

Open Resonator in the Microwave Region

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Abstract

By taking into account both the diffraction and ohmic losses, a complete analysis of the modes in an open resonator is developed. The coupling between the rectangular feeding waveguide and the cavity is fully considered, by taking into account all the relevant modes in the waveguide and the open cavity, in order to derive the coupling coefficients. Preliminary experimental results are presented.

1 INTRODUCTION

A Beat Wave Accelerator in the microwave region has been recently proposed [1]. It uses an open cavity to overcome the problem of excessive power requirements. Open resonators have also been proposed as cavities for quasi-optical gyrotrons, stressing the advantages of their use, as, f.i., the optimization of the electronic efficiency and minimization of undesirable effects as mode mixing or ohmic losses [2]. At last, open resonators have been shown to provide an accurate and convenient tool for measuring the complex permittivity of flat or nonplanar dielectric at microwave frequencies [3]. There are several papers [4], [5] dealing with the coupling through a small hole or through a tapered rectangular waveguide, taking into account only the fundamental mode. In this work a first experimental validation of the theory, [6] that takes into account all the relevant eigenfunctions in the rectangular waveguide and the open cavity, is presented.

2 WAVEGUIDE-CAVITY COUPLING

The electromagnetic field inside the cavity can be expressed in terms of (quasi) transverse electromagnetic modes (TEM):

$$\mathbf{E} = \sum_n V_n \mathbf{e}_n \quad (1)$$

$$\mathbf{H} = \sum_n I_n \mathbf{h}_n \quad (2)$$

The expressions for the \hat{y} polarized modes are:

$$\mathbf{e}_n = \phi_{mp}$$

$$\cos \left[k_n z - (m + p + 1) \tan^{-1} \frac{z}{z_R} + \frac{\pi x^2 + y^2}{\lambda R(z)} + \frac{q\pi}{2} \right] \hat{y} \quad (3)$$

$$\mathbf{h}_n = -\phi_{mp}$$

$$\sin \left[k_n z - (m + p + 1) \tan^{-1} \frac{z}{z_R} + \frac{\pi x^2 + y^2}{\lambda R(z)} + \frac{q\pi}{2} \right] \hat{x} \quad (4)$$

and similarly for the \hat{x} polarized ones. In (3-4) the index n summarizes the mode indexes (mpq), k_n is the propagation constant of the n -th mode,

$$\phi_{mp}(x, y, z) = \frac{1}{w(z)} \sqrt{\frac{2}{\pi 2^{m+p} m! p!}}$$

$$H_m \left(\sqrt{2} \frac{x}{w} \right) H_p \left(\sqrt{2} \frac{y}{w} \right) \exp \left[-\frac{x^2 + y^2}{w^2(z)} \right] \quad (5)$$

H_m is the Hermite polynomial of order m , z_R is the Rayleigh distance, $R(z)$ is the radius of curvature of the wavefront intersecting the axis at z and $w(z)$ is the measure of the decrease of the field amplitude with the distance from the axis. The decrease is Gaussian, and the beam "diameter" contracts to a minimum value $2w_0$ at the beam waist, where the phase front is plane [7].

