Longitudinal Space Charge Effects in Cooled Bunched Beams

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Abstract

The study of coherent beam instability limits in space charge dominated bunches is important to achieve high intensity bunched beams for heavy ion fusion. Effects such as growth of bunch length and reduction of synchrotron frequency due to potential well flattening have been observed and evaluated. The origin of these collective phenomena is the longitudinal complex coupling impedance, which describes the interaction (coupling) of the ion beam with its surroundings. In the heavy ion cooler ring ESR two components of the total coupling impedance are dominant, the broadband space charge impedance and the narrowband impedance of the accelerating cavity. The spectrum of the longitudinal coupling impedance has been measured by analyzing beam transfer functions of a 150 MeV/u coasting $^{20}Ne^{10+}$ beam with 1 mA intensity. The reduction of the focusing rf potential has been measured by evaluating the shift of synchrotron satellites in Schottky-bands of bunched beams with more than 10^8 particles per bunch. Experimental results are compared with numerical results from the envelope equation and computer simulations including space charge. Even better results are expected in future with an extended computer code, considering the entire measured impedance spectrum of the machine.

1 INTRODUCTION

In the heavy ion cooler ring ESR very high phase space densities can be achieved by means of rf-stacking and electron cooling. Further subject of machine development is the formation of high intensity bunches, suitable for experiments on high energy density in matter. The final compression to the minimum bunch length shall be performed by a fast rotation in the synchrotron plane. Our goal is to predict and interpret collective effects and instabilities that may occur during the compression process. This requires a precise analysis of longitudinal motion including stability analysis of beams with extremely high local particle currents and correspondingly high space charge forces. Two models, suitable for theoretical examination of space charge dominated bunches shall be dicussed and compared with experimental results. The first one is analytic and known as the *Envelope Equation* [1]. It requires a couple of idealizations, but is easy to handle. The other model is a numeric computer simulation in six dimensional phase space. It can be fitted to reality much better, but is much more complex, too. Both models assume a more or less idealized accelerator environment, that plays a very important role when looking at collective effects. The quantity describing the coupling of the ion beam with its environment is the longitudinal coupling impedance $Z_{\|}(\omega)$, which can directly be measured for the ESR. The closer the spectrum of the coupling impedance used by a model to the real situation in the ESR, the more realistic are the results obtained.

2 ENVELOPE EQUATION

The envelope equation for longitudinal motion describes the behaviour of the envelope z_0 of the elliptic self-consistent particle distribution

$$f(z, z', s) = D\sqrt{1 - \frac{z^2}{z_0^2} - \frac{z_0^2}{\epsilon_L^2}\left(z' - \frac{z_0'}{z_0}z\right)^2}. \qquad (1)$$

Since the line density $\lambda(z)$ of this distribution is always parabolic, all particles in the bunch experience a purely linear space charge force. Assuming a linearily ramped external electric field, we obtain a harmonic equation of motion for a single particle. As a solution of the Vlasov equation in longitudinal phase space we obtain the envelope equation for z_0

$$\frac{d^2 z_0}{ds^2} = \frac{\epsilon_L^2 \eta^2}{z_0^3} + \frac{3q^2 r_0 g|\eta|N}{2A\beta^2\gamma^3}\frac{1}{z_0^2} - \frac{qeh|\eta|V_{rf}}{2\pi R^2 A m_u \gamma\beta^2 c^2} z_0 \quad (2)$$

where ϵ_L is the longitudinal emittance of the beam ($\epsilon_L \equiv z_0 \cdot \Delta p/p$), q is the charge state of the ions, r_0 is the classical proton radius, g is a geometrical factor of order unity, N is the number of ions per bunch, h is the number of bunches in the ring and R is the ring radius. The initial conditions $z_0(s = 0)$ and $z_0'(s = 0)$ may be chosen arbitrarily. The motion of the envelope is determined by the three r.h.s. terms. The external rf voltage that tries to focus the bunch, the debunching space charge force that tends to extend the bunch and the emittance term, which is associated with the momentum spread of the beam. Since the bunch is considered to be moving along the axis of a cylindric, perfectly conducting chamber wall no other longitudinal self fields acting on the beam (e.g. beam loading) are taken into account. Of particular interest is the stationary case, which is simply the special solution of equation (2) where ϵ_L, z_0, V_{rf} and N are chosen in a combination such that z_0'' is set equal to zero.

