

Impedance measurements for the pumping holes in the LHC liner

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Abstract

We report the results of impedance measurements, performed by the triaxial wire method, on a 2 m long model of the LHC vacuum chamber. The model consists of an inner screen (liner) with pumping holes and of the outer vacuum chamber. The transmission coefficient for different numbers of holes with various radii confirms the role played by TEM modes, propagating at the same speed of the beam particles in the coaxial structure formed by the liner and the outer vacuum chamber. We also compare the results of these measurements to analytical estimates of the coupling impedance.

1 EXPERIMENTAL SETUP AND MEASUREMENT RESULTS

The design of LHC includes a beam liner to screen the external beam pipe at 1.9 K from synchrotron radiation. Owing to vacuum requirements, a fraction f (in the order of a few per cent) of the screen surface should be covered by holes or slots. Using the triaxial wire method [1], we measured the forward transmission coefficient α up to a frequency of 2 GHz, through a double row of 150 circular holes (on top and bottom of the inner conductor) with a longitudinal spacing $d_h = 1$ cm. The transmission coefficient α is the ratio between the power transmitted through the holes into the external coaxial structure and the input power in the inner conductors. The radii of the inner and outer conductors were $R_I = 2$ cm and $R_O = 5$ cm, respectively, and the thickness of the beam screen $t = 2$ mm. A typical result, corresponding to a hole radius $r = 1.5$ mm, is shown in Fig. 1. In Table 1 we report the measured values of the transmission coefficient (in dB) at 1 GHz as well as the values predicted using Eq. (14), discussed in the next section.

The measurement corresponding to a single hole is probably affected by a large error, since the backward wave due to reflections was not negligible. Therefore the measured transmission coefficient is compatible with our noise level of $-100 \div 110$ dB. The ratio between the transmission coefficient $G = 10^{(\alpha/20)}$ for 300 holes and for a single hole (with $r = 2$ mm) is 92 and, in view of the previous remark, it is compatible with 300. As discussed in the next section, this confirms that over the length of 1.5 m of our model the holes contribute *coherently* to the build up of the TEM wave in the external coaxial region. If the effect of the holes were incoherent, the measured ratio should be $\sqrt{300} \simeq 17$.

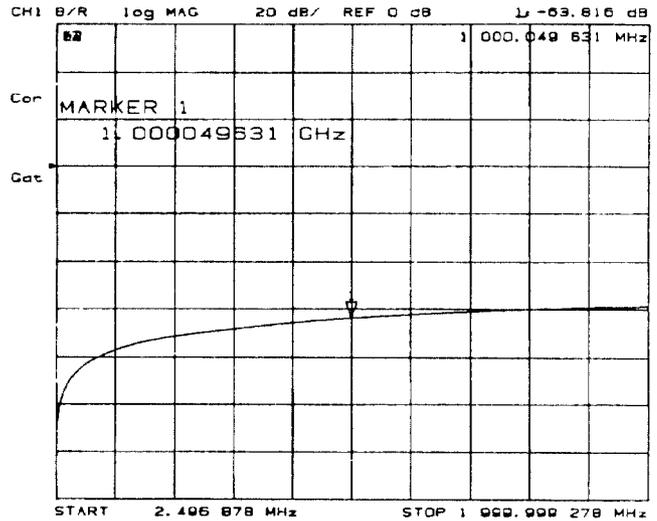


Figure 1: Forward transmission coefficient α (in dB) up to 2 GHz measured with 2×150 holes of radius $r = 1.5$ mm. The marker gives the value of α at 1 GHz.

The dependence of the transmission coefficient on frequency and on the hole radius r is well fitted by the analytic expression Eq. (14), but the predicted values of G are systematically smaller than the measured values by a factor between two and three. In the case of a single hole, the discrepancy is by a factor five. It is worth noting that a better agreement between measurements and predictions is obtained for larger hole radii. This can be partly explained by geometrical imperfections associated with the drilling process, giving rise to countersunk holes whose effective depth is smaller than the thickness t of the inner screen. On the other hand, this will be a general feature of the (hundreds of millions) holes drilled by laser in the

Table 1: Measured forward transmission coefficient α (in dB) at 1 GHz for N_h holes of radius r . In the last two columns we also report the measured and computed values of the transmission coefficient $G = 10^{(\alpha/20)}$.

