REFERENCE DATA ON DISC LOADED GUIDE SPACE HARMONICS

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The use of non-fundamental space harmonics in linac accelerating structures, particulaly in X, Y and Q bands, appears to be profitable due to essentually less attenuation in buncher sections. Since the data on + 1 harmonic are scarce in scientific literature the calculations were carried out on the basis of the complete electromagnetic field data [1] for $\pi/2$ mode, β_{ph}= 1,2,3, t/λ = 0,04 and 0,08.

The axial electric field in a disc loaded guide at a fixed moment can be written as

$$E(z) = |E(z)|\cdot e^{-j\psi(z)} , \qquad (1)$$

where $\psi(z)$ is the field phase shift along Z axis. The E-field is a periodic function, i.e

$$E(z + \lambda_w) = E(z) \qquad (2)$$

Besides

$$\psi(z + D) = \psi(z) + \theta, \qquad (3)$$

where D is structural period.

$$\theta = \frac{2\pi}{\lambda_w}\cdot D = k_{z_o} \cdot D \qquad (4)$$

Let's expand the electric field expressed by formula (1) in Fourier series

$$E(z) = \sum_{p=-\infty}^{\infty} E_p \cdot e^{-j\frac{2\pi}{\lambda_w}\cdot pz} , \qquad (5)$$

where number p harmonic amplitude is

$$E_p = \frac{1}{\lambda_w} \int_{-\lambda_{w/2}}^{+\lambda_{w/2}} E(z)\cdot e^{+j\frac{2\pi}{\lambda_w}pz} dz \qquad (6)$$

Substituting E_z from (1) one can obtain

$$E_p = \frac{1}{\lambda_w} \int_{-\lambda_{w/2}}^{+\lambda_{w/2}} |E(z)|\cdot e^{j\left[\frac{2\pi}{\lambda_w}pz-\psi(z)\right]} dz \qquad (7)$$

The first nonvanishing harmonic at p=1 can be written as

$$E_1(z) = E_1\cdot e^{-j\frac{2\pi}{\lambda_w}\cdot z} = E_1\cdot e^{-jk_{z_o}\cdot z} \qquad (8)$$

This is the fundamental harmonic, which has the highest amplitude. For example, let's consider the $\pi/2$-mode and expand the complete axil E-field by means of Fourier analysis. It can be shown that the second nonvanishing harmonic occurs at p=5 (if p≥0). Due to strict axial symmetry of E(z) function in the integral (7) at p=0,2,3,4 the corresponding space harmonics amplitudes are equal zero ($E_o = E_2 = E_3 = E_4 = 0$).

If p<0 the nearest nonvanishing space harmonic occurs at p=-3 because at p=-1,-2 the corresponding amplitudes equal zero due to the integral function (7) symmetry.

In general for any mode θ nonvanishing space harmonics appear if

$$\frac{2\pi}{\lambda_w}(p-1) = n\frac{2\pi}{D} , \qquad (9)$$

where is integer (n=0,±1,±2...). Taking into account (4) one can obtain

$$p=1+n\frac{2\pi}{\theta} \qquad (10)$$

The approach [2,3,4,5] to the expansion of the electrical field is as follows. The field is beeing expressed as

$$E(z) = \tilde{E}(z)\cdot e^{-jk_{z_0}\cdot z} \tag{11}$$

The function $\tilde{E}(z)$ is a periodical one with period D. The expansion of $\tilde{E}(z)$ yields

$$\tilde{E}(z) = \sum_{n=-\infty}^{\infty} E_n \cdot e^{-j\frac{2\pi}{D}\cdot nz}, \tag{12}$$

where

$$E_n = \frac{1}{D}\int_{-D/2}^{+D/2} \tilde{E}(z)\cdot e^{j\frac{2\pi}{D}nz}\,dz \tag{13}$$

Combining Eqs. (11), (12), (13) yields

$$E(z) = \sum_{n=-\infty}^{\infty} E_n \cdot e^{-jk_{zn}\cdot z} \tag{14}$$

and

$$E_n = \frac{1}{D}\int_{-D/2}^{+D/2} |E(z)|\cdot e^{j\left[(k_{z_0}+\frac{2\pi}{D}n)z-\psi(z)\right]}dz\ , \tag{15}$$

where

$$k_{z_n} = k_{z_0} + n\frac{2\pi}{D} = k_{z_0}(1 + n\frac{2\pi}{\theta}) = k_{z_0}\cdot p. \tag{16}$$

