BEAM OPTICS AT RESONANT EXTRACTION SEPTA

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Abstract

Several recently designed slow extraction schemes deliver a zero emittance beam at the first septum (which is usually electrostatic). This beam is obtained by a special adjustment of dispersion and chromaticity. The same parameters can be tuned to optimize the separation between circulating and extracted beams at the second septum in the case when the transfer between septa is not achromatic (e.g. septa in different straight sections). The method and the relevant calculations are presented for the third-order resonance and the new slow extraction scheme from the CERN PS is given as an example.

1. INTRODUCTION

Resonant extraction is widely used to deliver spills of long duration to experimenters working on secondary beams originating from external fixed targets. In this study, we shall restrict ourselves to third-order resonances. Such a resonance is driven by a sextupolar perturbation. In the horizontal phase space, it defines a stable area and separatrices on which particles move away from the closed orbit. A first septum (usually electrostatic) divides the part of the beam to be extracted from the circulating one. Its deflection is converted into a physical separation later on, where a second thicker septum deflects the beam further towards the extraction channel.

2. PHASE PLANE BEHAVIOUR AT RESONANCE

We shall deal with trajectories in the normalized horizontal phase plane (x,x'), for the sake of simplicity, using the classical normalization matrix:

$$\begin{vmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{vmatrix}$$

where α and β are the Twiss parameters at the considered point.

Let us consider the third-order resonance driven by a single sextupole of normalized strength S given by:

$$S = \frac{\beta_s^{3/2}}{B\rho} \int B'' dl$$

where $B\rho$ is the magnetic rigidity of the beam, β_s the horizontal betatron amplitude function at the sextupole and where the integral of the second horizontal derivative of the vertical component of the field, B'', is evaluated over the length of the sextupole.

In the more general case of several sextupoles, one has to consider the third harmonic of the sextupolar distribution in betatron phase around the machine. If there are M sextupoles of strength S_m and normalized strength S_{nm} at phase φ_m, the resulting localized sextupole has a strength S and a phase φ_s with:

$$Se^{3j\varphi_s} = \sum_{m=1}^{M} S_{nm} e^{3j\varphi_m} = \sum_{m=1}^{M} S_m \beta_m^{\frac{3}{2}} e^{3j\varphi_m}$$

In this equation, phases can be taken from whichever common reference is most convenient.

The first-order classical theory[1] states that the stable areas are triangles and the separatrices are straight lines (fig.1). In the phase plane at the sextupole, one of the separatrices is parallel to the x' axis and is given by:

$$x - \left(\frac{8\pi Q_x'}{S} + D_{no} \right) \frac{\Delta p}{p} = 0 \qquad (1)$$

where we have introduced the horizontal chromaticity Q_x' (including the sextupole contribution) defined as p dQ_x/dp, the relative momentum offset from resonance $\Delta p/p$ and the local normalized dispersion D_{no}.

In the general case of chromatic extraction, Q_x' is not zero and the size of the stable area varies with energy. It starts from zero on resonance and grows as the betatron frequency (and the energy) moves away from the resonance value. Moreover, the position of the stability triangle depends on energy if D_{no} is not zero.

The emittance of the stable area is given by:

$$\varepsilon = \frac{\pi}{\sqrt{3}} \left(\frac{24 \Delta Q_x}{S} \right)^2 = \frac{\pi}{\sqrt{3}} \left(\frac{24 Q_x'}{S} \frac{\Delta p}{p} \right)^2 \qquad (2)$$

where $\Delta Q_x = Q_x - Q_{xr}$, the difference between the horizontal betatron oscillation number Q_x and its value at resonance.

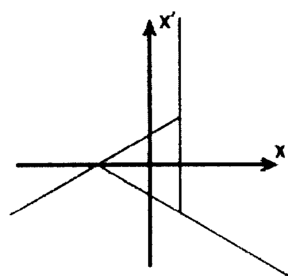

Fig.1

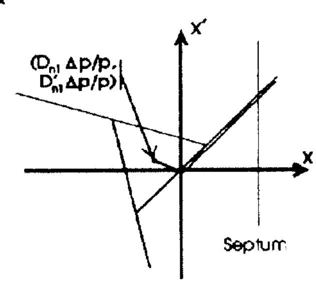

Fig.2

The constant of equation (1) can thus also be expressed in terms of the emittance, ε_0 (in π mm mrad), and the instantaneous extracted momentum bite, $(\Delta p/p)_E$:

$$\frac{8\pi Q_x'}{S} = \frac{\sqrt{\pi\varepsilon_0 \sqrt{3}}}{3\left(\frac{\Delta p}{p} \right)_E}$$

3. Beam optics at the first septum

Efficiency and low losses are important requirements for most machines, so that the apparent septum thickness must be minimized. Electrostatic septa have been proposed with wires down to 10 µm in diameter[2].