N_h	r (mm)	measured		computed
		α (dB)	G	G
2×150	1	-81.1	8.8×10^{-5}	3.1×10^{-5}
2×150	1.5	-63.8	6.5×10^{-4}	3.2×10^{-4}
2×150	2	-52.7	2.3×10^{-3}	1.3×10^{-3}
1	2	-92	2.5×10^{-5}	4.4×10^{-6}

LHC liner and one should keep in mind that drilling small holes by laser is technically easier than drilling large holes.

2 ANALYTICAL ESTIMATES OF THE COUPLING IMPEDANCE

The analytical estimate of the impedance for a single hole or slot (in Bethe approximation) was first carried out by Kurennoy [2] and then extended to the case of a thick wall liner with many holes by Gluckstern [3, 4]. Here we present a simplified analysis, valid in weak field approximation, with special emphasis on the beam power loss and the transmission coefficient.

We start by computing the resistive wall power loss in the external beam pipe *without liner*. This gives an (unacceptable) upper limit for the losses through the pumping holes. Then we define the attenuation length ℓ_{att} of the coaxial wave guide formed by the liner and the outer vacuum chamber. By equating the growth rate of TEM coaxial modes excited by the pumping holes (in weak field approximation) and their attenuation due to resistive losses, we obtain the transmission coefficient G_∞ at steady state. Our simplified derivation is valid when $G_\infty \ll 1$, i.e., when the field level in the outer coaxial structure is much lower than inside the liner. This is the regime corresponding to a liner which is "doing its job". This result allows us to estimate the power loss in LHC for different choices of hole radius and liner thickness. Finally we estimate the transmission coefficient $G(\omega)$ for lengths much smaller than the attenuation length ℓ_{att} and compare it to the measured values.

The resistive wall power loss P_W for a beam consisting of k_b Gaussian bunches with r.m.s. length σ and N_b protons per bunch can be written

$$P_W = \Gamma \left(\frac{3}{4} \right) \frac{k_b}{b\sigma^{3/2}} \left(\frac{N_b e c}{2\pi} \right)^2 \sqrt{\frac{1}{2} Z_o \rho}, \quad (1)$$

where $Z_o \simeq 376.73 \Omega$, b is the beam pipe radius and ρ the wall resistivity. Using LHC parameters [5], namely $k_b = 4725$, $N_b = 10^{11}$ and $\sigma = 7.5$ cm (at top energy), we obtain a loss $P_W = 4$ kW in the .1 mm copper layer inside the liner, with resistivity $\rho = 5.5 \times 10^{-10} \Omega\text{m}$ at $5 \div 10$ K in presence of a 10 Tesla magnetic field and assuming a circular liner with radius $b = 1.3$ cm. The main purpose of the liner is to screen the external beam pipe at 1.9 K from synchrotron radiation (9.1 kW at top energy). If no liner were present, the resistive wall power loss in the external beam pipe (assuming a radius $b = 2.2$ cm and a resistivity $\rho = 5 \times 10^{-7} \Omega\text{m}$ for stainless steel) would be $P_W = 72.7$ kW. This gives an upper bound for the losses through pumping holes in the liner.

We now consider the resistive power loss P_W^{ext} in the external coaxial structure and denote by R_I , ρ_I the radius and resistivity of the inner conductor (liner) and by R_O , ρ_O those of the outer conductor (beam pipe):

$$P_W^{ext} = k_b c \int_{-\infty}^{\infty} d\omega \sqrt{\frac{|\omega| \mu_o}{2}} \left[\frac{\sqrt{\rho_I}}{2\pi R_I} + \frac{\sqrt{\rho_O}}{2\pi R_O} \right] \left| \tilde{I}_b^{ext}(\omega) \right|^2, \quad (2)$$

where $\left| \tilde{I}_b^{ext}(\omega) \right|^2$ is the spectral density at frequency ω of the current induced by a bunch in the external coaxial structure. It can be written in terms of the field amplitude $\tilde{a}(\omega)$ in the coax and of its characteristic impedance Z_{char}

$$\tilde{I}_b^{ext}(\omega) = \frac{\tilde{a}(\omega)}{\sqrt{Z_{char}}}, \quad Z_{char} = \frac{Z_o}{2\pi} \ln \left(\frac{R_O}{R_I} \right). \quad (3)$$