3 EXPERIMENTAL RESULTS

The reduction of the single particle synchrotron frequency due to potential well flattening in space charge dominated

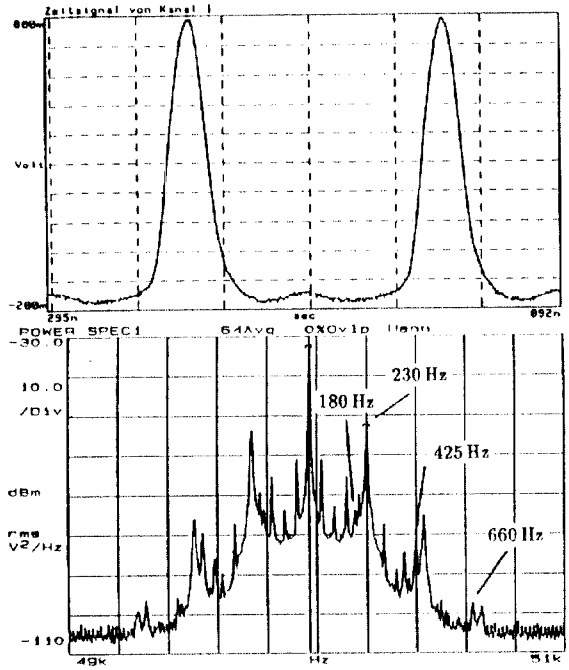

Figure 1: Cooled $^{20}Ne^{10+}$ beam (250 MeV/u, V_{rf}=100 V, I_{beam}=850 μA, τ_{bunch}=79ns and z_0=14m) in a) time and b) frequency domain (Schottky band at 10th harmonic). The 50 Hz spaced peaks are due to the rf power supply.

bunches has been observed and evaluated. Fig. 1 shows a $^{20}Ne^{10+}$ bunch at 250 MeV/u in time and frequency domain. Beam loading effects can be neglected in this case, since the cavity was slightly off-resonance and the beam current relatively low. Solving the envelope equation with this set of beam parameters, including the measured bunch length z_0, yields $\epsilon_L = 6.4 \cdot 10^{-4}$ m ($\Delta p/p = 8.8 \cdot 10^{-5}$) and an effective focusing potential of 63 V. The longitudinal oscillating ions hence experience a space charge potential of 37 V, which reduces the external 100 V potential, giving an effective synchrotron frequency of 184 Hz. This corresponds fairly well to the directly measured synchrotron frequency of 180 Hz, that can be determined from the synchrotron satellites in Fig. 1b. The Schottky band shows an incoherent peak at 180 Hz, which we interpret as the single particle synchrotron motion, and coherent peaks at 230, 425 and 660 Hz, which correspond to the multipole modes of the bunch. Another result is, that the maximum local beam current in the bunches determines the minimum momentum spread that can be obtained by electron cooling. Preleminary coasting beam measurements for various ion species in the ESR all agree with the scaling law

$$\frac{\Delta p}{p} \propto N^{1/3} \tag{3}$$

for intrabeam scattering dominated behaviour, which can also be confirmed by theoretical calculations [2] and is typical for the equilibrium between electron cooling and intrabeam scattering. Fig. 2 shows that for bunched beams the total intensity divided by the bunching factor N/B_f, as a measure for the peak intensity in the bunch, replaces N in equation (3). It is therefore possible to extrapolate cooled beam equilibria from measured data to higher intensities required for the experiments discussed above.