It is also important to minimize the angular dispersion of the separatrices. Superimposing the separatrices for all momenta has been implemented in the LEAR machine at CERN [3] and is now frequently used [2,4,5].

From (1), we can derive the equation of the separatrices in the phase plane at the first septum:

$$x\cos\varphi_1 - x'\sin\varphi_1 - \left(\frac{8\pi Q'_x}{S} + D_{n1}\cos\varphi_1 - D'_{n1}\sin\varphi_1\right)\frac{\Delta p}{p} = 0 \quad (3)$$

where φ_1 is the betatron phase angle between the sextupole and the septum, D_{n1} and D'_{n1} the normalized dispersion coefficients at the first septum.

They all superimpose (fig.2) when:

$$\frac{8\pi Q'_x}{S} + D_{n1}\cos\varphi_1 - D'_{n1}\sin\varphi_1 = 0 \qquad (4)$$

4. Beam optics at the second septum

The deflection given by the first septum separates the extracted beam from the circulating particles and a second septum is located where this separation is large enough to accomodate the thickness of the septum blade. However, in the general case of chromatic transfer between the septa, the separation phase plane location at the second septum depends on the extracted particle energy, even when condition (4) is satisfied at the first septum.

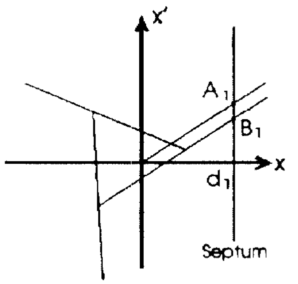

Fig.3

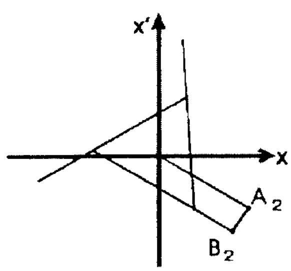

Fig.4

The best way to avoid the problem is to fulfill condition (4) and to place both septa in the same straight section: the emittance is thus zero in both septa and the gap is optimized at the second one. Unfortunately, this is not always possible, but one can still adjust the chromaticity and dispersions to optimize the separation at the second septum.

Let us first consider the situation at the electrostatic septum in the general case when equation (4) is not satisfied. All separatrices are described by equation (3). Let us consider two separatrices, the first one on resonance with $\Delta p/p = 0$, and a second one with any $\Delta p/p$. They intersect the septum, located at distance d_1 from the machine central orbit (see fig.3), at points A_1 and B_1 with x' ordinates:

$$x'_{A1} = d_1\frac{\cos\varphi_1}{\sin\varphi_1}$$

$$x'_{B1} = \frac{1}{\sin\varphi_1}\left[\left(d_1 - D_{n1}\frac{\Delta p}{p}\right)\cos\varphi_1 - \frac{8\pi Q'_x}{S}\frac{\Delta p}{p}\right] + D'_{n1}\frac{\Delta p}{p}$$

Points A_1 and B_1 become A_2 and B_2 at the second septum, situated at phase φ_2 from the sextupole, after a rotation around their respective closed orbits, namely the origin for A_1 and A_2, the point $(D_{n1}\Delta p/p, D'_{n1}\Delta p/p)$ for B_1, and the point $(D_{n2}\Delta p/p, D'_{n2}\Delta p/p)$ for B_2 (see fig.4).

The x ordinates of A_2 and B_2 are then:

$$x_{A2} = d_1\cos(\varphi_2 - \varphi_1) + d_1\frac{\cos\varphi_1}{\sin\varphi_2}\sin(\varphi_2 - \varphi_1)$$

$$x_{B2} = \left(d_1 - D_{n1}\frac{\Delta p}{p}\right)\cos(\varphi_2 - \varphi_1) + \frac{\sin(\varphi_2 - \varphi_1)}{\sin\varphi_1}\left[\left(d_1 - D_{n1}\frac{\Delta p}{p}\right)\cos\varphi_1 - \frac{8\pi Q'_x}{S}\frac{\Delta p}{p}\right] + D_{n2}\frac{\Delta p}{p}$$

The separation between circulating and extracted beam is optimized when:

$$x_{A2} = x_{B2}$$

$$D_{n2}\sin\varphi_1 - D_{n1}\sin\varphi_2 = \frac{8\pi Q'_x}{S}\sin(\varphi_2 - \varphi_1) \qquad (5)$$

This result can also be derived by stating that x_{B2} is independent of $\Delta p/p$.

Note that a suitable choice of parameters D_{n1}, D'_{n1}, D_{n12} and Q'_x may be found to satisfy both conditions (4) at the first septum and (5) at the second.

5. Application to the CERN PS

The slow extraction to the East Area of the CERN PS has just been overhauled for several reasons:

- improvement of the vacuum for future lead ion acceleration required a reduction of the number of septa under vacuum together with a change in their technology,
- synchrotron radiation during the acceleration of leptons on other cycles for the LEP machine damages the septa, so that it was decided to place them towards the inside of the machine.