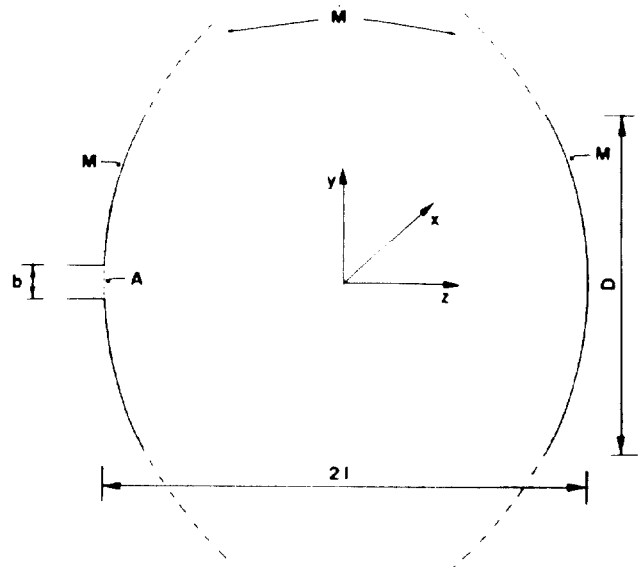


Figure 1: Open resonator powered by rectangular waveguide

Let us divide the cavity surface S in three parts: the coupling aperture A , the mirror M and the (ideal) cavity

boundary external to the the mirrors, \hat{M} (see Fig. 1). Hence:

$$I_n = \frac{j\omega\epsilon_0}{k^2 - k_n^2} \frac{1}{l}$$

$$\left\{ \int_A \hat{n} \times \mathbf{E} \cdot \mathbf{h}_n^* dS + \int_M \hat{n} \times \mathbf{E} \cdot \mathbf{h}_n^* dS + \int_{\hat{M}} \hat{n} \times \mathbf{E} \cdot \mathbf{h}_n^* dS \right\} \quad (6)$$

where l is half the cavity length. The Leontovič boundary condition $\hat{n} \times \mathbf{E} = \frac{1+j}{\sigma\delta} \hat{n} \times \mathbf{H} \times \hat{n}$ wherein σ is the electric conductivity of the mirrors and $\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$ is the penetration depth, can be applied to express the (tangential) electric field over the mirrors in terms of magnetic one, given by (2). Outside the mirrors we can assume that the field is an outgoing locally plane wave (Fresnel-Kirchhoff approximation), so that on \hat{M} , $\hat{n} \times \mathbf{E} = \zeta_0 \hat{n} \times \mathbf{H} \times \hat{n}$, where ζ_0 is the free space impedance.

The diffraction losses of a cavity can be calculated by taking into account the diffraction effects produced by the finite size of the mirrors. Under the simplifying assumption of quasi-optic nature of the problem (dimensions of the resonator large compared to wavelength and quasi-transverse electromagnetic fields) the Fresnel-Kirchhoff formulation can be invoked for the diffracted field from the mirrors.

Taking into account that $\sigma\delta\zeta_0 \gg 1$ and $\delta_n/\delta \simeq 1$ for all relevant modes, equation (6) becomes

$$I_n = \frac{j\omega\epsilon_0}{\left(k^2 - k_n^2 + \frac{kk_n}{Q_{rn}}\right) - j\frac{kk_n}{Q_{rn}}} \frac{1}{l} \left\{ \iint_A \mathbf{E} \times \mathbf{h}_n^* \cdot \hat{n} dS - 2\zeta_0 \sum_m' I_m \alpha_{nm} \right\} \quad (7)$$

wherein $\sum' \equiv \sum_{n \neq m}$, $\frac{1}{Q_{rn}} = \left(\frac{1}{Q_{dn}} + \frac{1}{Q_{rn}}\right)$ with Q_{rn} quality factor for the ohmic losses and Q_d that one for the diffraction losses,

$$2\alpha_{nn} = \iint_M |\mathbf{h}_n|^2 dS = \frac{\omega_n \mu_0}{Q_{rn}} \sigma \delta_n l \quad (8)$$

A rectangular metallic waveguide, with transverse dimensions $a \times b$, is assumed to feed the cavity. The waveguide field on the coupling aperture A is represented as:

$$\mathbf{E}^g = \sum_n V_n^g \mathbf{e}_n^g \quad (9)$$

$$\mathbf{H}^g = \sum_n I_n^g \mathbf{h}_n^g \quad (10)$$

where \mathbf{e}^g and \mathbf{h}^g are either TE electromagnetic modes of the metallic rectangular waveguide.