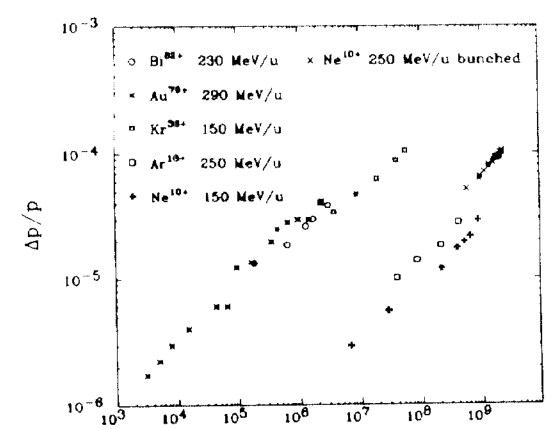

Figure 2: Experimental data for momentum spread as a function of peak ion current in the ring. $\Delta p/p$ increases with $(N/B_f)^{1/3}$, where B_f = average current/peak current and B_f=1 for coasting beam. The bunched beam data have been evaluated by using the envelope equation.

4 IMPEDANCES IN THE ESR

A direct measure for the beam-wall coupling strength is the complex coupling impedance, which describes the interaction of the ion beam with its surroundings. Collective self fields acting back on the beam are proportional to the beam current as well as to the coupling impedance itself and could lead to coherent beam instabilities, especially when dealing with high intensity bunched beams [3]. According to expectation confirmed by preliminary measurements [4] two components of the total longitudinal coupling impedance are dominant in the ESR, the broadband space charge impedance and the narrowband impedance of the accelerating cavity. The actual impedance seen by the beam is the sum of these components. The spectrum now has been evaluated over a wide frequency range by analyzing measured beam transfer functions (BTF) of a cooled 150 MeV/u coasting $^{20}Ne^{10+}$ beam with 1 mA intensity.

The ESR cavity represents a narrow band, high Q resonator, which makes it necessary to examine the vicinity of the resonance. Since BTF measurements can only be performed at harmonics of the particle revolution frequency, and the acceptance of the machine is not large enough to cover the expected width of the resonance by shifting the ion energy, we had to sweep the tune of the cavity over the desired frequency span. Two series of measurements with different cooling electron current have been taken. The result as shown in Fig. 3 is in perfect agreement with theory, which demands for the coupling impedance $Z_{\parallel}^{reson}$ of a resonator

$$\frac{Z_{\parallel}^{reson}}{h} = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)} \quad . \tag{4}$$

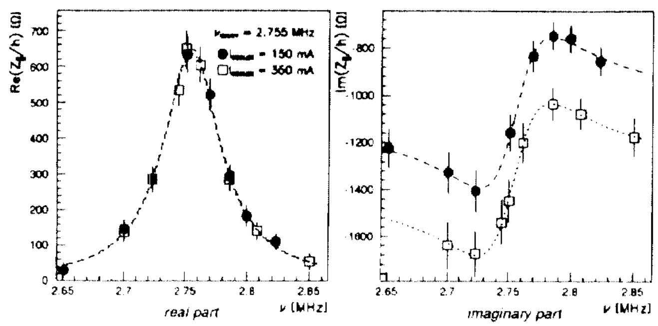

Figure 3: Longitudinal coupling impedance of the ESR cavity, measured with a cooled $^{20}Ne^{10+}$ beam (150 MeV/u, 1 mA) at h=2. The cooling electron current was 150 mA (upper trace) and 350 mA (lower trace).