In the regular alternating gradient combined function lattice of the PS, the transfer between the two septa is obviously chromatic. We should therefore optimize the clearance at the first magnetic septum and tune the chromaticity according to equation (5).

One needs a large enough horizontal chromaticity to avoid too large a momentum dispersion of the extracted beam. The left-hand side of equation (5) must therefore be as large as possible.

Since both septa are on the same side of the closed orbit, $\sin\varphi_1$ and $\sin\varphi_2$ have the same sign. One should then aim at a large dispersion at the magnetic septum and a small one at the electrostatic septum.

The two quadrupoles used to bring the horizontal tune to the resonance value of 6+1/3 distort the dispersions as well as the β-functions all around the machine. A careful choice of the locations and strengths of these elements produced the desired effect and increased the β-functions at both septa according to the values of table 1.

Table 1: Perturbed parameters with 2 quadrupoles in PS

Location	Unperturbed machine		Machine with 2 quads	
	β horiz.	Dispersion	β horiz.	Dispersion
Electrostatic septum	22.2 m	3.04 m	36.2 m	1.27 m
Magnetic septum	22.6 m	3.04 m	35.5 m	5 .01 m

After the left-hand side of equation (5) is made larger, we still had to lower the right hand side of the equation to fulfill the condition. This was achieved through lowering the absolute value of the horizontal chromaticity in 2 ways:

- choice of a large zero harmonic distribution for the set of 2 sextupoles, together with the third harmonic needed to drive the resonance,
- introduction of a large sextupole componant in the combined function magnets by means of the pole face windings.

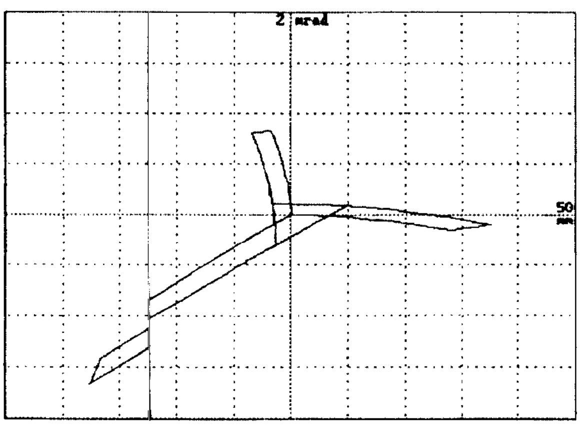

Fig.5: Horizontal phase plane at the electrostatic septum

Results from analytical calculation, computer simulation and actual experimention are compared in table 2.

Table 2: Machine chromaticity from different methods

Equation (5)	Tracking simulation	Machine Experiment
−3.14	−3.87 (−3.28)	≅ −3.2

The agreement between the three figures is satisfying, considering that the theory used relies on the approximation of linear separatrices. The simulation result in brackets has been calculated with the electrostatic septum much nearer to the orbit, where the curvature of separatrices is negligible.

The horizontal phase plane diagrams at the electrostatic and first magnetic septum can be seen in fig. 5 and 6 with the local orbit distortions not shown. For a 1 π mm mrad circulating beam emittance, the momentum bite is found to be Δp/p = .001 .

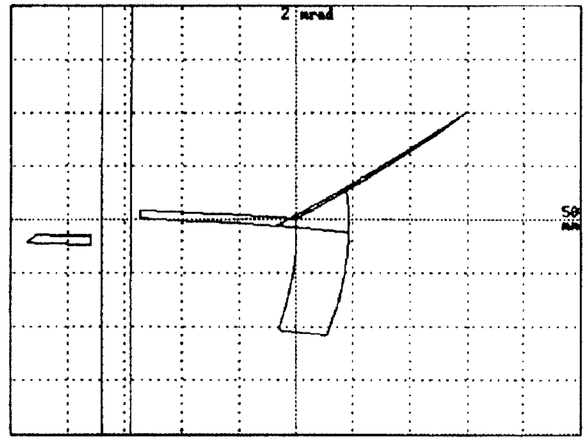

Fig.6: Horizontal phase plane at the first magnetic septum

The angular dispersion could not be corrected at the electrostatic septum. The resulting extra loss is estimated to be .5% of the extracted beam, tolerable for this extraction, which is designed for relatively low intensities. The space available for the first magnetic septum blade thickness exceeds 8 mm, as compared to less than 3 mm in the former scheme.

6. Conclusion

Using the method outlined above, the gap created at the second septum by the first one becomes independant of the various particle energies within the beam. One can thus optimize the separation between extracted and circulating beams at the first magnetic septum. In the case of the CERN PS machine, this enabled us to decrease the number of magnetic septa from 3 in the former scheme to 2 in the new one, with only one in vacuum, for the same extraction momentum of 24 GeV/c.

7. References

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