Remarks about the Standard Formula of Beam-Beam Radiation in Electron-Positron Collision

Huschang Heydari
Institut für Theoretische Elektrotechnik
Technische Universität Berlin

Abstract

Since it is important for the design of electron-positron linear colliders to estimate beam-beam radiation this paper introduces a more exact formula for calculating the radiated energy per frequency interval for classical synchrotron radiation but also for highly-relativistic beam-beam radiation. Reference is made to formulas ocurring in other papers.

1. INTRODUCTION

During the collision of highly relativistic electrons of an electron bunch with an opposite-directional positron bunch the electrons (as well as the positrons) are accelerated towards the center of the axis of the cylindrical beam and emit electromagnetic radiation as a consequence of this acceleration. As a result of the ultra-relativistic movement of the colliding charges only normal acceleration is of importance for the radiation capacity, i.e. the radiation emitted consists mainly of synchrotron radiation due to longitudinal acceleration, its percentage is the larger the higher the center of mass energy is. Therefore it is useful among other things to make a more thorough analysis of the classical synchrotron radiation formula.

2. A BRIEF OUTLINE OF THE DERIVATION OF A MORE EXACT SYNCHROTRON RADIATION FORMULA

On the basis of the energy radiated per frequency interval (of the photon) and per solid-angle ([1] : 14.67)

$$\frac{dI}{d\Omega}=\frac{\hbar\alpha_s}{8\pi^2}\omega^2\left|\int_{-\infty}^{+\infty} dt_{ret}\vec{m}^0 x\left[\vec{m}^0 x\vec{\beta}(t_{ret})\right]e^{i\omega\left\{t_{ret}-\frac{1}{c}\vec{r}_q(t_{ret})\cdot\vec{m}^0\right\}}\right|^2 \quad (1)$$

and using a circular movement (as in [1]) with an angular velocity ω_k, a more exact analysis of the phase Φ in (1) results in the term:

$$\Phi=\frac{\omega}{2}\left[t\left\{\frac{2}{\gamma_k^2\left(1+\frac{\rho\omega_k}{c}\right)}+\frac{\rho\omega_k}{c}\theta^{*2}\right\}+\frac{\rho}{3c}\omega_k^3 t^3\right] \quad (2)$$

since here

$$1-\frac{\rho\omega_k}{c}=\frac{1}{\gamma_k^2\left(1+\frac{\rho\omega_k}{c}\right)}$$

is used and not

$$1-\frac{\rho\omega_k}{c}=\frac{1}{\gamma_k^2\cdot 2}$$

All other calculation methods are the same as for example in [1].

The classical formula for the power radiated as a result of synchrotron radiation was analyzed more thoroughly and a more exact term than indicated in ref. [1], [2] was found

$$I(\omega)=\alpha_s\hbar\frac{\omega}{\omega_c}2\sqrt{3}\gamma_k\sqrt{\frac{\rho\omega_k}{c}\left(\frac{1+\frac{\rho\omega_k}{c}}{2}\right)}\int_{2\frac{\omega}{\omega_c}}^{\infty} d\chi K_{\frac{5}{3}}(\chi) \quad (3)$$

with the Sommerfeld fine-structure constant α_s, the photon frequency ω and the 'critical freqency'

$$\omega_c=3\sqrt{\frac{\rho\omega_k}{c}}\omega_k\left[\frac{\gamma_k^2\left(1+\frac{\rho\omega_k}{c}\right)}{2}\right]^{\frac{3}{2}} \quad (4)$$

for the angular frequency ω_k $\quad \gamma_k:=\left(1-\frac{\rho^2\omega_k^2}{c^2}\right)^{-\frac{1}{2}}$

with ρ being the radius of curvature and $(\rho^2\omega_k^2/c^2)=\beta$.

The difference from the formula indicated in the references comes on the one hand from (3) in the presence of the extra factor $\{(\rho\omega_k/c)([1+(\rho\omega_k)/c]/2\}^{1/2}$ and on the other hand from (4) for the critical frequency ω_c in the extra factor occurring therein. Both extra factors, of course, approach 1 (in the sense of the results presented in the references) with $\rho\omega_k\to c$ (but $\gamma_k\to\infty$).

Hence, we are not at variance with the references quoted above, but we have a more exact formula in the highly-relativistic case at our disposal without giving up $\gamma_k\to\infty$, $\omega_c\to\infty$ and $\omega_k\rho\to c$.