Since the resistive loss P_W^{ext} is also given by

$$\frac{P_W^{ext}}{k_b c} = - \int_{-\infty}^{\infty} d\omega \left[\frac{d|\tilde{a}(\omega)|^2}{d\ell} \right]_{\text{resist}} = \int_{-\infty}^{\infty} d\omega 2 \frac{|\tilde{a}(\omega)|^2}{\ell_{att}(\omega)},$$

we obtain the attenuation length $\ell_{att}(\omega)$

$$\ell_{att}(\omega) = 2 \frac{Z_o \ln(R_O/R_I)}{\sqrt{\frac{|\omega| \mu_o}{2}} \left(\frac{\sqrt{\rho_I}}{R_I} + \frac{\sqrt{\rho_O}}{R_O} \right)}. \quad (4)$$

For $R_I = 1.5$ cm, $R_O = 2.2$ cm and assuming a resistivity $\rho_I = \rho_O = 5 \times 10^{-7} \Omega\text{m}$ (stainless steel), we have

$$\ell_{att}(\omega) = 57.9 \text{ m} \times \left(\frac{2\pi \text{ GHz}}{\omega} \right)^{1/2},$$

where $\omega/2\pi \simeq 1$ GHz is the typical frequency in the bunch spectrum. Therefore all the pumping holes over this distance will contribute to the *coherent* build up of energy in the coaxial structure.

In weak field approximation, the excitation $\Delta \tilde{a}(\omega)$ due to a *single* hole can be written

$$\Delta \tilde{a}(\omega) = \frac{\omega(p_e - p_m)}{4\pi R_I^2 \sqrt{2\pi \frac{\epsilon}{\mu_o}} \ln(R_O/R_I)} \tilde{I}_b(\omega), \quad (5)$$

where $\tilde{I}_b(\omega) = (N_b e / \sqrt{2\pi}) \exp \left[-\frac{1}{2} \left(\frac{\omega \sigma}{c} \right)^2 \right]$ is the bunch current spectrum and p_e , p_m denote the electric and magnetic "outside" polarizabilities of the hole, respectively. For a circular hole of radius r in a wall of thickness t they become [6, 7]

$$p_e = -\frac{2}{3} r^3 e^{-2.405t/r} \left[1 - 0.14 \left(1 - e^{-4.81t/r} \right) \right],$$

$$p_m = -\frac{4}{3} r^3 e^{-1.841t/r} \left[1 - 0.19 \left(1 - e^{-3.682t/r} \right) \right]$$

and thus we have

$$p_e - p_m = \frac{2}{3} r^3 F(t/r), \quad (6)$$

where the function $F(t/r)$ is associated with attenuation from the "inside" to the "outside" of the hole, through the circular wave guide of radius r and length t equal to the hole depth. Let us remark that each hole contributes linearly to the field amplitude in the coax (since, in weak field approximation, we neglect the opposite process of field excitation inside the liner) and thus the energy in the coax, proportional to $|\tilde{a}(\omega)|^2$, has initially a *quadratic* build up.

If d_h is the longitudinal distance between holes and n_h the number of holes per cross section, the growth rate $\tilde{g}(\omega)$ of the TEM coaxial mode is

$$\tilde{g}(\omega) = \frac{\Delta \tilde{a}(\omega)}{d_h/n_h}. \quad (7)$$

The time evolution of the field energy is therefore governed by the following differential equation:

$$\frac{d|\tilde{a}(\omega)|^2}{d\ell} = 2\tilde{a}\tilde{g} - 2\frac{|\tilde{a}(\omega)|^2}{\ell_{att}}. \quad (8)$$

