Classical motion of a charged particle in the presence of a static, homogeneous magnetic field and a linearly frequency shifted electromagnetic plane wave

S. Varró and G. Kocsis
Central Research Institute for Physics
H-1525 Budapest P.O.Box 49 Hungary

Abstract

The relativistic motion of a charged particle in the field composed by a static, homogeneous magnetic field and a linearly frequency shifted (chirped) electromagnetic plane wave propagating along the magnetic field is studied. The classical relativistic equations of motion are used where the radiation reaction is neglected. The analytic solution shows that the energy of the particle is systematically increased due to the frequency shift. This result illustrates the theoretical possibility of a new type of laser accelerator.

1 INTRODUCTION

Recently considerable effort have been devoted to the search for fundamentally new methods for particle acceleration (see e.g. [1] and [2]). The most promising methods proposed rely on (direct or indirect) applications of the large electric fields present in high intensity laser beams. However, as is well-known [3], the motion of a charged particle in a monochromatic plane wave is periodic, and the energy gained by the particle in the first half of the period of the oscillation is lost in the second half of the period. There have been several ways proposed to overcome the above difficulty. For reference, we mention some of them below.

- If the dispersion relation of an electromagnetic plane wave is suitable modified, for example by propagation *in a medium* or on an interface of two media, then the field configuration generated this way can result in a monotonic acceleration along the particle's trajectory. Examples for this scheme are the inverse Cherenkov accelerator [4], the grating–linac [5] and the thin layer dielectric laser accelerator [6].

- High–intensity lasers can generate strong secondary fields in plasmas, and then the latter fields can accelerate charged particles. This is the principle of the beat–wave laser accelerator [7].

- *In vacuum*, if the trajectory of the particle is constrained by the presence of a static (spatially periodic or homogeneous) field, then under the joint action of this field and of a strong laser field a systematic increase of the particle's energy can occur. Examples for this scheme are the inverse free electron laser [8] and the autoresonance laser accelerator [9].

In the present paper we give an analytic treatment of the relativistic motion of a charged particle moving *in vacuum* in the presence of the field composed by a static homogeneous magnetic field and a linearly frequency shifted (chirped) electromagnetic plane wave propagating along the magnetic field. The present study was motivated by the expectation that the presence of the chirp (which destroy the periodicity of the wave) can result in an accumulation of the energy gained by the particle over time intervals much larger than the optical period. We note that, because of the nonlinear optical processes taking place in any laser amplifier the chirp (or in general, phase modulation) is always present (to some extent) in high power lasers. Perhaps the most important application of chirped pulses is the CPA (chirped pulse amplification) technique invented recently [10], with the help of which intensities up to 10^{18} W/cm^2 have been produced [11].

2 GENERAL EQUATIONS

Neglecting the radiaton reaction, the four–dimensional equations of motion of a particle of charge e and mass m read

$$\frac{d\vec{u}}{d\tau} = \frac{e}{mc}[u_0\vec{E} + \vec{u} \times \vec{B}], \quad (1)$$

$$\frac{du_0}{d\tau} = \frac{e}{mc}\vec{u}\vec{E}, \quad (2)$$

where τ is the proper time ,$u^i = \frac{dx^i}{d\tau} = (c\gamma, \gamma\vec{v}) = (u_0, \vec{u})$ is the four velocity of the moving charge, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$, c is the velocity of light in vacuum , $\vec{v}$ is the ordinary three–dimensional velocity.

The external electric and magnetic fields in equations (1) and (2) are composed by a static, homogeneous magnetic field and a linearly frequency shifted (chirped), circularly polarized electromagnetic plane wave propagating along the magnetic field:

$$\vec{E} = E_0\left[\vec{\epsilon}_x \sin(\omega\Theta - \sigma\Theta^2) + \vec{\epsilon}_y \cos(\omega\Theta - \sigma\Theta^2)\right], \quad (3)$$

$$\vec{B} = \vec{\epsilon}_z \times \vec{E} + \vec{\epsilon}_z B_0, \quad (4)$$

where $\Theta \equiv t - \frac{\vec{\epsilon}_z\vec{r}}{c}$, $B_0 = const.$, and $\vec{\epsilon}_{x,y,z}$ are mutually perpendicular unit vectors defining our Cartesian coordinate system of reference. In eq. (3) we have introduced ω, the central frequency of the laser field, and σ, which is the chirp parameter.

