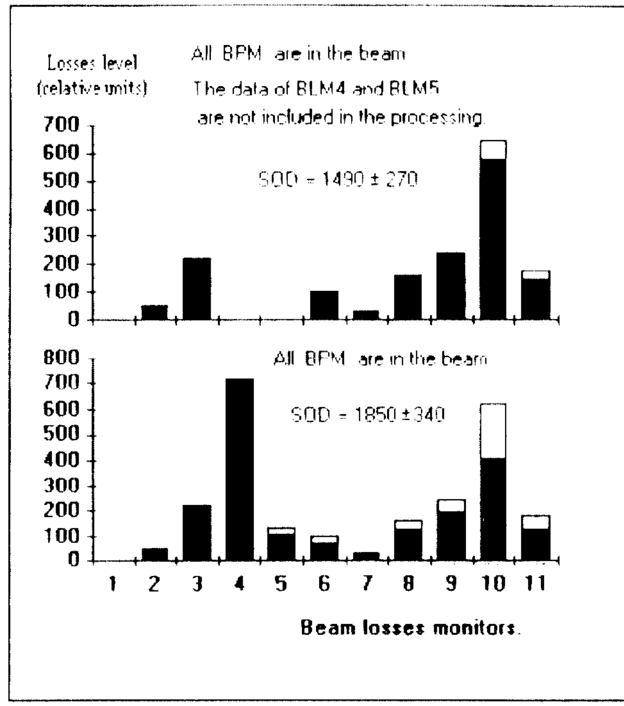

Figure 1: Results of measurements and processing (1).

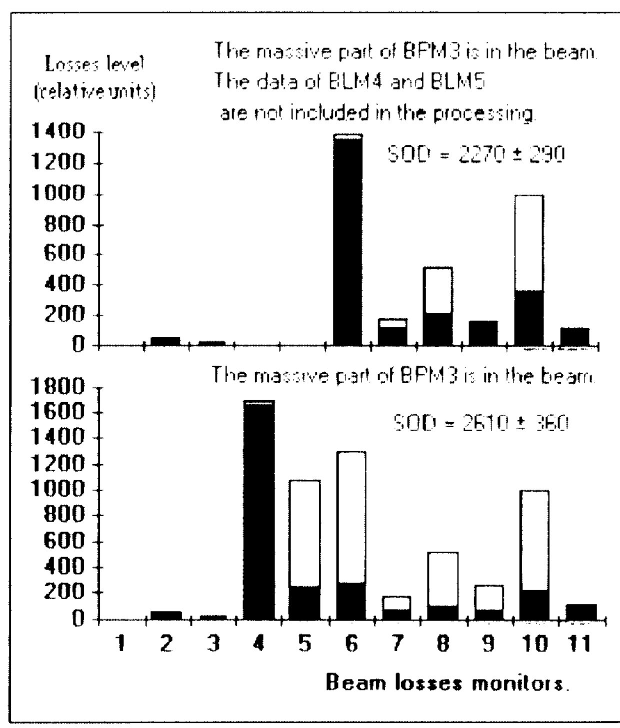

Figure 2: Results of measurements and processing (2).

Table 1: An example of the distribution of the data of radiation monitors.

BLM number	2	3	4	5	6	7	8	9	10	11
2	100%									
3	9%	91%								
4	2%	2%	96%							
5	-	1%	53%	46%						
6	-	1%	61%	1%	37%					
7	-	1%	60%	1%	2%	36%				
8	-	1%	64%	1%	-	2%	32%			
9	-	1%	38%	1%	-	-	1%	59%		
10	-	1%	58%	1%	-	-	1%	1%	38%	
11	-	1%	10%	1%	-	-	1%	1%	20%	66%

Tens of working regimes of this beam extraction subsystem were processed with the use of the proposed method.

The histograms in fig.1-2 show the results obtained experimentally at some regimes and processed (the dark part of each histogram) with the use of linear model (1). To examine the properties of our measuring system the results of all regimes were also processed without the data of some BLM. The most critical area of the beam transport channel was controlled by BLM4 and BLM5 (see physical part of this work - part 1). That is why we want to demonstrate here some results obtained without the data of these radiation monitors. Fig.1-2 are the illustrations of the fact that the disposition and the physical characteristics of our radiation monitors permits BLMS to remain "hermetic" in the worst possible case for this diagnostic system.

We note that in all cases represented here the total relative procedural and measurement errors were assumed to be equal to 5-10%. The resulting noise errors of the obtained solutions were 10-25% and typical "false signals" $(R_k A_k - I_k)x_k$ were 20-30% of the corresponding $y_k(i)$ values. A simple analysis of the obtained coefficients a_{km} allowed us to locate the primary losses sources which formed the observed radiation field along the channel. Consider an example presented in Table 1. The rows of this table describe the distribution of the total signal of each BLM obtained in one of the processed regimes. The columns show the shares of this signal which are interpreted as the results of the actions of the primary losses sources localised in the areas controlled by the corresponding radiation monitors.

4 CONCLUSION

One of the main purposes of our paper was to demonstrate one more time that the use of mathematical methods based on the physical picture of a studied phenomenon can add new features to a measuring system.

In our view the future of the mathematical part of this work can be formulated in the next form:

a) to test the proposed technique for circular accelerators and their separate parts;

b) to construct and to test a mathematical model (or models) which not only is adequate to the physical picture of the beam losses distribution in accelerators and to the physical characteristics of BLMS but also allow the most effective application of the ideas of the method of the reduction connected with the use of a various additional information: data of all possible diagnostic devices, results of modelling calculations etc.

5 REFERENCES

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