Calculation of Undulator Radiation from Measured Magnetic Fields and Comparison with Measured Spectra

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Abstract

We have developed a new code that calculates the spectrum, the state and the degree of polarization of the radiation emitted by any insertion device. It integrates along an arbitrary electron trajectory without any approximations. The trajectory can be derived either from a model undulator including random field errors or from a measured field distribution from a real undulator. The calculated one-electron pattern is convoluted with the electron beam emittance. Spectral calculations for the crossed field undulator U-2 at BESSY employing measured magnetic field data agree with absolute measurements of the spectra.

1 INTRODUCTION

The third generation of synchrotron radiation facilities are now being built or planned in many institutes all over the world. Undulators and wigglers (insertion devices, ID) will play the dominant role for the generation of radiation in these facilities. Using these periodic magnetic structures an enhancement of photon brilliance and flux of several orders of magnitude in comparison with the dipole radiation is theoretically expected under ideal circumstances. However, brilliance can be strongly reduced by the emittance of the storage ring and by magnetic field errors of the ID's. In order to design an ID it is necessary to be able to predict with reasonable accuracy the characteristics of undulator radiation it will produce under realistic circumstances. Here we describe a computer code that enable one to perform such calculations. It is based on the basic formula of Jackson [1] and employing no approximations for the trajectory calculation.

Numerical simulation programs like SMUT [2] or URGENT [3] include the electron beam emittance, but use sinusoidal trajectories with a Bessel function formulation [4] and therefore do not include magnetic field errors and fringe field effects. Additionally these codes use the far field approximation. Discrepancies between the measured and calculated spectra have been observed [5].

The theoretical study of the influence of the near field effect and magnetic field error on undulator radiation without considering the emittance effect has been discussed in detail elsewhere (see e.g. [6,7]). A full numerical simulation of these influences including the emittance effect has, however, not yet been comprehensively performed. To cover this insufficiency, a Fortran simulation code [8] has been developed which allows for the first time to do a comparison of measured and calculated spectra on an absolute scale, where the calculation is based on measured magnetic field data. Similar efforts [9] are under way at other synchrotron radiation centers.

A study of the 110 period undulator U-3 to be built in the planned third generation synchrotron radiation facility BESSY II showed that the influence of the emittance effect is still dominant compared to the effect of a 0.1 % remanence spread [8]. This value seems to be achievable by standard procedures of measuring and sorting of blocks. The brightness performance of an undulator can thus be predicted before installation in the storage ring, and the influence of the remaining field errors on the spectral quality can be estimated.

2 ELECTRON TRAJECTORY

In our program the electron trajectory in an undulator can be approximately described in terms of a quasi sinusoidal function. This excludes magnetic field errors and is useful if the computation time should be *minimized*. Alternatively the trajectory may be obtained from an exact integration of the equation of motion of the particle in a given magnetic field. In this case the magnetic field distribution is needed.

The field of an undulator made exclusively of permanent magnets without iron can be calculated to a good approximation by using the current sheet equivalent method [10]. This includes some new types of proposed undulators including crossed overlapped undulator and (modified) planar-helical undulator [11] for producing circularly polarized undulator radiation.

Magnetic field errors can be divided up into the following categories: (1) spread in magnetization of the material, (2) spread in the direction of the easy axis of the blocks, and (3) incorrect positioning of the blocks inside the periodic structure. Inhomogeneities in individual blocks can be handled by subdividing such blocks into small, homogeneous blocks. In the code these three kinds of field errors are modelled by different remanence values of individual blocks and by rotating and shifting the blocks as a whole with respect to the ideal position. These errors may be assumed to have any distribution including a Gaussian. It is thus possible to study the dependence of the on-axis brightness and of the maximal brightness on the different kind of errors by comparing the brightness reduction due to field errors to the reduction due to the electron beam emittance.

For a hybrid magnetic structures of the Halbach 2 type [12] a detailed description of the three dimensional magnetic field is required. This can be calculated with the help of 2D or 3D codes. For quantitative results the field must be determined approximately by Hall probe measurement on a test period or more exactly by successive measurements on the entire structures.