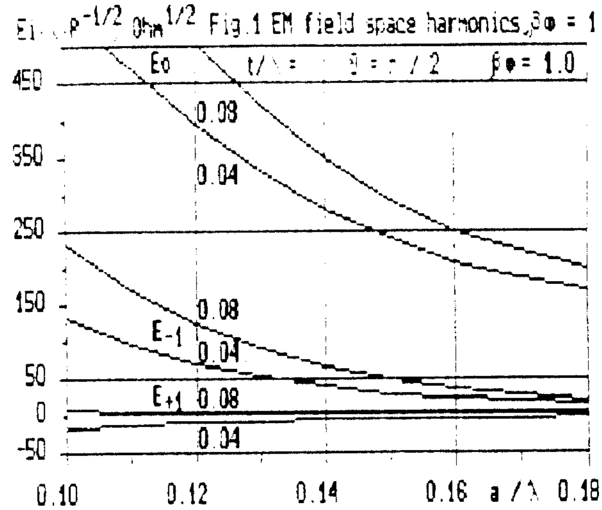
It should be noted that n is not a space harmonic number but it identifies terms in $\tilde{E}(z)$ expansion series or interval number in Brillouin dispersion relation $1/\lambda = F(1/\lambda_w)$ (so called Hartley harmonics).

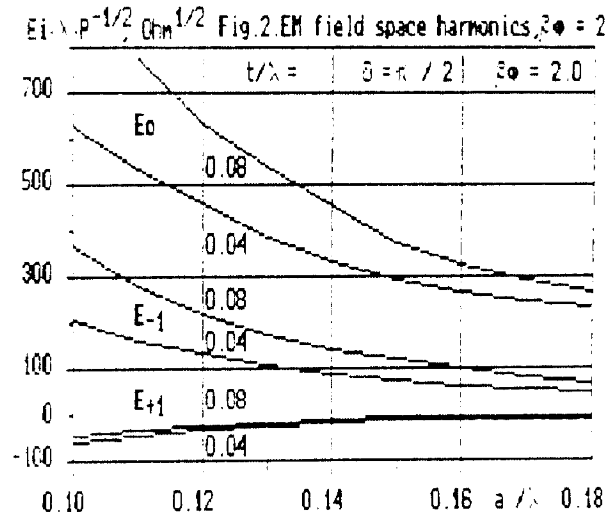
The correlation between space harmonics numbers p and dispersion intervals numbers n is shown in Table.

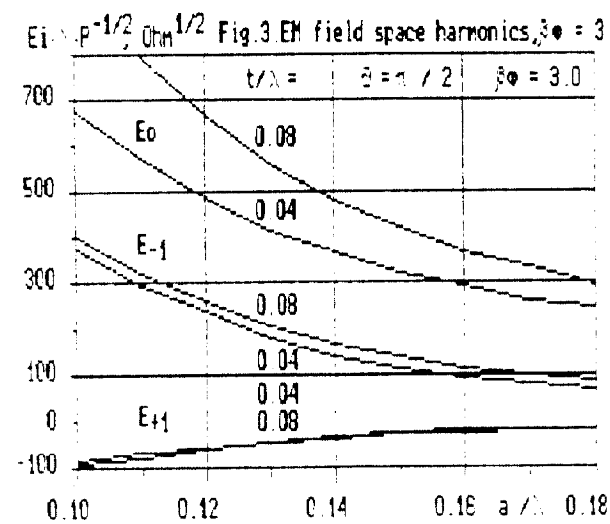
Table

Correlation between numbers p and n

n	p					
	θ=π/4	θ=π/3	θ=π/2	θ=2π/3	θ=3π/4	θ=π
0	1	1	1	1	1	1
-1	-7	-5	-3	-2	-	-1
1	9	7	5	4	-	3
-2	-15	-11	-7	-5	-	-3
2	17	13	9	7	-	5
-3	-23	-17	-11	-8	-7	-5
3	25	19	13	10	9	7







From (7) and (15) it's clear that $E_p = E_n$ if $p = 1 + n\frac{2\pi}{\theta}$.

The space harmonics reference data for disc loaded guides operating at $\pi/2$ mode are shown in fig.1 ($\beta_{ph} = 1$), fig.2 ($\beta_{ph} = 2$) and fig.3 ($\beta_{ph} = 3$). The curves are plotted for two values of relative disc thickness (0,04 and 0,08) and the interval of relative bore diameters from 0,10 to 0,18. For convenience reasons the scales for E_0 and $E_{\pm 1}$ amplitudes are different.

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