Particle Acceleration in Electromagnetic Wave Fields

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A magnitized body rotating with its vector of magnetic dipole moment $\vec{\mu}$ inclined by an angle χ against its vector of angular velocity $\vec{\omega}$ carries electromagnetic fields which beyond the range of near field contributions, that is outside the transition zone, are pure spherical wave fields.

For practical reasons it is of advantage to substitute parameters ω and μ by two other parameters, each with the dimension of a length. One is the *light radius*

$$r_L = c/\omega, \tag{1}$$

characterizing the state of rotation of the magnet.

The other parameter is the *typical radius*

$$r_T = (e\,\mu/m\,c^2)^{1/2} \tag{2}$$

which characterizes the electromagnetic properties of the system constituted of a magnetic dipole and an electric monopole.

In the case of vacuum fields when there is no matter surrounding the rotating magnet, the spherical wave fields may locally be described in dimensionless form by plane wave fields of amplitude

$$f_0 = r_T^2/r_L r_0, \tag{3}$$

where r_0 is the distance from the rotating magnet.

Within the equatorial plane of rotation the local plane wave is linearly polarized with the electric vector parallel to the axis of rotation, whereas along the axis of rotation the local plane wave is circularly polarized. In between these two limiting orientations the local plane wave is eliptically polarized with the ratio of the minor to the major axis $\sin\alpha/\cos\alpha$ equal to the cosine of the latitudinal angle $\cos\vartheta$ measured against the axis of rotation.

Spherical vacuum wave fields generated by rotating magnets can constitute powerful accelerators for electrically charged particles. To some extend these configurations may be helpful as models to understand mechanisms at work in cosmic accelerators, for example in pulsar magnetospheres.

For this reason I have performed theoretical studies on relativistic particle dynamics within spherical vacuum wave fields of rotating magnets(K. O. Thielheim, 1991a). These investigations have revealed a rich variety of features characterizing the development of particle orbits and energy. In what follows I will give a short review of some of these findings.

The notion of *acceleration boundary* refers to the decoupling of particles from the spherical wave field (K. O. Thielheim, 1987) under the premises that the particle is initially at rest ($\vec{u}(\Phi_0) = 0$) where the fields strength has its maximum ($\Phi_0 = \pi/2$).

The dynamics of the particle under consideration is subject to a competition between two quantities: One is the decay length of the spherical wave amplitude in radial direction, i.e. the interval of radial coordinate in which the spherical wave amplitude decays by the factor 1/2 which obviously is just equal to the initial value of the radial coordinate r_0.

The other is the interval by which the particle would be shifted through drift motion within the corresponding plane wave field in direction of wave propagation during one half period

$$\delta x^1 = \pi r_L \overline{\mathbf{u}^1} \tag{4}$$

so that with $\Phi = \pi/2$ one is lead to

$$\delta x^1 = (\pi/4) r_L (r_T^2/r_L r_0)^2 (2 - \cos(2\alpha))\,. \tag{5}$$

By definition, the acceleration boundary is located where these two quantities become equal which is the case for

$$r_B = r_0 = \delta\, x^1 \tag{6}$$

so that

$$\begin{aligned} r_B &= (\pi/4)^{1/3} r_L (r_T/r_L)^{4/3} (2 - \cos(2\alpha))^{1/3} \\ &= (\pi/4)^{1/3} r_L (r_T/r_L)^{4/3} (1 + 2\sin^2(\alpha))^{1/3}\,. \end{aligned} \tag{7}$$

The notion of acceleration boundary may also be related to certain features of energy development.

As far as particles are concerned starting from within a spherical wave field, a monotonous development of energy is expected, as long as these particles originate from inside either of the two coni $\vartheta < \pi/4$ or $\vartheta > 3\pi/4$ around the axis of rotation.

But the situation may be different for particles originating from outside these two coni, i.e. from within the range $\pi/4 < \vartheta < 3\pi/4$ including the equatorial plane of rotation. In that case, there is a competition between the decay length r_0 of the spherical wave amplitude and (twice, by definition) the interval by which the particle would be transported in the direction of wave propagation within the corresponding plane wave field before entering from the accelerating into the deccelerating region, $\delta\tilde{x}^1$.

In the special case of linear polarisation, corresponding to initial positions within the equatorial plane of rotation, $\delta\tilde{x}^1/2 = \delta\Phi r_L \overline{\mathbf{u}^1}$ with $\delta\Phi = \pi/2$ i.e. $\delta\tilde{x}^1/2 = (\pi/2) r_L \overline{\mathbf{u}^1}$ so that $r_B = (\pi/4)^{1/3} r_L (r_T/r_L)^{4/3}$ in agreement with what was found before.

For $r_0 \ll \delta\tilde{x}^1$ the particle is thus expected to exhibit a monotonous development of energy since it essentially decouples from the spherical wave field before entering into the decelerating region.