Expressing the field over the coupling aperture A by means of expression (9) we obtain from (7):

$$I_n + 2F_n \sum_m' \alpha_{nm} I_m = \frac{F_n}{\zeta_0} \sum_m \beta_{nm} V_m^g \quad (11)$$

where

$$F_n = \frac{jk/l}{\left(k^2 - k_n^2 + \frac{kk_n}{Q_{rn}}\right) - j\frac{kk_n}{Q_{rn}}} \quad (12)$$

and

$$\beta_{nm} = \iint_A \mathbf{e}_m^g \times \mathbf{h}_n^* \cdot \hat{n} dS = - \iint_A \mathbf{h}_n^* \cdot \mathbf{h}_m^g dS \quad (13)$$

By introducing the matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$, whose elements are

$$a_{nm} = \begin{cases} \frac{1}{F_n} & n = m \\ 2\alpha_{nm} & n \neq m \end{cases} \quad (14)$$

and β_{nm} respectively and the vectors $\underline{\underline{I}} \equiv \{I_n\}$ and $\underline{\underline{V}}^g \equiv \{V_n^g\}$, the relation (11) can be written in a compact form as:

$$\zeta_0 \underline{\underline{A}} \cdot \underline{\underline{I}} = \underline{\underline{B}} \cdot \underline{\underline{V}}^g = \underline{\underline{B}} \cdot (\underline{\underline{V}}^+ + \underline{\underline{V}}^-) \quad (15)$$

wherein $\underline{\underline{V}}^+$ and $\underline{\underline{V}}^-$ are the vectors of the incident and reflected waveguide mode amplitudes respectively. By enforcing the continuity of the magnetic field tangential component over the coupling aperture, we get from (2), (10) and (11):

$$-\underline{\underline{B}}^+ \cdot \underline{\underline{I}} = \underline{\underline{I}}^g = \frac{1}{\zeta_0} \zeta^{-1} \cdot (\underline{\underline{V}}^+ - \underline{\underline{V}}^-) \quad (16)$$

wherein $\underline{\underline{B}}^+$ is the adjoint (i.e., the transpose, being $\underline{\underline{B}}$ a real matrix) of $\underline{\underline{B}}$ and ζ is the diagonal matrix whose elements are the modes characteristic impedances, normalized to ζ_0 . From (48) and (49) we immediately obtain:

$$\left(\underline{\underline{I}} - \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}} \cdot \zeta \cdot \underline{\underline{B}}^+\right) \cdot \underline{\underline{I}} = \frac{2}{\zeta_0} \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}} \cdot \underline{\underline{V}}^+ \quad (17)$$

$$\left(\underline{\underline{I}} - \zeta \cdot \underline{\underline{B}}^+ \cdot \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}}\right) \cdot \underline{\underline{V}}^- = \left(\underline{\underline{I}} + \zeta \cdot \underline{\underline{B}}^+ \cdot \underline{\underline{A}}^{-1} \cdot \underline{\underline{B}}\right) \cdot \underline{\underline{V}}^+ \quad (18)$$

wherein $\underline{\underline{I}}$ is the unit matrix and $\underline{\underline{A}}^{-1}$ the inverse of the matrix $\underline{\underline{A}}$.

Solution of eq. (17) or (18) provides the answer to our problem.

3 EXPERIMENTAL RESULTS AND CONCLUSIONS

An experimental set-up has been developed in order to validate the previous theory.

The sample cavity used has a length $2l=266$ mm. The curvature radius of the mirrors is $R=351$ mm, the diameter $D=144$ mm. Copper mirrors are assumed, with a conductivity $\sigma = 5.71 \times 10^7$ s/m. The rectangular feeding waveguide has standard dimensions $a=10.668$ mm, $b=4.318$ mm, which ensure that only the fundamental mode TE_{10} can propagate in the working band 18-26 GHz.