3. FURTHER RESULTS IN VIEW OF QUANTUM MECHANICS

In the same way in this case the fractional energy loss can be indicated making use of the Airy function [3], [4] and taking the extra factor into account as:

$$\frac{d\delta}{dx}:=\frac{I}{\hbar}=\frac{\alpha_s m^2\Delta z\omega}{p_{II}^2c^3}\cdot\sqrt{\frac{\rho\omega_k}{c}\left(\frac{1+\frac{\rho\omega_k}{c}}{2}\right)}\int_{U^*}^{\infty}dp\left[2\frac{p}{U^*}-1\right]A_i(p) \quad (5)$$

and with ($k_{II} = c\omega$),

$$1 - x = \frac{\hbar k_{II}}{p_{II}} = \frac{\hbar c\omega}{p_{II}}.$$

Thus with the electron energy $p_{II}c$, the electron rest energy $m := m_0c^2$ and the force $\vec{K}_\perp$(caused by the positrons) standing vertically on the path

$$U^* := \left(3\frac{\omega}{\omega_c}\right)^{\frac{2}{3}} = \left(\frac{m^3\omega}{|K_\perp| p_{II}^2 c^3}\right)^{\frac{2}{3}} \cdot \left[\left(\frac{\rho\omega_k}{c}\right)^{\frac{5}{3}} \cdot \frac{2c}{c+\rho\omega_k}\right] \quad (6)$$

was introduced using

$$\omega_k = \frac{|K_\perp|}{m\gamma_U \frac{\rho\omega_k}{c}} c \quad (7)$$

In the quantum mechanical case and in the case of low field variability (small field gradients) the equivalent to the equation above (calculated for 'scalar' electrons, i.e. solutions of the Klein Gordon equation) is:

$$\frac{I}{\hbar} = \frac{\hbar\omega\, dp_{rob}}{cp_{II}dx} = \frac{m^2\alpha_s\omega}{c^3 p_{II}^2} \int_{-}^{-} d\bar{z} \int_{U} dv \left[2\frac{V}{U} - 1\right] A_1(v) \quad (8)$$

with

$$U := \left[\frac{1}{\bar{\gamma}}\left(\frac{1-x}{x}\right)\right]^{\frac{2}{3}}; \qquad \bar{\gamma} := \frac{p_{II}\hbar c^2 |K_\perp(z)|}{m^3} \quad (9)$$

The classical case is resulting from the approximation for 'soft photons' $x \rightarrow 1$. The exact application, i.e. the use of the Dirac equation leads to [2]

$$\frac{I}{\hbar} = \frac{m^2\alpha_s\omega}{c^3 p_{II}^2} \int_{-}^{-} d\bar{z} \int_{U} dv \left\{\left[2\frac{V}{U} - 1\right]\left(\frac{1-x^2}{2x}\right) + \frac{(1-x)^2}{2x}\right\} A_1(V). \quad (10)$$

4. REMARKS

The photons coming into being as a result of beam curvature and whose energy radiated per frequency intervall is given by the improved formula (3) in the classical case or by (10) in the quantum mechanical case are in a position on the one hand to use pulse transfer to form pairs with the positrons $\gamma \rightarrow e^+ + e^-$ which can interact with their environment and can on the other hand emit compton radiation $\gamma + e^+ \rightarrow \gamma + e^+$ as well as $\gamma + e^+ \rightarrow +\gamma + \gamma + e^+$ e.t.c. (using quantum electrodynmic calculation).

The motivation to publish this paper is to know in how far in colliders (e.g. e^-e^+ colliders) which become more and more complicated, in particular in the collision region, the above mentioned extra factor is of importance (e.g. for the processes described above).

In other words, this paper is only meant to be a basis for discussion. Besides that it is clear that the incident electron can also interact in terms of quatum electrodynamics with one of the positrons directly whereby additional γ_t (protons) and e^-e^+ pairs can come into being. **This shows that the** investigation of beam-beam interaction or beamstrahlung, respectively, can be of importance for the selection of the collider type with referece to the estimation of losses in high energy experiments in the final analysis.

5. REFERENCES

[1] J.D. Jackson, Klassische Elektrodynamik, Berlin: de Gruyter, 1983.

[2] D.V. Schröder, Ph.D. Thesis DE-ACO3-76SF00515, Stanford Linear Accelerator Center, Stanford, California, USA, Ph.D. Dissertation, 1990.

[3] H. Heydari, "Determination of the Luminosity Spectrum", presented at the 3rd International Workshoo on Linear Colliders CL 91, Protvino, Russia, 1991

[4] H. Heydari, "A simple method to solve the Airy Function", presented at the 3rd International Workshoo on Linear Colliders CL 91, Protvino, Russia, 1991