Alternatively, for $r_0 \gg \delta\tilde{x}^1$ the particle is expected to exhibit an oscillatory development of energy (before approaching its asymptotical value of energy) since it has to pass through various accelerating and decelerating stages until finally the spherical wave amplitude has decayed substantially. This behaviour is confirmed by results of numerical integration.

The notion of a ***plasma border*** refers to the scattering of particles of mass m, electric charge e and given energy $\gamma m c^2$ – which for practical reasons is taken as the energy at the point of reflection – by the spherical wave field of a magnet rotating with its vector of angular velocity $\vec{\omega}$ inclined by a given angle ($\chi = \pi/2$ in this case) against its vector of magnetic dipole moment $\vec{\mu}$ (K. O. Thielheim, 1989b). The plasma border $r_P = r_P(\vartheta, \varphi)$ by definition then is the surface generated by all points of reflection which for all pairs of angular coordinates ϑ and φ are the points of nearest approach to the rotating magnet.

Another way of looking at this phenomenon is to consider a rotating magnet within an ambient fully ionized plasma. For appropriately chosen values of the frequency of rotation ω and of the magnetic dipole moment μ, the rotating magnet is expected to be able to sweep away all charged particles from its immediate vicinity and thereby to evacuate a region of space up to a certain radial distance, in other words, up to the plasma border if the ambient plasma is of sufficiently low density.

The calculation of the plasma border r_P as a function of parameters r_L and r_T is based on the suggestion that the actual particle orbit within a given ***spherical*** wave field at its point of reflection is osculated by a closed particle orbit in the corresponding local ***plane*** wave field (K. O. Thielheim, 1989b).

One is lead to

$$r_P = (2^{-1/2})(\beta\gamma)^{-1} r_L (r_T/r_L)^2. \tag{8}$$

In what follows I will discuss the development of energy of particles initially at rest

$$\vec{u}(\Phi_0) = 0 \tag{9}$$

within the interval $r_B < r_0 < r_P$ of the spherical wave field of a magnet rotating with its vector of magnetic dipole moment $\vec{\mu}$ perpendicular to its vector of angular velocity $\vec{\omega}$.

The *constant latitude approximation*

$$\vartheta(\mathbf{s}) \cong \vartheta_0 = const. \tag{10}$$

implying that each particle orbit essentially remains confined to the neighbourhood of a circular conus of constant latitude ϑ_0 will be adopted for a first estimate of the energy of particles from the wave zone. For the present purposes it will be sufficient to consider particle motion near the equatorial plane of rotation

$$\vartheta_0 = \pi/2. \tag{11}$$

It should be well understood that the constant latitude approximation ***does not*** imply the assumption that the latitudinal component of the velocity vector $\mathbf{u}_\vartheta$ vanishes.

Interest is focused on the mean development of dynamical variables as functions of phase and especially of the asymtotic value of the Lorentz factor for large values of phase.

Straight forward integration of the equations of motion leads to the conclusion that the relativistic kinetic energy $mc^2(\gamma - 1)$ is proportional to the inverse to the square of the initial radial coordinate r_0 with the average for arbitraly choosen initial phase Φ_0

$$< (\gamma - 1)mc^2 > = (1/2)(r_0/r_L)^{-2}(r_T/r_L)^4 . \tag{12}$$

These and the aforementioned features of particle dynamics within the wave zone is shown schematically in figure 1.

Seed particles produced inside the wave zone, e. g. by ionisation of neutral particles, which may have invaded that region from outside, with a constant spatial source function then exhibit a differential energy spectrum proportional to the $-2.5th$ power of energy, which is not far from what is observed for cosmic ray particles. Discrete objects through this mechanism can be non-monoenergetic sources of high energy particles.

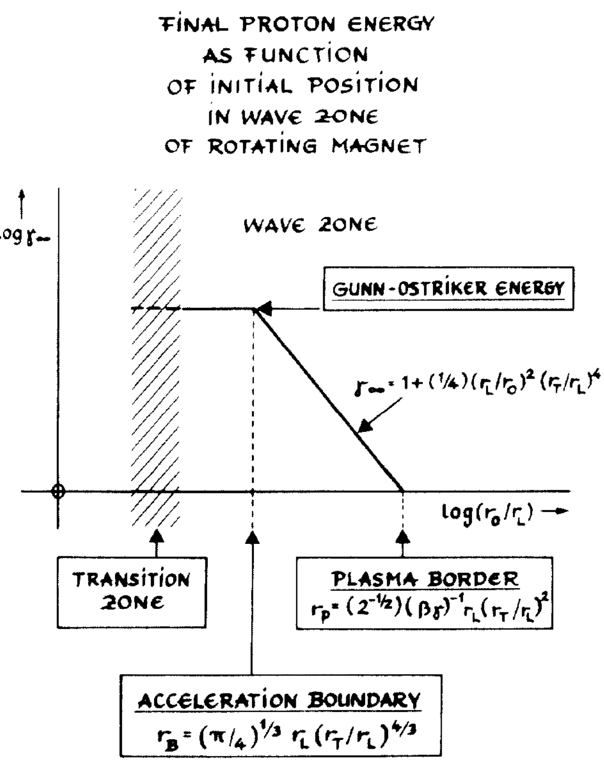

Figure 1

In what follows I shall outline first approaches to more general investigations on ***particle dynamics in plasma fields*** which may exist around rotating magnets.

