RESONANCES IN THE BEAM-PLASMA INTERACTION DUE TO A FINITE CONDUCTOR CYLINDER

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Abstract

The effects of the Beam - Plasma interaction on a finite radial and Transversal conductor was studied. We obtained the resonances for different density profiles at the plasma and beam. We used different temperatures too. The method is based on the Vlasov-Maxwell system equations with first order perturbations. The discussions was restrited to the adiabatic regime.

1 INTRODUCTION

The beam plasma interactions is one of the best known interactions for understanding many phenomena as wake field[1,2] for instances and others too. The particles are assumed to be inhomogeneous radial distribute and confined by strong magnetic field. In this paper we discuss the relation dispersion for a bounded beam plasma system in the regime below for linear unstability. In system as beam plasma interactions appears density gradients in the axial and radial directions, here we only treat witch radial inhomogeneities, besides, we suppose a electronic plasma , neglected the effects of binary collisions, e.i, the collisions frequency is much lower than the wave frequency.

2 THEORETICAL MODEL

In order to solve the Vlasov Maxwell system, we used the small perturbations about the Vlasov equilibrium[3], and due to strong magnetic field that allow us to consider only axial velocity distribution functions f_μ (r,v,t) for the plasma and beams:

$$f_\mu(r,v_z,t) = \int d^2v\, f_\mu(\vec{r},\vec{v}_\perp,t) \qquad (1)$$

Where $v_\perp$, v_z is the transversal, longitudinal velocity and the equilibrium distributions may be written as $f\mu(r,v) = g\mu(r)F\mu(v)$. μ denotes the kind of particles in the plasma (in this case, electron for the plasma and beam). $g_\mu(r)$ is the radial density (see fig 1), and $F\mu(v)$ is the axial velocity equilibrium distribution functions.

The model adopted and the former assumptions yield to the system Vlasov Maxwell equations written as follow:

$$\mathrm{Div}\,\vec{E} = \sum_\mu 4\pi e_\mu n_\mu(0)\int d^2v\, f_\mu(\vec{r},\vec{v},t) \qquad (2)$$

$$\mathrm{Rot}\,\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \qquad (3)$$

$$\mathrm{Div}\,\vec{B} = 0 \qquad (4)$$

$$\mathrm{Rot}\,\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{1}{c}\sum 4\pi e_\mu n_\mu(0)\int v\,dv\, f_\mu(\vec{r},v,t)\hat{\varepsilon}_z \qquad (5)$$

$$\frac{\partial f_\mu'}{\partial t} + v\frac{\partial f_\mu'}{\partial z} + g_\mu(r)\frac{e_\mu}{m_\mu}E_z\frac{\partial F_\mu^0}{\partial v} = 0 \qquad (6)$$

$n_\mu(0)$ is the particles density on axis.

In the equation 5,we can to identify the current density as[4]:

$$J_z = \sum_\mu e_\mu n_\mu(0)\int v\,dv\, f_\mu(\vec{r},v,t) \qquad (7)$$

$\hat{\varepsilon}_z$ is the z component of the dielectric tensor, which may be obtained by expanding the Vlasov equation in spacial coordinates and using the Laplace transform for the time variable,we obtain in this way:

$$\hat{\varepsilon}_z(r,k,\omega) = 1 - \sum_\mu g_\mu\frac{\omega_{p\mu}^2(0)}{k^2}\int\frac{dv\left(\frac{dF_\mu^0}{dv}\right)}{v-\frac{\omega}{k}} \qquad (8)$$

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where:

$$\omega_{p\mu}^2(0) = \frac{4\pi e_\mu^2 n_\mu(0)}{m_\mu} \qquad (9)$$

We assume the same value of w_p (0) for the plasma and beam.

If the fields have the form

$$E(\vec{r},t) = E(r)\exp i(m\theta)\exp i(kz - \omega t) \qquad (10)$$

We can to find from the Maxwell's equation the following differential equation for the z coordinate:

$$\frac{d^2}{dr^2}E_{zm}(r) + \frac{1}{r}\left[\frac{d}{dr}E_{zm}(r)\right] - \left\{\left[1 - g_\mu I_\mu(k,\omega)\right]\kappa + \frac{m^2}{r^2}\right\}E_{zm}(r) = 0 \qquad (11)$$

Here

$$I_\mu(k,\omega) = \frac{\omega_{p\mu}^2(0)}{k^2}\int \frac{dv\left(\frac{dF_\mu^0}{dv}\right)}{v - \frac{\omega}{k}} \qquad (12)$$

$$\kappa^2 = -\left(\frac{\omega^2 - k^2c^2}{c^2}\right) \qquad (13)$$

And

$$g_\mu = \left[1/\left(1 - \mathrm{Exp}\left(1 - \left(\frac{1}{2}\right)\left(\frac{r}{b} - 1\right)\right)\right)\right] \qquad (14)$$

If we used the Maxwelliam velocity distribution functions for the plasma and beam:

$$F_p^0 = \left(\frac{2\pi K_B T_p}{m_e}\right)^{-\frac{1}{2}} e^{-\left(\frac{m_e v^2}{K_B T_p}\right)} \qquad (15)$$

$$F_p^0 = \left(\frac{2\pi K_B T_b}{m_e}\right)^{-\frac{1}{2}} e^{-\left(\frac{m_e(v - v_0)^2}{K_B T_b}\right)} \qquad (16)$$

v_0 is the velocity of the beam.