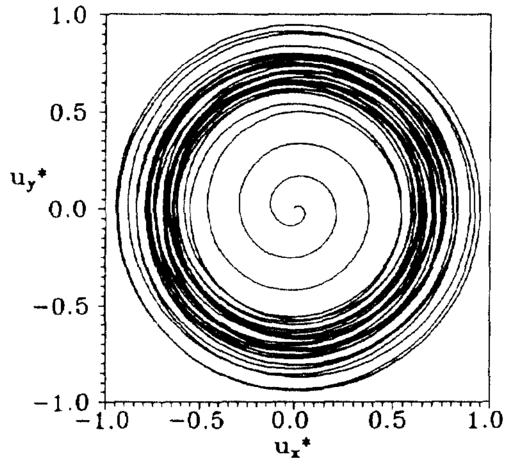

Fig. 1. Parametric plot of the transversal velocities of the particle. Here $0 \le \alpha\sqrt{\sigma}\tau \le 7$, K=0, $\omega/\sqrt{\sigma} = 30$, $\vec{\epsilon}_{x,y}\vec{u} = u^*_{x,y} c\Upsilon\sqrt{\pi/2}$.

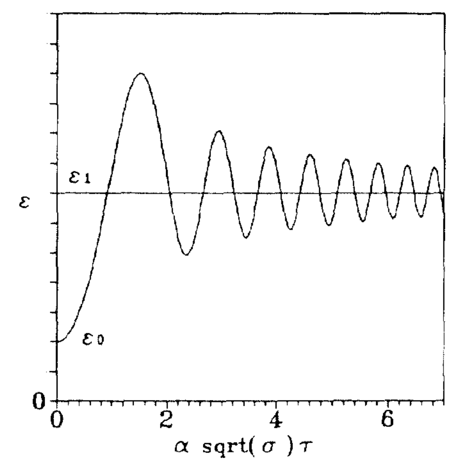

Fig. 2. The energy of the particle as function of the proper time. Here K=0, $\omega/\sqrt{\sigma} = 30$, $\varepsilon_0 = \varepsilon(\tau = 0)$, $\varepsilon_1 = mc^2\Upsilon\pi/(4\alpha)$.

It can be shown, that in the above described field configuration there is a constant of the motion, i.e. $u_0 - \vec{\epsilon}_z\vec{u} = \alpha c = constant$ whose existence plays an important role in reducing the solutions to quadratures. Integrating this expression over the proper time of the particle the Θ argument of the plane wave can be given as a function of the proper time: $\Theta = \alpha\tau + \delta$. α and δ are constants determined by the initial conditions. In the followings, for the sake of the simplicity, we set $\vec{\epsilon}_x\vec{u}(\tau = 0) = \vec{\epsilon}_y\vec{u}(\tau = 0) = 0$ and $\delta = 0$.

Taking into account these considerations, equations (1) and (2) can be easily integrated in terms of the well-known Fresnel integrals $S(x) = \sqrt{\frac{2}{\pi}}\int_0^x \sin(y^2)\,dy$ and $C(x) = \sqrt{\frac{2}{\pi}}\int_0^x \cos(y^2)\,dy$:

$$\vec{\epsilon}_x\vec{u} = c\Upsilon\sqrt{\frac{\pi}{2}}\sin\left(\omega_c\tau + K^2\right)\left[C(\Phi) + C(K)\right]$$
$$-c\Upsilon\sqrt{\frac{\pi}{2}}\cos\left(\omega_c\tau + K^2\right)\left[S(\Phi) + S(K)\right], \qquad (5a)$$

$$\vec{\epsilon}_y\vec{u} = c\Upsilon\sqrt{\frac{\pi}{2}}\cos\left(\omega_c\tau + K^2\right)\left[C(\Phi) + C(K)\right]$$
$$+c\Upsilon\sqrt{\frac{\pi}{2}}\sin\left(\omega_c\tau + K^2\right)\left[S(\Phi) + S(K)\right], \qquad (5b)$$

$$\vec{\epsilon}_z\vec{u} = \frac{c}{2\alpha}\Upsilon^2\frac{\pi}{2}\left(C(\Phi) + C(K)\right)^2 +$$
$$+\frac{c}{2\alpha}\Upsilon^2\frac{\pi}{2}\left(S(\Phi) + S(K)\right)^2 + u_{z0}, \qquad (5c)$$

Here we have introduced the notations: $\Upsilon \equiv \frac{eE_0}{mc\sqrt{\sigma}}$ and $K \equiv \frac{\omega\alpha - \omega_c}{2\alpha\sqrt{\sigma}}$ are dimensionless constant, $\omega_c \equiv \frac{eB_0}{mc}$ is the cyclotron frequency and $\Phi = -K + \alpha\sqrt{\sigma}\tau$.