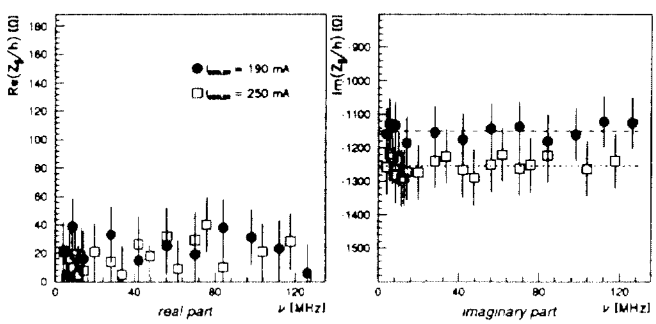

Figure 4: Space charge impedance of the ESR in the range from 4 MHz (3rd harmonic) to 130 MHz (90th harmonic).

The impedance has a distinct real maximum at the resonance frequency and is capacitive below and inductive above. A fit yields 640 Ω for the shunt impedance Rs and a quality factor of $Q = 50$. The imaginary offset of -1070 Ω (I_{cooler} =150 mA) respectively -1360 Ω (I_{cooler} =350 mA) is due to the space charge impedance $Z_{\parallel}^{sc}$. The variation of the beam radius and hence the space charge impedance with the cooling force shifts the offset of the imaginary component, while the real part remains constant.

The next step was to sweep over the entire frequency spectrum up to 130 MHz, with which we envisaged two goals, the systematic search for parasitic resonances and the verification of the theoretically predicted frequency independence of the purely imaginary space charge impedance. Again we took two series of measurements with different cooling electron currents, in order to observe the increase of the space charge impedance with the reduction of the beam radius (Fig. 4). Using the simple model of a homogeneous cylindrical beam in a perfectly conducting cylindrical vacuum chamber, and considering frequencies below the cut-off frequency of the chamber (≈ 750 MHz for the ESR) we obtain the space charge impedance

$$\frac{Z_{\parallel}^{sc}}{h} = -i\frac{Z_0}{2\beta\gamma^2}\left(\frac{1}{2} + 2\ln\frac{r_{wall}}{r_{beam}}\right) \tag{5}$$

which corresponds fairly well to our results. Assuming an average vacuum chamber radius of 5.9 cm it is found that the beam radius varies from 9 mm to 6 mm, if the electron current varies from 150 mA to 350 mA.

The real part of the impedance is approximately a factor 100 smaller than the imaginary part and is of the size of the error. It was therefore not possible to determine it exactly, but its value lies somewhere below 50 Ω. It is composed of the resistive wall impedance (according to theory several Ohms) and broad band impedances (low Q resonators) due to numerous irregular cross-section variations in the ring. As mentioned in the beginning these two components do not play a role. Nevertheless other high Q resonators (e.g. kicker) positioned between measured frequencies can not be excluded.

5 COMPUTER SIMULATION

The multi-dimensional nature of space charge force calculations in connection with the strong influence of close conducting boundaries and the possibility of collective oscillations driven by space charge forces limit the possibilities of analytic approaches like the envelope equation to a few special situations. Computer simulation has the important role of filling the gap between tractable analytic models and experiments. Our currently used code SCOPRZ is for $2\frac{1}{2}$-dimensional simulation. Although particles are traced in 6-dimensional phase space, we assume r-z geometry for the density distribution and hence for the space charge potential. Poisson's equation $\vec{\nabla}\vec{E} = \rho/\epsilon_0$ is solved with a fast Poisson solver on a rectangular mesh in r-z with boundary conditions on a conducting pipe of infinite conductivity and periodic boundary conditions in z. The charge density on the mesh points is determined using area weighting. Assuming a particle distribution as given by equation (1), the results obtained with this computer code are in perfect agreement with the ones obtained from the envelope equation (2).

As the previous section showed, for a realistic simulation of the conditions in the ESR we have to consider the beam loading effect in addition to space charge forces. This is done by fast Fourier transforming the beam current and calculating the self field using the measured impedance of the cavity. Practically important results are expected from comparing these simulations including beam loading with experimental results during bunching and fast bunch compression. This is planned in the near future.

6 REFERENCES

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