The equation 10 is solved by expanding Em(r) in Fourier - Bessel series. Finally we get the following dispersion relation in dimensionless values.

$$D_{mn}\widetilde{X} = 0 \qquad (17)$$

Which $\widetilde{X}$ is a vector associated with the eigen functions for the electric field

$$D_{mn} = \frac{\bar{c}^2\bar{k}^2 - \bar{\omega}^2}{\bar{c}^2\left(\bar{k}^2 + x_{ml}^2\right)}\left\{\frac{C_{mp}^{ll'}Z'(\beta)}{2\bar{\lambda}_{dp}^2\bar{k}^2} + \frac{C_{mb}^{ll'}Z'(\alpha)}{2\bar{\lambda}_{db}^2\bar{k}^2} - \Pi\right\} \qquad (18)$$

$$C_{mp,b}^{ll'} = \frac{2}{a^2 J_{m+1}(x_{ml}) J_{m+1}(x_{ml'})}\int_0^r dr\, r\, J_{m+1}(r p_{ml}) J_{m+1}(r p_{ml'}) g_{p,b}(r) \qquad (19)$$

pml = Xml/a zeros of the Bessel´s functions.

$$\bar{k} = ka \; ; \; \bar{\lambda}_\mu^2 = \frac{K_B T_\mu}{m\omega_{p\mu}^2(0)a^2} \; ; \; \beta^2 = \frac{\bar{\omega}^2}{\bar{k}^2}\frac{1}{2\bar{\lambda}_{dp}}$$

$$\bar{\omega} = \frac{\omega}{\omega_p} \; ; \; \alpha = \frac{\bar{\omega}^2}{\bar{k}^2}\frac{1}{2\bar{\lambda}_{dp}} - \left(\frac{E}{2K_B T_B}\right)$$

a is the wave guide, k is the wave number, Π the unity matrix, E is the beam energy, Z´ is the derivate of the dispersion function. Due to the transversely boundery at z=L for the electric field give us:

$$\overline{K_+} + \overline{K_-} = \frac{2\tau a\pi}{L} \quad \text{with } \tau = 0,1,. \qquad (20)$$

K+(-) is the wave number when the electromagnetic wave travel to right (left).

3 RESULTS.

The equations (17) is solve[5] for the fundamental mode m = 0, and the figure 2 illustrated it´s results. Two modes appear low and high frequencies, remember that for the case Vlasov-Poisson system only appear the modes for low frequency[6]. The axial quantizations give us a decoupling of the beam from the imperturbable modes and hence a decrease of the growth rates.the figure 2 show the relation of k_+ and k_-

4 REFERENCES

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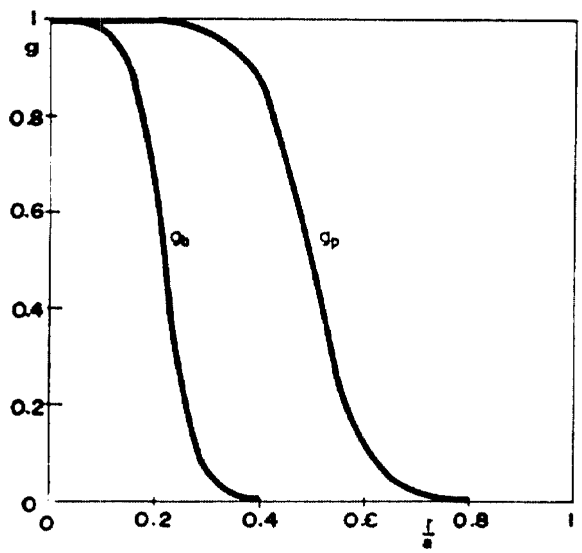

Fig 1. Density profiles for the plasma (p) and the electron beam(b).
In this case a=16,b_p=0.4 and b_b=.2 In general this values are determined by the experiment.

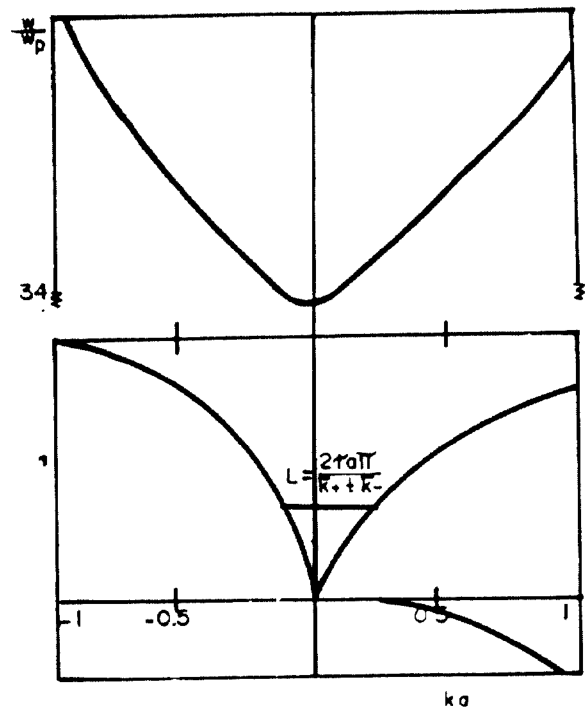

Fig 2 Show the low and high frequency branch The low branch including the Landau dampig (the lower branch)