3 SPECTRA

The spectral brightness from a relativistic electron is given by [1]

$$\frac{\partial^2 I_{elec}}{\partial\omega\partial\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\frac{\hat{n}\times\left\{(\hat{n}-\vec{\beta})\times\dot{\vec{\beta}}\right\}}{(1-\vec{\beta}\cdot\hat{n})^2}e^{i\omega(t'+R/c)}dt'\right|^2 . \tag{1}$$

Here the symbols have the usual meaning. From experience we recommend using this formula without any further approximation together with Simpson's rule for the integral for reliable results. However, in the code other faster options are provided. The loss of accuracy may be tolerable in a given study and can be determined by checking against the result obtained from eq. (1). The options included are the far field approximation, the neglect of fringe field effects, and a purely periodic harmonic motion of the electron. The last option allows the use of the Fast Fourier Transformation (FFT) or alternatively Tatchyn's formulation [13] providing an additional saving of computation time.

The emittance effect is evaluated through a convolution formula according to the off-axis-approximation [2]:

$$\frac{\partial^2 I_{beam}}{\partial\omega\partial\Omega} = \frac{\partial^2 I_{elec}}{\partial\omega\partial\Omega} * G_{beam} \tag{2}$$

with

$$G_{beam} = \frac{e^{-\frac{x^2}{2\sigma_{S_x}^2}-\frac{y^2}{2\sigma_{S_y}^2}}}{2\pi\sigma_{S_x}\sigma_{S_y}} \quad\text{and}\quad \sigma_{S_{x,y}} = \sqrt{\sigma_{x,y}^2 + d^2\sigma_{x',y'}^2}. \tag{3}$$

Here d is the distance between the center of the undulator and the plane of observation. $\sigma_{x,y}$ and $\sigma_{x',y'}$ are the transverse electron beam dimension and divergence, respectively, evaluated in the center of the undulator; this implies a symmetrical focussing of the electron beam. The convolution is efficiently performed with help of the FFT algorithm or by Monte Carlo techniques. The integration over a pinhole of finite size is performed by the FFT algorithm. Finally, the state of polarization of the radiation is described in terms of the Stokes parameters S_0, S_1, S_2, S_3 [14]. - An explicit derivation of algorithm and formula will appear elsewhere [8]. A user's guide is in preparation.

4 SPECTRUM OF THE BESSY UNDULATOR U-2

Undulator U-2 is a so-called crossed undulator proposed independently by Nikitin et al.[15] and by K.J.Kim [16] for the generation of circularly polarized VUV radiation. It consists of two planar undulators arranged one after the other with mutually perpendicular fields. A three pole magnetic structure (modulator) is located between the two undulators and enables the relative phase of the radiation from the two undulator to be varied. A monochromator with a sufficient spectral resolution causes a superposition of the two linearly polarized waves generating circularly polarized radiation. In U-2 the different magnetic components are separated by iron field clamps. U-2 is a hybrid design with NdFeB permanent magnets and Vanadium-Permendur pole shoes. Both undulators have a period length of 84 mm and a pole width of 90 mm. The horizontal deflecting undulator consists of 13 poles while the vertical deflecting undulator consists of 15 poles. Because of the small number of poles and the effect of iron field clamps the effect of fringe magnetic fields (end pole effect) cannot be neglected. The undulator U-2 was installed into the BESSY storage ring in December 1990 [17].

Measured magnetic field data of U-2: After proper setting of the end pole correctors [18] the on-axis field was measured with a spatial resolution of 1 mm for different gaps using a Hall probe. Consistent with the requirement of coherent overlap of the two radiation fields, the field integrals are found to be

$$|I_1| = \left|\int_{z_1}^{z_2} B(s)ds\right| \leq 0.5 \text{ Gauss-m} \tag{4}$$

and

$$\begin{aligned} |I_2| &= \left|\int_{z_1}^{z_2}\int_{z_1}^{s} B(s')ds'ds\right| \\ &\leq 0.4 \text{ Gauss-m}^2 \text{ (Gap=43 mm)} \\ &\leq 0.2 \text{ Gauss-m}^2 \text{ (Gap=83 mm)}. \end{aligned} \tag{5}$$

Here z_1, z_2 are far enough away from the undulator so that the contribution of the remaining fringe magnetic field can be neglected.