Special attention will be given to a scenario in which a stationary outflow of *plasma particles* cooperates with an outgoing ***spherical electromagnetic wave*** so that plasma particles contribute to *and* react on the 'resulting' electromagnetic plasma wave field in a *collisionless non-linear relativistic magnetohydrodynamic spherical wave.* The problem, under which condition particles can or cannot cooperate in this sense — at least locally, within a certain range of radial coordinate — with a spherical electromagnetic wave is one of the questions which has to be investigated.

Another problem of interest will then be, which of the aforementioned features of particle dynamics in vacuum wave fields survive within the 'resulting' field of a non-linear relativistic magnetohydrodynamic spherical wave, and if so, what the modifications are.

In any case the situation of an individual charged particle within the 'resulting' plasma wave field can be seen in analogy to that of a particle within the 'vacuum' wave field considered before. In a first approach it is then useful to perform parametric studies, for example, to calculate the acceleration boundary r_B as a function of the phase velocity β (in units of the velocity of light, with $\beta > 1$) without referring explicitly to dispersion relations and wave profiles.

Within the range of *comparatively small phase velocity* characterized by

$$k^2 f_0^2/2 \ll 1 \tag{13}$$

with

$$k^2 = (\beta^2 - 1)/\beta^2 \tag{14}$$

one again is lead to to the vacuum acceleration boundary

$$r_B = (\pi/4)^{1/3} r_L (r_T/r_L)^{4/3}$$

in agreement with what has been found earlier for $\alpha = 0$.

But within the range of *comparatively large phase velocity* characterized by

$$k^2 f_0^2/2 \gg 1 \tag{15}$$

the result is

$$r_B = (\pi/2) r_L/(\beta - 1) \tag{16}$$

which is remarkable in so far as it is found to be independent from r_T and therefore independent from the electromagnetic properties of both the rotating magnet and the particle under consideration.

Still this phenomenom can be understood as produced by the competition of two quantities: One, again, is the decay length of the field amplitude, r_0. The other is the interval $\delta\tilde{x}^1$ by which the particle has to travel until it falls back relative to the wave profile by $\delta\Phi = \pi/2$ which in this case is governed by the increment of phase velocity $\beta - 1$. For illustration the acceleration boundary r_B as a function of the phase velocity β is shown schematically in figure 2.

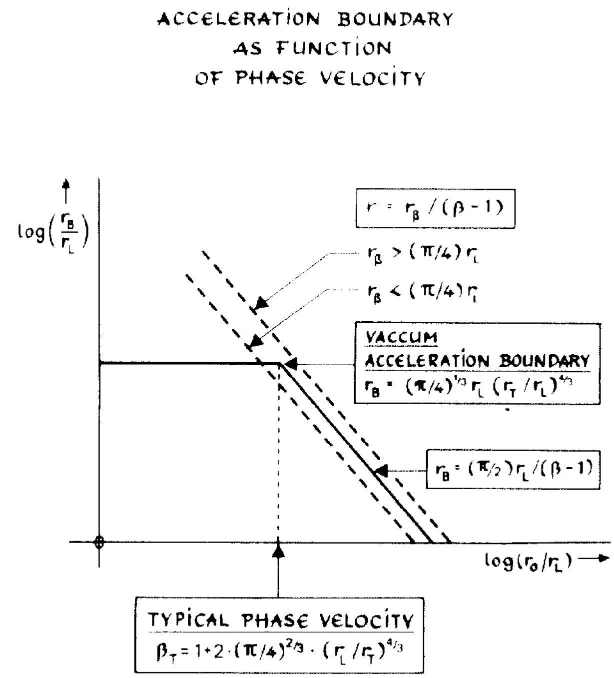

Figure 2

1 REFERENCES

[1] Thielheim, K. O. (1987).
Acceleration Boundary for an Electrically Charged Particle within the Field of a Rotating Magnetic Dipole
J. Phys. 20 , L203–L206.

[2] Thielheim, K. O. (1989a).
Particle Acceleration by Rotating Magnets – Pulsars as Cosmic Accelerators
Fundamentals of Cosmic Physics 13 , 357–399.

[3] Thielheim, K. O. (1989b).
Pulsars as particle accelerators – Some features of the machine become visible
from: Shapiro, M. M. and Wefel, J. P. (eds.): Cosmic Gamma Rays, Neutrinos and Related Astrophysics, 523-536, Kluwer Academic Publishers.

[4] Thielheim, K. O. (1991a).
Particle Acceleration by Rotating Magnetized Bodies and the Problem of the Origin of High Energy Cosmic Particles
Nucl. Phys. B (Proc. Suppl.) 22 B , 60–79.