Then the energy of the particle is given by

$$\varepsilon(\tau) = mcu_0 = mc\left[\vec{\epsilon}_z\vec{u} + \alpha c\right], \qquad (6)$$

The evolution of the transverse velocity and the particle energy can be seen on Figs. 1–2. for $K = 0$ and $\omega/\sqrt{\sigma} = 30$.

By integrating equations (5a–b–c) the trajectories as functions of the proper time can be obtained

$$\vec{\epsilon}_x\vec{r}(\tau) = \vec{\epsilon}_x\vec{r}(\tau = 0) - \frac{1}{\omega_c}\vec{\epsilon}_y\vec{u}(\tau) + \qquad (7a)$$
$$+\frac{c}{\omega_c}\Upsilon\sqrt{\frac{\pi}{2}}\cos\left(L^2\right)\left[C(\Psi) + C(L)\right] +$$
$$+\frac{c}{\omega_c}\Upsilon\sqrt{\frac{\pi}{2}}\sin\left(L^2\right)\left[S(\Psi) + S(L)\right],$$

$$\vec{\epsilon}_y\vec{r}(\tau) = \vec{\epsilon}_y\vec{r}(\tau = 0) + \frac{1}{\omega_c}\vec{\epsilon}_x\vec{u}(\tau) + \qquad (7b)$$
$$+\frac{c}{\omega_c}\Upsilon\sqrt{\frac{\pi}{2}}\cos\left(L^2\right)\left[S(\Psi) + S(L)\right] -$$
$$-\frac{c}{\omega_c}\Upsilon\sqrt{\frac{\pi}{2}}\sin\left(L^2\right)\left[C(\Psi) + C(L)\right],$$

$$\vec{\epsilon}_z\vec{r}(\tau) = \vec{\epsilon}_z\vec{u}(\tau)\frac{\Phi}{\alpha\sqrt{\sigma}} - \qquad (7c)$$
$$-\frac{c}{2\alpha}\Upsilon^2\sqrt{\frac{\pi}{2}}\frac{1}{\alpha\sqrt{\sigma}}\sin(\Phi)^2\left[C(\Phi) + C(K)\right] +$$
$$+\frac{c}{2\alpha}\Upsilon^2\sqrt{\frac{\pi}{2}}\frac{1}{\alpha\sqrt{\sigma}}\cos(\Phi)^2\left[S(\Phi) + S(K)\right],$$

where $L = \frac{\omega}{2\sqrt{\sigma}}$ and $\Psi = -L + \alpha\sqrt{\sigma}\tau$. These trajectories are illustrated on Figs. 3-4.

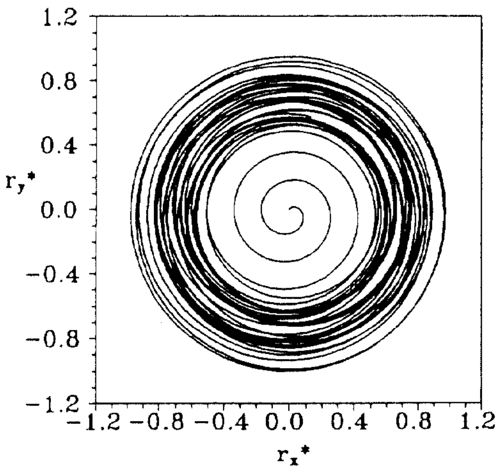

Fig.3. Parametric plot of the transversal trajectories of the particle. Here $0 \le \alpha\sqrt{\sigma}\tau \le 7$, *K=0,* $\omega/\sqrt{\sigma} = 30$, $\vec{\epsilon}_{x,y}\vec{r} = r^*_{x,y} c\Upsilon\sqrt{\pi/2}/\omega_c$.

