REFERENCE DATA ON DISC LOADED GUIDE SPACE HARMONICS

I.S. SHCHEDRIN

Moscow Engineering Physics Institute
Kashirsheoskoe shosse, 31, 115409, Moscow, Russia.

The use of non-fundamental space harmonics in linear accelerating structures, particularly in X, Y and Q bands, appears to be profitable due to essentially less attenuation in buncher sections. Since the data on + 1 harmonic are scarce in scientific literature the calculations were carried out on the basis of the complete electromagnetic field data [1] for +2 mode, $\beta_{ph} = 1, 2, 3$, $t/\lambda = 0.04$ and $0.08$.

The axial electric field in a disc loaded guide at a fixed moment can be written as

$$E(z) = |E(z)| \cdot e^{-j\psi(z)}$$

where $\psi(z)$ is the field phase shift along Z axis. The E-field is a periodic function, i.e.

$$E(z + \lambda_E) = E(z)$$

Besides

$$\psi(z + D) = \psi(z) + \theta$$

where $D$ is structural period.

$$\theta = \frac{2\pi}{\lambda_E}D = \frac{k_E \cdot D}{2\pi}$$

Let's expand the electric field expressed by formula (1) in Fourier series

$$E(z) = \sum_{p=-\infty}^{\infty} E_p \cdot e^{-j\frac{2\pi p}{\lambda_E}z}$$

where number $p$ harmonic amplitude is

$$E_p = \frac{1}{\lambda_E} \int |E(z)| \cdot e^{j\frac{2\pi p}{\lambda_E}z} \cdot dz$$

Substituting $E_z$ from (1) one can obtain

$$E_p = \frac{1}{\lambda_E} \int |E(z)| \cdot e^{j\frac{2\pi p}{\lambda_E}z} \cdot dz$$

The first nonvanishing harmonic at $p=1$ can be written as

$$E_1(z) = E_1 \cdot e^{-j\frac{2\pi}{\lambda_E}z} = E_1 \cdot e^{-j\cdot \frac{k_E}{2\pi}z}$$

This is the fundamental harmonic, which has the highest amplitude. For example, let's consider the +2 mode and expand the complete axial E-field by means of Fourier analysis. It can be shown that the second nonvanishing harmonic occurs at $p=5$ (if $p>0$). Due to strict axial symmetry of $E(z)$ function in the integral (7) at $p=0, 2, 3, 4$ the corresponding space harmonics amplitudes are equal zero ($E_0 = E_2 = E_3 = E_4 = 0$).

If $p<0$ the nearest nonvanishing space harmonic occurs at $p=-3$ because at $p=-1, -2$ the corresponding amplitudes equal zero due to integral function (7) symmetry.

In general for any mode $\theta$ nonvanishing space harmonics appear if

$$p(p-1) = n\pi \cdot \frac{D}{\lambda_E}$$

where is integer ($n=0, 1, 2, ...$). Taking into account (4) one can obtain

$$p=1, n\pi \cdot \frac{D}{\lambda_E}$$

The approach [2,3,4,5] to the expansion of the electrical field is as follows. The field is being expressed as
The function $\hat{\beta}(z)$ is a periodic one with period $D$. The expansion of $\hat{\beta}(z)$ yields

$$\hat{\beta}(z) = \sum_{n=-\infty}^{\infty} E_n e^{-j2\pi nz/D}. \quad (12)$$

where

$$E_n = \frac{1}{D} \int_{-D/2}^{+D/2} \hat{\beta}(z) e^{j2\pi nz/D} dz. \quad (13)$$

Combining Eqs. (11), (12), (13) yields

$$E(z) = \sum_{n=-\infty}^{\infty} E_n e^{-j\kappa_n z} \quad (14)$$

and

$$E_n = \frac{1}{D} \int_{-D/2}^{+D/2} \hat{E}(z) e^{j\frac{2\pi n z}{D}} dz \quad (15)$$

where

$$k_{zn} = k_z - n\frac{2\pi}{D}; \quad k_z = k_z_0 (1 + n^2 \frac{\pi^2}{D^2}) = k_{zp} \quad (16)$$

It should be noted that $n$ is not a space harmonic number but it identifies terms in $\beta(z)$ expansion series or interval number in Brillouin dispersion relation $1/\lambda = \Gamma(1-\lambda)$. (so called Hartley harmonics).

The correlation between space harmonics numbers $p$ and dispersion intervals numbers $n$ is shown in Table 1.

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Table 1: Correlation between numbers $p$ and $n$.
From (7) and (15) it's clear that $E_p = E_n$ if $p = 1 + n\frac{2\pi}{\beta}$.

The space harmonics reference data for disc loaded guides operating at $n/2$ mode are shown in fig. 1 ($\beta_{ph} = 1$), fig. 2 ($\beta_{ph} = 2$) and fig. 3 ($\beta_{ph} = 3$). The curves are plotted for two values of relative disc thickness (0.04 and 0.08) and the interval of relative bore diameters from 0.10 to 0.18. For convenience reasons the scales for $E_0$ and $E_{\pm}$ amplitudes are different.

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REFERENCES