Spectrometer: A calibrated pinhole transmission grating spectrometer (PTGS) [5] was used to measure the spectra of U-2. Because of the small angular acceptance of the pinhole ($0.1 * 0.1 mm^2$ at a distance of about 10 m) as compared with the natural emission angle of the radiation of 100 μrad the integration over the pinhole is trivial. The pinhole transmission grating has a line density of 5000 1/mm and can be laterally translated in both directions with respect to the undulator axis. Fig. 1 shows the calculated and measured spectra of the vertical undulator (vertical oscillations of the electrons) of U-2 at a gap 68 mm ($K \sim 0.95$) in the BESSY storage ring (electron energy 800 MeV).

Discussion: (1) Due to the small number of poles and the iron field clamps the fringe field effects have a strong influence on the spectrum. The Fourier transform of the magnetic field indicates a relatively strong component at the third harmonic of the main frequency which is partially due to fringe fields. In the spectrum an interference

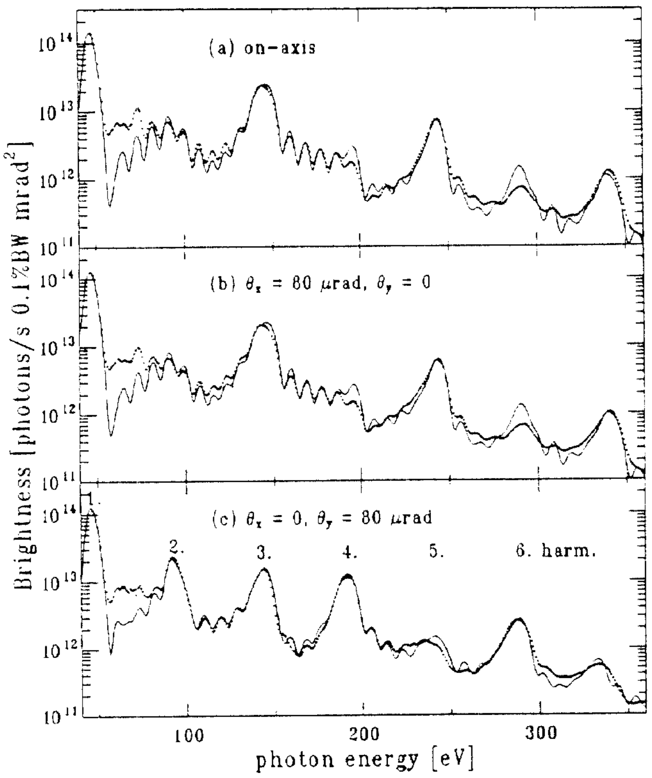

Figure 1: On- (a) and Off-axis (b,c) brightness of the vertical undulator of U-2 at BESSY for $K \simeq 0.95$ (gap 68 mm). The solid line represents the calculated spectrum including the emittance effects; the symbols (+) represent the measured data (ring current 100 mA).

occurs between the 'natural' third harmonic and the intensity due to the third harmonic of the magnetic field causing a broadening of the peak near 150 eV. **(2)** For a undulator which deflects the electrons in the plane of the storage ring, the spectral brightness of the even harmonics can become quite strong on-axis due to the emittance effect. For the vertical deflecting undulator of U-2 the situation is different. At BESSY the vertical electron beam divergence in the long straight section is below 20 μrad and is thus significantly smaller than the natural divergence of the radiation. This implies for the vertical undulator an undisturbed variation of the intensity in the vertical direction. Fig. 1.a and 1.c show that the vertical undulator spectrum near even harmonics varies strongly when the observation point is moved along the y-axis which is perpendicular to the main component of the magnetic field. Due to the higher electron beam divergence in the horizontal plane of 130 μrad such a strong variation is not observed in the spectrum of the horizontal undulator. **(3)** Deviations between calculation and measurement are due to uncertainties of the magnetic field data, the energy spread of the electron beam, the uncertainty in the energy calibration of the PTGS, corrections for higher orders of the transmission grating, and perhaps other effects. Further discussions are found in [5], where an agreement of calculation and measurement within 10 % is shown.

5 CONCLUSION

A FORTRAN code has been developed that calculates the spectrum of an undulator including the effects due to magnetic field errors, fringe fields, emittance, polarization, and a finite aperture. The code is used to reproduce the measured spectrum of the BESSY undulator U-2 on an absolute scale, and an agreement within 10 % is found. This good agreement between experimental and theoretical result opens up the possibility to use undulator radiation as a radiometric source in the vacuum ultraviolet spectral range. Moreover, the code allows one to specify an insertion device that will maintain the high brightness of the storage ring.

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