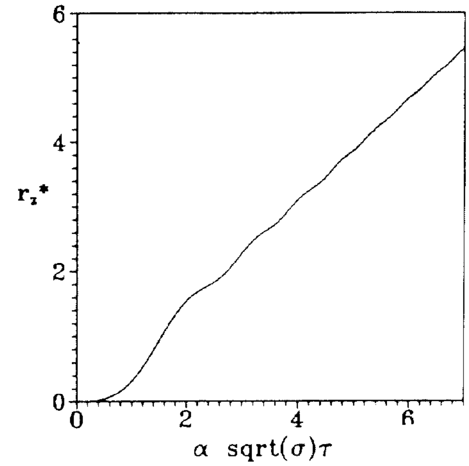

Fig.4. The longitudinal trajectory of the particle as function of the proper time. Here K=0, $\omega/\sqrt{\sigma} = 30$, $\vec{\epsilon}_z\vec{r} = r^*_z c\Upsilon^2/(2\alpha^2\sqrt{\sigma})$.

If one wishes to parametrize the motion by the time measured in the laboratory reference system, then using eq.(7c) the time observed in the laboratory reference system can be express in term of the proper time:

$$t(\tau) = \alpha\tau + \delta + \frac{\vec{\epsilon}_z\vec{r}(\tau)}{c}, \tag{8}$$

3 DISCUSSION

According to the non–periodic behaviour of the Fresnel integrals, on the basis of eqs. (5–7) we can state, that in contrast to the case of the monochromatic plane wave, the motion of the particle is non–periodic and the energy of the particle is systematically increased and tends toward an asymptotic value. This non–periodic behaviour is caused by the linear frequency shift of the electromagnetic wave and is independent of the magnitude of the static, honogeneous magnetic field (similar non–periodicity can be found without this magnetic field too).

Because $\lim_{x \to \infty} S(x), C(x) = \frac{1}{2}$ and because the Fresnel integrals have a fast convergence to their asymptotic value, $S(x), C(x)$ can be approached by their asimptotic value when $x > A$ (depending on the accuracy A is typically in the interval of [4,8]). Consequently, if $\tau \ge \frac{A}{\alpha\sqrt{\sigma}} + \frac{\omega}{2\alpha\sigma} - \frac{\omega_c}{2\alpha^2\sigma}$, the particle energy (and of course the other characteristics of the motion) can be substituted by its asymptotic value, and in this case the frequency shift of the electromagnetic plane wave is given by

$$\Delta\omega \equiv \sigma\Theta = \sigma\alpha\tau \ge A\sqrt{\sigma} + \frac{\omega}{2} - \frac{\omega_c}{2\alpha} \tag{9}$$

From this formula one can see the role of the static, homogeneous magnetic field. If this magnatic field is zero ($B_0 = 0$), then the $\Delta\omega$ frequency shift, which is required to reach to the asymptotic regime, is higher than $\frac{\omega}{2}$. However increasing the B_0 magnetic field the $\Delta\omega$ frequency shift can be reduced to an experimentally manageable value.

4 REFERENCES

[1] P.J. Channel (Editor): Laser Acceleration of Particles (Los Alamos, 1982) AIP Conference Proceedings No. 91, American Institute of Physics (New York, 1982)

[2] Ch. Joshi and Th. Katsouleas (Editors): Laser Acceleration of Particles (Malibu, California, 1985) AIP Conference Proceedings No. 130, American Institute of Physics (New York, 1985)

[3] L.D. Landau and E.M. Lifsic , The Classical Theory of Fields (Pergamon Press, Oxford, 1962)

[4] K. Shimoda , Appl. Optics 1 , 33 (1962)

[5] Y. Takeda and I. Matsui , Nucl. Instrum. Methods 62, 306 (1968)

[6] T. Weiland , p 203 , AIP Conf. Proc. No. 91 , (Los Alamos , 1982)

[7] T. Tajima and J.M. Dawson , Phys. Rev. Lett. 43, 276 (1979)

[8] R.B. Palmer , J. Appl. Phys. 43 , 3014 (1972)

[9] A. Loeb and L. Friedland , Phys. Rev. A33 , 1828 (1986)

[10] D. Strickland and G. Mourou, Opt. Commun. 56 , 219 (1985)

[11] K. Yamakawa, H. Shiraga, Y. Kato and C.P.J. Barty, Opt. Lett. 16 , 1593 (1991)