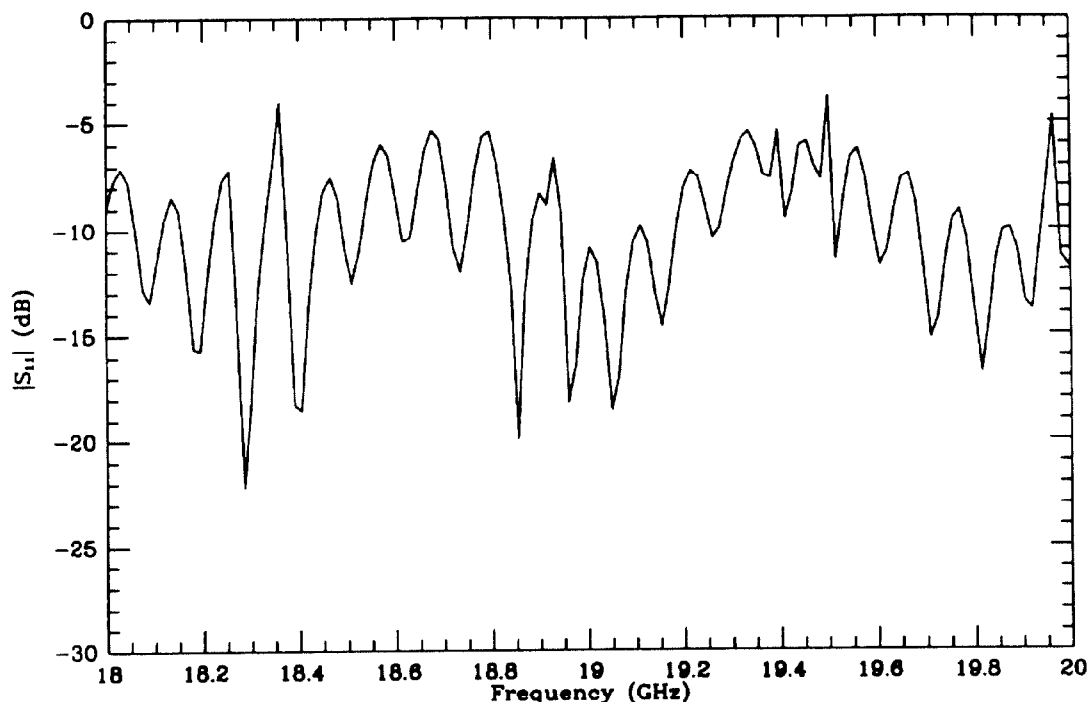


Figure 2: Reflection coefficient of the sample cavity

The amplitude of the reflection coefficient has been measured with a Network Analyzer, Hp 8757C. Fig. 2 shows the experimental results.

The theoretical values computed, according to section 2, for the same cavity are reported in Table 1. In the table q is the number of nodes of the axial standing wave pattern, i.e. the longitudinal mode number, f is the cavity resonant frequency for the assumed $(q,0,0)$ modes, Δf the frequency band and Γ the amplitude of the reflection coefficient.

Table 1: Theoretical results

q	f (GHz)	Δf (kHz)	$ \Gamma $
31	18.28	67.7	.942
32	18.84	68.7	.950
33	19.41	69.7	.957
34	19.47	70.7	.962

From a comparison between the theoretical and the experimental data a quite good agreement is evident. On these results the following comments are in order:

(a) Mismatching of the experimental apparatus and cavity misalignments produce several unwanted reflections, that are evident in fig.2, and a little shift of the resonance peaks;

(b) for the reasons pointed out in (a) and for the cavity narrow band, due to the high quality factor ($Q \sim 0.66 \times 10^5$), plus the limits on the instrument resolution the expected resonances are not very well resolved.

The comparison between the theoretical and the experimental results shows that the first is reliable, but the second requires some improvements either on the measuring set-up, either considering different coupling structures, already taken into account in the theory. Future work is in progress.

4 REFERENCES

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