The steady state field amplitude \tilde{a}_∞ corresponds to an energy balance between hole excitation and resistive losses and is thus given by

$$\tilde{a}_\infty(\omega) = \tilde{g}(\omega)\ell_{att}(\omega). \quad (9)$$

We can now compute the transmission coefficient $G_\infty(\omega)$ at steady state

$$G_\infty(\omega) \equiv \frac{\tilde{I}_\infty^{ext}(\omega)}{\tilde{I}_b(\omega)} = \frac{2}{3\pi} f \frac{\sqrt{2\mu_o|\omega|} r F(t/r)}{\sqrt{\rho_I} \left(1 + \frac{R_I}{R_O} \sqrt{\frac{\rho_O}{\rho_I}}\right)}, \quad (10)$$

where f is the fraction of liner surface covered by holes.

$$f = \frac{\pi r^2 n_h}{2\pi R_I d_h}. \quad (11)$$

For a hole radius $r = 1$ mm and a liner thickness $t = 2$ mm, using the same parameters adopted to evaluate the attenuation length ℓ_{att} , we obtain

$$G_\infty(\omega) = 0.038 \times \left(\frac{f}{5\%}\right) \left(\frac{\omega}{2\pi \text{ GHz}}\right),$$

showing that the weak field approximation $G_\infty \ll 1$ is well justified. At steady state, the power loss in the external coaxial structure is given by Eq. (2), where $\tilde{I}_b^{ext}(\omega)$ can be replaced by the product $G_\infty(\omega)\tilde{I}_b(\omega)$. Performing the integral over frequency yields

$$P_W^{ext} = \Gamma \left(\frac{5}{4}\right) \frac{k_b}{R_I} \left[\frac{N_b e}{3\pi^2} f r F\left(\frac{t}{r}\right) \right]^2 \frac{Z_o \sqrt{2\mu_o/\rho_I}}{\left(1 + \frac{R_I}{R_O} \sqrt{\frac{\rho_O}{\rho_I}}\right) \left(\frac{c}{v}\right)^{5/2}}. \quad (12)$$

Using LHC parameters and assuming a liner thickness $t = 2$ mm, this corresponds to a beam power loss of 121 W for a hole radius $r = 1$ mm. A fraction $R_I/(R_I + R_O) \simeq 40\%$ of this power is dissipated in the external vacuum chamber at 1.9 K. This is again consistent with the weak field approximation, since P_W^{ext} is much smaller than the upper bound of 72.7 kW previously obtained in the absence of a beam screen and also of the 4 kW dissipated in the copper layer inside the liner.

For a length ℓ much smaller than the attenuation length ℓ_{att} , the current induced in the coax is

$$\tilde{I}_b^{ext}(\omega) = \frac{\tilde{g}(\omega)\ell}{\sqrt{Z_{char}}} \quad (13)$$

and the transmission coefficient can be written

$$G(\omega) = N_h \frac{r^3 F(t/r)\omega}{6\pi R_I^2 c \ln(R_O/R_I)}, \quad (14)$$

where $N_h = \ell n_h/d_h$ is the number of holes within the length ℓ .

3 CONCLUSIONS

The results of our measurements confirm the (adverse) effect of TEM coaxial modes travelling in synchronism with the beam. They also show the critical dependence of the transmission coefficient (and thus of the beam power loss) on the ratio r/t of the hole radius to the wall thickness of the liner. We plan to perform further impedance measurements, using a longer model of the LHC vacuum chamber with circular or elliptical holes. By artificially enhancing the ohmic losses in the coaxial structure, we could reduce the attenuation length and thus check our asymptotic formula for the beam power loss.

Analytical estimates of the coupling impedance for elliptical holes or slots in a thick wall liner are being developed. However, although the low frequency impedance of a long slot is much smaller than that of a circular hole with the same area [2, 3], the transmission coefficient at high frequency is necessarily larger. Indeed the corresponding cutoff frequency becomes lower. An interesting compromise could be to drill a long series of small circular holes with a separation comparable to their diameter (say a 2 cm series of 8 circular holes with 1 mm radius). Such a geometry could share the advantages of slots (at low frequency) and of circular holes (high cutoff frequency). Of course this should be demonstrated by further impedance measurements.

4 REFERENCES

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